The Joint Determination of Haircuts and Interest Rates for Collateralized Loans in Shadow Banking

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This Version: December 30, 2023

Abstract

This paper investigates the joint determination of haircuts and interest rates for collateralized loans in shadow banking. The model uncovers that higher collateral quality does not necessarily reduce haircuts. Instead, higher downside quality reduces haircuts only under specific conditions, while higher upside quality always increases them. The model demonstrates that the relative insensitivity of interest rates compared to haircuts arises when depositors have logarithmic preferences, causing their marginal rate of substitution to change proportionally with the loan principal, or when shocks solely affect upside collateral quality. The model finally reveals that collateral scarcity resulting from increased depositor saving needs or reduced bank collateral supply is mitigated through lower haircuts but higher interest rates.

Keywords: Haircut, Interest Rate, Asset Price, Collateral Quality, Saving Needs, Collateral Supply
JEL Classification Code: E43, G12, G21

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1 INTRODUCTION

The global non-financial corporations and institutional investors have maintained substantial cash pools since before the 2007-2008 financial crisis (Pozsar, 2011). Similar to consumers, these cash holders also require effective liquidity management. However, the demand deposits at commercial banks do not serve their purpose due to limited deposit insurance. Additionally, the financial markets face a scarcity of secure and liquid assets, such as short-term government securities (Caballero et al., 2017). Consequently, short-term collateralized loans, including securitized instruments and repurchase agreements (repo), have emerged as the preferred alternative investments for these cash investors. These assets created by shadow banks are considered safer than their underlying collateral assets, but they are not entirely safe. Cash investors are exposed to the credit risk that the value of collateral may decrease over the transaction's duration and become insufficient for recovering the principal and interest if the counterparty defaults. Nonetheless, investors can opt for protection through a collateral haircut (overcollateralization) or seek compensation via a higher interest rate.[1]

However, the understanding of how these two alternative tools, haircuts and interest rates, are employed to address credit risk remains limited.[2] It is plausible that cash investors would require higher haircuts for collateral of lower quality. Indeed, during the 2007–2008 financial crisis, when the quality of private-sector collateral significantly deteriorated, haircuts experienced substantial increases (Gorton and Metrick, 2012; Krishnamurthy et al., 2014; Copeland et al., 2014). However, the evidence regarding the impact of collateral quality on haircuts is mixed (Gorton and Metrick, 2012; Auh and Landoni, 2016; Hu et al., 2019; Baklanova et al., 2019). Moreover, it is puzzling that in practice, interest rates adjust to a much lesser extent than haircuts in response to shocks affecting collateral fundamentals or market conditions (Geanakoplos, 2003; Krishnamurthy et al., 2014).

[1] The term "haircut" is commonly used in the repo market. For instance, if a repo investor lends $80 to a bank, which provides $100 collateral, a 20% haircut on the collateral is applied. The $20 difference represents the equity component of the transaction, while the $80 represents the debt component.

[2] Baklanova et al. (2019) commented, "There is little agreement in the theoretical academic literature on the main determinants of the level of haircut and interest rate in collateralized loans" (p. 244). Gorton and Metrick (2012) also noted, "It could seem natural that repo spreads and repo haircuts should be jointly determined. Unfortunately, the theory is not sufficiently developed to provide much guidance here" (p. 446).

Furthermore, the empirical literature has sought evidence for a presumed negative relationship between haircuts and interest rates—that is, investors would demand lower interest rates if they are protected by higher haircuts. However, Baklanova et al. (2019) find that such a trade-off is surprisingly absent in the data.

This paper shows that an improved collateral quality does not necessarily reduce haircuts, even when the downside and upside quality (i.e., the distribution of the future payoff of collateral below and above the loan principal) are appropriately measured separately due to their asymmetric effects. Specifically, improved downside quality reduces haircuts only when the downside quality is low or the upside quality is high, while improved upside quality always increases haircuts. Additionally, the study reveals two reasons why interest rates may respond less than haircuts to shocks. Firstly, interest rates do not adjust to shocks solely on the upside quality of collateral, whereas haircuts do. Secondly, interest rates remain unaffected by certain shocks when cash investors approximately have logarithmic preferences. Furthermore, this paper demonstrates that when investors possess excess cash or banks face limited collateral, resulting in relative scarcity of collateral, the tension is resolved through lower haircuts on collateral but higher interest rates.

These insights are derived from a model of imperfect risk sharing between depositors and banks through collateralized loans.[3] Depositors are endowed with consumption goods at the beginning; they are risk averse and want to save to smooth consumption. Banks are risk neutral, seeking maximal expected returns. In a frictionless economy, depositors would achieve perfect consumption smoothing, with banks issuing risk-free debt and depositors being fully insured. However, a friction exists in this economy—the only thing banks can pledge is the payoff of a risky asset; their future consumption good endowment is fully divertible. In the second-best allocation of this economy, depositors can achieve consumption smoothing only in states where banks' pledgeable income, i.e., the risky asset payoff, is high enough; otherwise, their future consumption is bound by banks' maximum pledgeable income. This allocation can be implemented as a financial market equilibrium,

[3] The setup is consistent with the shadow banking model of Gennaioli, Shleifer, and Vishny (2013). The model feature that banks issue collateralized debt to investors is also present in Krishnamurthy and Vissing-Jorgensen (2015) and Moreira and Savov (2017).

where banks issue risky debt to depositors.

Specifically, banks issue a bond with a face value that is endogenously determined against each unit of the risky asset as collateral. If the risky asset's payoff falls below the bond face value in the future, the risky asset is effectively transferred to depositors; otherwise, banks pay the face value. This transaction effectively tranches the risky asset into a debt component (the bond) that banks sell to depositors and an equity component that banks retain. The asset price consists of the valuation of depositors for the bond part and that of banks for the equity part. The haircut represents the value of the equity part as a proportion of the asset price, while the interest rate reflects the ratio of the bond face value to the bond price.

We can first observe the asymmetric effects of the downside and upside quality of collateral on haircuts. The downside quality affects depositors' valuation for the bond, while the upside quality affects banks' valuation for the equity. Importantly, an improvement in collateral quality does not necessarily reduce haircuts. Improved upside quality simply increases banks' valuation for the equity and thus raises haircuts. Although improved downside quality directly increases the bond price, which reduces haircuts, it also prompts banks to endogenously lower the bond face value, indirectly raising haircuts. This is due to the negative income effect of a lower interest rate on depositors, which reduces their demand for bonds, leading banks to respond by reducing the bond face value. Overall, whether an improved downside quality reduces haircuts is contingent on the downside and upside quality of collateral. These findings offer specific guidance for empirical tests.

This analysis also reveals the first scenario where interest rates are insensitive to shocks. When the upside quality of collateral changes, both the bond face value and price remain constant. As a result, interest rates do not change; only haircuts adjust accordingly. Nevertheless, the model demonstrates another, more general reason why interest rates may be insensitive to shocks. Whenever the bond face value increases upon shocks,[4] depositors' consumption in future states with a high asset payoff increases, so does their consumption in the current state due to consumption smoothing. Consequently, their marginal rate of substitution between the current state and future

[4] Similarly when the bond face value decreases upon shocks.

states with a low asset payoff increases. However, if depositors approximately have logarithmic preferences, this increase in their marginal rate of substitution, and thus the valuation of the bond payoff, is proportional to the increase in the bond face value, resulting in unchanged interest rates while haircuts still decrease.

Finally, this paper sheds light on the basics of how haircuts and interest rates respond to changes in the demand and supply of collateralized loans. When depositors are initially endowed with more consumption goods, their increased demand for collateralized bonds intensifies the scarcity of collateral. In response, banks raise the bond face value. Consequently, the haircut decreases due to a reduced equity component and an increased debt component of the asset price, while the interest rate increases due to higher default risk. On the other hand, when banks finance the purchase of more assets with collateralized loans, it alleviates the scarcity of collateral, resulting in opposite effects: haircuts increase and interest rates decrease.

These findings generate testable predictions, linking the cash holdings of money market funds (MMFs) and security lenders (SLs), the two main creditors to the shadow banking system, and the collateral asset holdings of dealer banks with the behavior of repo haircuts and interest rates. Moreover, these findings suggest that a spurious negative relationship between haircuts and interest rates can endogenously emerge due to omitted variable problems, but it would disappear if underlying driving variables such as depositors' saving needs and banks' collateral supply are controlled for in the empirical designs. Additionally, these predictions have policy implications. When the Federal Reserve conducts certain monetary policy operations that change the cash holdings of investors or the amount of collateral assets circulating in the financial markets, they also impact haircuts and interest rates in private repo markets.

1.1 Related Literature

I now explain the position of this paper in the literature by reviewing the existing studies that speak to the haircut and interest rate of collateralized loans. Note that most of these studies focus on different questions like the origin, stability or efficiency of collateralized securities, but they generate predictions on the haircut and interest rate as by-products. Moreover, most of these studies are from the perspective of cash investors providing liquidity to dealer banks (borrowers) which have liquidity shortfall or better investment opportunities, while this paper is from the perspective of banks providing liquidity to depositors (lenders) for consumption smoothing.

This paper is closely related to Simsek (2013), Gottardi et al (2019), and Biais et al. (2021).[5] These papers also adopt a general equilibrium approach and essentially assume the same incentive constraint as my paper. In Simsek (2013), both depositors and banks are risk neutral, but banks are more optimistic (or less pessimistic) than depositors regarding the payoff of the risky asset. Moreover, banks are cash constrained, while depositors are cash rich. The key difference arising from these features in Simsek (2013) is that the equilibrium bond face value is determined by banks' budget constraint in holding the equity component of the asset, whereas in my model, it is determined by depositors' budget constraint in holding the debt component. This difference implies that, unlike in my model, an improvement in downside quality of collateral does not affect the bond face value, thus consistently reducing haircuts. Conversely, an improvement in upside quality of collateral increases the bond face value, leading to opposite predictions for haircuts as well as different predictions for interest rates. This difference also indicates that the equilibrium remains unaffected by depositors' savings needs, in contrast to my model. Finally, it suggests that, unlike in my model, an increased collateral supply raises the bond face value, resulting in opposite predictions for both haircuts and interest rates. Overall, none of the three main results of this paper—the impact of collateral quality on haircuts, the relative insensitivity of interest rates, and the effects of collateral scarcity on haircuts and interest rates—can be obtained in Simsek's model.

Gottardi et al. (2019) differs from this paper in several aspects, leading to results that are not directly comparable,[6] In this paper, I focus on one feature of their model—specifically, the

[5]Fostel and Geanakoplos (2015) show that within a binomial economy, any collateral equilibrium involving traded risky debt contracts is equivalent (in terms of real allocations and prices) to an alternative equilibrium featuring solely a riskless debt contract. However, the unique equilibrium characterized in my paper with a continuum of states corresponds to, among the multiple equilibria in a binomial version of the economy, the one where there is default and the risk-tolerant agents hold all the risky assets. Therefore, the analysis of haircut and interest rate in these two papers are based on debt contracts traded within different equilibria: riskless debt contract versus risky debt contract.

[6]Distinct from this paper and the practice, Gottardi et al. (2019) define the haircut as the dollar amount of the down payment (i.e., the equity component of the asset price) rather than as a percentage of the collateral value and consider loan contracts with state-contingent promises instead of a fixed face value. Gottardi et al. (2019) also consider collateral re-use, non-pecuniary costs of default, and recourse loans.

quasilinear preference of depositors—and show that this single feature alone generates substantial differences in terms of model outcomes and predictions.[7] The fundamental distinction resulting from quasilinear preferences lies in the equilibrium bond face value, which is then determined by the condition that the future marginal utility of depositors in non-defaulting states equals 1. This condition implies that, unlike in my model, an improvement in downside quality of collateral consistently reduces haircuts because changes in bond prices have no income effect and do not impact depositors' future consumption and the bond face value. It also implies that, unlike in my model, depositors' saving needs do not influence the equilibrium. Moreover, quasilinear preferences rule out logarithmic preferences, based on which my model reveals the more general reason behind the relative insensitivity of interest rates. Although the model with quasilinear preferences still indicates that the upside quality of collateral has no impact on the bond face value and that an increase in banks' collateral supply reduces the bond face value, yielding the same predictions as my model for haircuts, interest rates, and asset prices, the underlying reasons for these predictions differ.

Biais et al. (2021) also study incentive constrained risk sharing, where liabilities are backed by collateral assets. A key assumption in their paper is that collateral assets are imperfectly pledgeable. It limits arbitrage and generates a basis between prices of an asset and its replicating portfolio of Arrow securities. It also leads to differential asset valuations among agents and thus markets segmentation. However, in this paper, the collateral asset is fully pledgeable. Consequently, the law of one price holds, and agents value the asset equally.

I next review the literature that emphasizes particular frictions of the repo market, such as the intermediation role of banks (Eren, 2015; Infante, 2019), the microstructure (Martin et al., 2014), the moral hazard (Kuong, 2021), the fire sale discounts (Dang et al., 2013; Kuong, 2021), the adverse selection in collateral use (Ozdenoren et al., 2023; Bigio and Shi, 2020), and the exogenous probabilities of default or liquidity shocks (Dang et al., 2013). Since this literature has mostly focused on the issue of fragility of the repo market, its predictions on the patterns of haircut and

[7] Phelan (2017) also assumes quasilinear preferences for lenders.

interest rate are not as rich as in this paper. For example, in Eren (2015) and Infante (2019) the haircut and interest rate between cash investors and dealer banks are zero. In Martin et al. (2014) the interest rate equals the risk-free rate and the collateral level is indeterminate. In Bigio and Shi (2020) the adverse selection in collateral use affects haircuts and interest rates, but it is only through the loan value with the collateral level and collateral asset price unaffected. Kuong (2021) relates the collateral level (rather than the haircut) and interest rate to a moral hazard friction and a fire sale discount. With a two-point payoff distribution for the collateral asset, Ozdenoren et al. (2023) relate the haircut and interest rate to its expected payoff in addition to the degree of adverse selection and the persistence of private information. In contrast, except for the limited commitment which motivates the use of collateral, this paper does not impose any other frictions. Instead, the analysis is purely based on the endowments and preferences in a general equilibrium, which allows us to identify a rich set of primitive determinants of haircuts and interest rates at the same time. In particular, none of these determinants has ever been identified in this literature.

The paper is organized as follows. The next section describes the model and and characterizes the equilibrium. Section 3 investigates the impacts of collateral quality. Section 4 analyzes the impacts of depositor's saving needs and banks' collateral supply. Section 5 discusses the empirical implications, and Section 6 concludes.

2 THE MODEL

Consider an economy with two dates, denoted by $0, 1$, and a single type of perishable consumption good. There is a continuum of possible states at date 1, denoted by $\omega \in \Omega \equiv [\underline{\omega}, \bar{\omega}] \subset (0, \infty)$. There is a mass K of a single type of risky asset. The risky asset pays ω units of the consumption good at date 1 if state ω is realized, so the state captures the uncertainty in the risky asset's payoff. The probability distribution of states at date 1 is commonly known, with a distribution function $F(\omega)$ and a density function $f(\omega)$ over Ω.

There is a continuum of depositors and a continuum of banks, each with a mass of one. Depositors are risk averse, with a date-0 expected utility given by

$$u(c_0^d) + \mathbb{E}\big[u(c_1^d(\omega))\big],$$

where c_0^d and c_1^d represent depositors' consumption at date 0 and date 1, respectively. The utility function u is strictly increasing and strictly concave. On the other hand, banks are risk neutral, with a date-0 expected utility given by

$$c_0^b + \mathbb{E}\big[c_1^b(\omega)\big],$$

where c_0^b and c_1^b represent banks' consumption at date 0 and date 1, respectively. For simplicity, I set discount factors to 1.

At date 0, depositors and banks are endowed with e_0^d and e_0^b units of the consumption good, respectively, and banks are endowed with all the risky assets. At date 1, depositors and banks are also endowed with e_1^d and e_1^b units of the consumption good, respectively, where e_1^d and e_1^b are constant. Importantly, $e_0^d > e_1^d$, indicating that depositors need to save for consumption smoothing, while banks can provide insurance for risk sharing. However, there is a friction in the economy.

I assume that banks face an incentive constraint in that they cannot commit to repaying debt. Nevertheless, banks can use their risky assets as collateral for debt, but not their consumption good endowment.[8] To model the risk-sharing between depositors and banks, I employ a competitive market for collateralized bonds.[9] Specifically, each collateralized bond is backed by one unit of the risky asset. The incentive constraint is thus modeled as a collateral constraint in that whenever banks issue one unit of bond, they must hold one unit of the risky asset as collateral. An appropriate notion of equilibrium would be the collateral equilibrium of Geanakoplos (1997, 2003). However, the complication with this notion of equilibrium is that ex-ante we need to consider a continuum of types of bonds with different face values.

[8]In the model, depositors will always give the collateral back when the loans are repaid. This aligns with the settlement practice of tri-party repos, where securities posted as collateral remain in the custody of the clearing bank (Baklanova et al., 2015, p. 8). Note that tri-party repos, which connect broker-dealers and nonbank cash investors, primarily serve as a source of external funding to the shadow banking system, aligning with the setting of this paper.

[9]In this paper, I confine the contract space to collateralized bonds when addressing the limited commitment problem. This choice is supported, for example, by Hébert (2018), who explores why debt-type contracts are prevalent in practice. In scenarios involving securitized lending, where the lender faces an agency problem due to the borrower's ability to privately modify the quality of underlying assets, Hébert shows that a debt-type contract is optimal as it balances the moral hazards of excessive risk-taking and lax effort from the borrower.

Instead, I will consider a notion of equilibrium that is very similar to the one in Biais et al. (2021). I first augment the collateralized bond market by allowing Arrow securities to be tradable at date 0 as in Biais et al. (2021).[10] Again, banks can only sell Arrow securities collateralized by the payoff of the risky assets they hold. After characterizing the equilibrium allocation, we will see that it can be implemented by trading only on the risky asset and a single type of bond collateralized by it. Hence the Arrow securities will actually be redundant, but their availability simplifies the analysis. I then can apply the notion of a classic time-zero Arrow-Debreu equilibrium,[11] in which agents only trade claims to consumption in all future states, suppressing any explicit reference to agents' positions in not only Arrow securities but also collateralized bonds, which are just portfolios of Arrow securities. Importantly, it also suppresses any explicit reference to agents' positions in the risky assets. This is in contrast to Biais et al. (2021), where the definition of equilibrium must be explicit about agents' positions in the risky assets. The key distinction is that in my model, the risky asset's payoff is fully pledgeable, whereas in Biais et al. (2021), it is imperfectly pledgeable. Consequently, selling a replicating portfolio of Arrow securities is equivalent to selling the risky asset directly in my model, whereas it is not in Biais et al. (2021).

With this formulation of the equilibrium, the collateral constraint is no longer specified for each individual bond as in Geanakoplos (1997, 2003). Instead, it is specified for each future state as in Biais et al. (2021): for each state ω at date 1, banks cannot issue more Arrow securities than the payoff in state ω of the risky assets they hold. In other words, in each state ω at date 1, banks cannot consume less than their consumption good endowment. This incentive or collateral constraint is formalized below.

Assumption 1. *Banks' date-1 consumption good endowment e_1^b is fully divertible, so the only thing they can pledge is the payoff of the risky assets:*

$$c_1^b(\omega) \geq e_1^b. \tag{1}$$

Note that Assumption 1 implies that recourse for loans is infeasible; there is nothing else to

[10]See also Rampini and Viswanathan (2010).

[11]See Definition 19.C.1 in Mas-Colell et al. (1995).

which depositors can turn for recourse other than the collateral assets.[12] To further simplify the analysis, when characterizing the competitive equilibrium, I will first find the equilibrium allocation, relying on a version of the welfare theorem, and then implement this allocation in a competitive equilibrium. Consequently, to save space, the formal definition of the equilibrium is provided in Section A in the appendix.

2.1 The First-Best Allocation

I first consider the first-best allocation in the absence of the friction introduced by Assumption 1, where banks' date-1 consumption good endowment e_1^b is fully pledgeable. Following Negishi (1960), I characterize the equilibrium allocation by solving a social planner's maximization problem with a weighted social welfare function. The coefficient for depositors' welfare is denoted by α, while the coefficient for banks' welfare is normalized to 1.[13] Namely, the social planner solves the following optimization problem:

$$\max_{\{c_0^d, c_0^b, c_1^d(\omega), c_1^b(\omega)\}} \alpha \mathbb{E}\big[u(c_0^d) + u(c_1^d(\omega))\big] + \mathbb{E}\big[c_0^b + c_1^b(\omega)\big], \tag{2}$$

subject to the resource constraints:

$$c_0^d + c_0^b \leq e_0^d + e_0^b, \tag{3}$$

and

$$c_1^d(\omega) + c_1^b(\omega) \leq e_1^d + e_1^b + K\omega, \tag{4}$$

which are binding. By substituting the constraints, the first-order conditions with respect to c_0^d and $c_1^d(\omega)$ yield:

$$u'(c_0^d) = u'(c_1^d(\omega)) = \frac{1}{\alpha}, \tag{5}$$

implying that depositors' date-1 consumption does not vary with the state of the world ω. This is intuitive since the risk-neutral banks fully insure the risk-averse depositors.

[12] In practice, repo contracts are subject to lender recourse. However, failures in repo markets are rarely taken to courts possibly because of the burdensome bankruptcy proceedings. In fact, Infante (2019) and Bigio and Shi (2020) explicitly assume nonrecourse for loans in their models. Thus, the infeasibility of recourse in the model of this paper may not preclude its application to the repo market. Nevertheless, see Footnote 16 for discussions on the relaxation of the constraint in (1).

[13] These coefficients of α and 1 are the inverses of the marginal utility of income of depositors and banks, respectively (Negishi, 1960).

The first-best allocation can be implemented with a risk-free bond market. At date 0, depositors exchange some of their date-0 consumption good for bonds issued by banks, and banks hold all the risky assets. At date 1, banks use the consumption good endowment e_1^b and the risky asset payoff $KD_1(\omega)$ to pay back the bonds to depositors. In this equilibrium, depositors achieve perfect consumption smoothing, bearing no risk.

2.2 The Second-Best Allocation

With the friction introduced by Assumption 1, the competitive equilibrium will not necessarily achieve the first-best allocation. Nevertheless, I show in Section A in the appendix that the equilibrium allocation can still be characterized using a generalized Negishi (1960) approach. Namely, for a particular choice of the coefficient α for the depositors' welfare in the social welfare function (2), the equilibrium allocation is a solution to the social planner's problem of maximizing the social welfare function (2) subject to the incentive or collateral constraint (1). This also implies that a welfare theorem still holds: the competitive equilibrium is constrained efficient, achieving the second-best allocation.[14]

With the social welfare function in (2) and substituting the binding resource constraints, the Lagrangian writes as follows:

$$\begin{aligned}\mathcal{L} =& \alpha\mathbb{E}\big[u(e_0^d + e_0^b - c_0^b) + u\big(e_1^d + e_1^b + K\omega - c_1^b(\omega)\big)\big] + \mathbb{E}\big[c_0^b + c_1^b(\omega)\big] \\ &+ \int_\omega \lambda(\omega)\big[c_1^b(\omega) - e_1^b\big]dF(\omega),\end{aligned} \tag{6}$$

where $\lambda(\omega)$ is the multiplier of the incentive constraint in state ω. The first-order condition with respect to c_0^d remains unchanged compared to the first-best case, i.e., $c_0^d = u'^{-1}\big(\frac{1}{\alpha}\big)$. The first-order condition with respect to $c_1^d(\omega)$ becomes:

$$u'(c_1^d(\omega)) = \frac{1}{\alpha} + \frac{\lambda(\omega)}{\alpha}. \tag{7}$$

In states ω where $\lambda(\omega) = 0$, i.e., states where the incentive constraint is slack, the second-best optimality condition (7) is the same as the first-best optimality condition (5), so that depositors'

[14]I thank the editor for suggesting that a welfare theorem holds in my model, which allows for characterizing the equilibrium by solving a social planner's problem. This insight greatly simplifies the exposition of the paper.

date-1 consumption is constant across these states and equal to:

$$c_1^d(\omega) = u'^{-1}\left(\frac{1}{\alpha}\right). \tag{8}$$

This conflicts with the incentive constraint when:

$$e_1^d + K\omega \leq u'^{-1}\left(\frac{1}{\alpha}\right). \tag{9}$$

There are three types of allocations depending on the scarcity of collateral:

(i) There is no scarcity of collateral: $e_1^d + K\underline{\omega} > u'^{-1}\left(\frac{1}{\alpha}\right)$. In this case, the collateral constraint does not bind in any state, and the second-best and first-best allocations coincide.

(ii) There is an extreme scarcity of collateral: $e_1^d + K\bar{\omega} < u'^{-1}\left(\frac{1}{\alpha}\right)$. In this case, the collateral constraint binds in all states, and depositors hold all the risky assets.

(iii) There is an intermediate scarcity of collateral: $e_1^d + K\underline{\omega} < u'^{-1}\left(\frac{1}{\alpha}\right) < e_1^d + K\bar{\omega}$. Let ω^* be the state for which (9) holds as an equality. The second-best consumption of depositors at date 1 is:

$$c_1^d(\omega) = e_1^d + K\omega \text{ for } \omega < \omega^* \text{ and } c_1^d(\omega) = c_0^d = u'^{-1}\left(\frac{1}{\alpha}\right) = e_1^d + K\omega^* \text{ for } \omega \geq \omega^*. \tag{10}$$

In this case, the risky asset is tranched into a debt component, $\min(\omega, \omega^*)$, held by depositors and an equity component, $\max(\omega - \omega^*, 0)$, held by banks.

In this paper, I focus on the more interesting case (iii) by making the following assumption.[15]

Assumption 2. *There is a scarcity of collateral in the economy in the sense that:*

$$2\underline{\omega} < \frac{e_0^d - e_1^d}{K}, \tag{11}$$

but the scarcity of collateral is not extreme in the sense that:

$$\bar{\omega} + \mathbb{E}\left[\frac{u'(e_1^d + K\omega)}{u'(e_1^d + K\bar{\omega})}\omega\right] > \frac{e_0^d - e_1^d}{K}. \tag{12}$$

2.3 The Financial Market Equilibrium

The second-best allocation can be implemented as a financial market equilibrium.

[15] If $\Omega = (0, \infty)$, we would always be in case (iii).

Theorem 1. *There exists a unique competitive equilibrium. The depositors' consumption plan is:*

$$c_0^d = e_1^d + K\omega^*, \quad c_1^d(\omega) = e_1^d + K\min(\omega, \omega^*), \tag{13}$$

and the banks' consumption plan is:

$$c_0^b = e_0^d + e_0^b - c_0^d, \quad c_1^b(\omega) = e_1^b + K\max(\omega - \omega^*, 0). \tag{14}$$

Banks hold all the risky assets and sell to depositors K units of a bond that has a face value of ω^* and is collateralized by one unit of the risky asset.

The Arrow security prices are given by depositors' marginal rate of substitution, $MRS^d(\omega)$, which equals banks' marginal rate of substitution, $MRS^b(\omega)$, for states $\omega \geq \omega^*$:

$$MRS^d(\omega) = \frac{u'(e_1^d + K\omega)}{u'(e_1^d + K\omega^*)} \text{ for } \omega < \omega^* \text{ and } MRS^d(\omega) = MRS^b(\omega) = 1 \text{ for } \omega \geq \omega^*. \tag{15}$$

The bond price is given by:

$$q = \mathbb{E}\big[MRS^d(\omega) \cdot \min(\omega, \omega^*)\big]. \tag{16}$$

The risky asset price is given by:

$$p = \underbrace{\mathbb{E}\big[MRS^d(\omega) \cdot \min(\omega, \omega^*)\big]}_{\text{debt component } q} + \underbrace{\mathbb{E}\big[MRS^b(\omega) \cdot \max(\omega - \omega^*, 0)\big]}_{\text{equity component } E}, \tag{17}$$

The equilibrium bond face value ω^* is uniquely determined by depositors' budget constraint at date 0:

$$\omega^* + q = \frac{e_0^d - e_1^d}{K}. \tag{18}$$

Proof. See the appendix. □

A few comments on the equilibrium are useful.[16] Firstly, the asset price is decomposed into the value of its debt component q and the value of its equity component E in (17). Secondly, the law of

[16] It is worth discussing two features of the model. First, one might wonder about the implications of the presence of a risk-free asset in the economy. Suppose that banks are endowed with K units of a risk-free asset with a payoff of ω^f at date 1, which is pledgeable. As long as the supply of this risk-free asset is low such that $\omega^f + \underline{\omega} < \frac{e_0^d - e_1^d}{2K}$, the same type of consumption good allocation focused on in the paper persists: depositors' date-1 consumption is $e_1^d + K\omega^f + K\min(\omega, \omega^*)$ for some $\omega^* \in (\underline{\omega}, \bar{\omega})$, and consumption smoothing is achieved for them only in states $\omega \geq \omega^*$ at date 1. However, this allocation can be implemented by multiple financial market equilibria. For instance, banks can issue K units of a risky bond with a face value of $\omega^f + \omega^*$, which is collateralized by a combination of risk-free and risky assets. The findings of this paper remain applicable with the collateral asset in the model interpreted as this asset bundle. Alternatively, banks can sell the risk-free assets to depositors and issue K units of a risky bond

one price holds, with the price of the asset and that of its replicating portfolio of Arrow securities being equal. Agents value the risky asset identically because banks are willing to pay a collateral premium for it. This stands in contrast to Biais et al. (2021), where a basis exists between the price of an asset and that of the replicating portfolio of Arrow securities, and agents potentially value assets differently. The key distinction arises from the fact that the risky asset is fully pledgeable in my model, whereas it is imperfectly pledgeable in their model.

Lemma 1. *We have* $\frac{\partial q}{\partial \omega^*} > 0$ *in (16).*

Lemma 1 and Assumption 2 together guarantee the existence and uniqueness of an equilibrium bond face value $\omega^* \in (\underline{\omega}, \bar{\omega})$ satisfying equation (18). This pins down the haircut and interest rate jointly. Define the quoted interest rate as

$$R \equiv \frac{\omega^*}{q} - 1, \tag{19}$$

which is subject to default risk. For example, if a borrower issues a bond with a face value of \$1.0 at a price of \$0.8, the interest rate is 25%.[17] Given the equilibrium prices, define the haircut as

$$H \equiv \frac{E}{p} = 1 - \frac{1}{1 + \frac{E}{q}}, \tag{20}$$

which, as a percentage of the collateral value, represents the equity component contributed by banks themselves.[18] In the example above, if the borrower puts up one share of a stock with a price of \$1.0 as collateral, the haircut is 20%. Note that the haircut H is positively related to the ratio of equity and debt components of the asset price, $\frac{E}{q}$, which will be more convenient to work with in the comparative statics.

with a face value of ω^* that is backed by the risky asset. Another implementation involves banks issuing a risky bond with a face value of $\omega^f + \omega^*$ that is collateralized by only the risky asset but permits depositors to fall back on banks' holdings of the risk-free asset for recourse. If the supply of the risk-free asset is high, the collateralized bond market would be eliminated, and the first-best allocation would be achieved.

Secondly, one might wonder about the implications of having recourse for collateralized bonds in the model. This consideration necessitates an initial assumption that banks' date-1 consumption good endowment is pledgeable; otherwise, recourse is infeasible. Suppose that banks have a pledgeable consumption good endowment of $K\omega^f$ at date 1. Under this assumption, the implications are the same as what was discussed above when banks are endowed with K units of the aforementioned risk-free asset.

[17]This definition of the interest rate is consistent with the practice in the repo market as well as in Fostel and Geanakoplos (2015).

[18]This definition of the haircut is the same as the definitions of margin in Geanakoplos (2003) (p. 179), Simsek (2013) (Equation (17)), and Fostel and Geanakoplos (2015).

Lemma 2. *We have* $\frac{\partial E}{\partial \omega^*} < 0$ *and* $\frac{\partial p}{\partial \omega^*} > 0$ *in (17), which implies* $\frac{\partial H}{\partial \omega^*} < 0$.

As the bond face value ω^* increases, indicating higher default risk, the bond price q also increases by Lemma 1. It is unclear whether the bond interest rate also increases. The assumption below on depositors' utility function helps to generate an unambiguous prediction.

Assumption 3. *Depositors' utility function* $u(\cdot)$ *satisfies*

$$\frac{-u''(e_1^d + c)c}{u'(e_1^d + c)} < 1 \tag{21}$$

for $c \in [K\underline{\omega}, K\bar{\omega}]$.[19]

First, note that if depositors are not endowed with any consumption good at date 1, i.e., $e_1^d = 0$, (21) is reduced to $\frac{-u''(c)c}{u'(c)} < 1$, which means that the relative risk aversion coefficient is less than 1. However, note that $\frac{-u''(e_1^d+c)(e_1^d+c)}{u'(e_1^d+c)} < 1$ for $e_1^d > 0$ implies (21), so (21) is a weaker assumption than the relative risk aversion coefficient being less than 1. In particular, for a CRRA utility $u(c) = \frac{c^{1-\eta}}{1-\eta}$, (21) means $\eta < 1 + \frac{e_1^d}{c}$. Given that the most commonly accepted estimates of risk aversion are around one, Assumption 3 is mild. This Assumption then implies that the bond interest rate is increasing with the bond default risk.

Lemma 3. *Under Assumption 3, we have* $\frac{\partial R}{\partial \omega^*} > 0$ *in (19).*

2.4 Two Benchmark Models

To clarify the link to the literature and highlight the contributions, I will contrast my model with those of two closely related papers in the literature: Gottardi et al. (2019) and Simsek (2013). Both of these papers adopt the collateral equilibrium approach and essentially assume the same limited commitment of borrowers as my paper. However, it is worth noting that I work with a simplified version of Gottardi et al. (2019) and explore a more extensive range of comparative statics compared to their original model. Furthermore, my comparative statics are completely different from those in Simsek (2013). Notably, neither of these papers studies the interest rate of

[19] What is really needed is that $u'(e_1^d+c)c$ is increasing in c. The other places where this assumption is also employed are the comparative statics with respect to the downside quality of collateral and banks' risky asset endowment.

collateralized loans. Therefore I will independently conduct all comparative statics for my paper within these two benchmark models.

2.4.1 Quasilinear Preferences for Depositors

The model in Gottardi et al. (2019) is distinct from my model in a number of ways, resulting in non-directly comparable results and predictions (see Footnote 6). Here, I consider a version of their model that differs from my model in only one way: depositors have quasilinear preferences with their date-0 expected utility given by

$$c_0^d + \mathbb{E}\big[u(c_1^d(\omega))\big].$$

To make the analysis comparable, I consider the same type of equilibrium allocation in this benchmark model; that is, banks hold all the risky assets and issue collateralized bonds.[20]

The key difference caused by quasilinear preferences is that the equilibrium bond face value ω^* is determined by

$$u'(e_1^d + K\omega^*) = 1, \tag{22}$$

which, in contrast to (18) in this paper, says that the depositors' marginal utility in states $\omega > \omega^*$ at date 1 equals their marginal utility of 1 at date 0. It implies that collateral quality does not affect the equilibrium bond face value ω^*, In particular, it implies that, unlike in my model, there is no income effect of bond price changes on depositors—bond price changes do not affect depositors' date-1 consumption and thus the equilibrium bond face value ω^*. Depositors consume what is left over after the purchase of bonds at date 0, $c_0^d = e_0^d - Kq$. Equation (22) also implies that, unlike in my model, depositors' initial consumption good endowment does not affect the equilibrium bond face value ω^*. Although Equation (22) does imply that the supply of assets affects the equilibrium bond face value ω^* as in my model, it is for a different reason. The other difference caused by quasilinear preferences is that depositors' marginal rate of substitution is now given by

$$MRS^d(\omega) = u'(e_1^d + K\omega) \text{ for } \omega < \omega^* \text{ and } MRS^d(\omega) = MRS^b(\omega) = 1 \text{ for } \omega \geq \omega^*, \tag{23}$$

[20] This requires assuming that there exists a solution $\omega^* \in [\underline{\omega}, \bar{\omega}]$ to (22).

which is in contrast to (15) in this paper. The bond and the risky asset are still priced according to (16) and (17), respectively.

2.4.2 Heterogeneous Beliefs

Compared with my model, there are two fundamental differences in Simsek (2013) that result in distinct model predictions. First, in Simsek's model, both depositors and banks are risk-neutral but have heterogeneous beliefs. They maximize their date-0 expected utility

$$c_0^i + \mathbb{E}_i\left[c_1^i(\omega)\right]$$

for $i = d, b$, respectively.[21] Banks are optimistic, while depositors are pessimistic. Their prior beliefs about the state at date 1 are given by the probability distribution F_i over Ω, respectively, which satisfy the hazard-rate order. This feature makes banks the natural buyers of the risky assets.

Second, in Simsek's model, at date 0, banks are cash constrained (e_0^b is small), while depositors are cash rich (e_0^d is large). These two features of the model motivate banks to buy the risky assets by issuing collateralized loans to depositors. The bond is priced by depositors with

$$q = \mathbb{E}_d[\min(\omega, \omega^*)]. \tag{24}$$

Note that banks' perceived cost of debt financing is

$$r_b^{per}(\omega^*) \equiv \frac{\mathbb{E}_b[\min(\omega, \omega^*)]}{\mathbb{E}_d[\min(\omega, \omega^*)]} - 1, \tag{25}$$

which is greater than 0 due to the disagreement about the probability of default. Therefore, if banks are endowed with all the risky assets as in my model, there would be no trade as banks would not be cash constrained. In Simsek's model, all the risky assets are instead initially endowed to unmodeled agents who sell their assets at date 0. Nevertheless, in my model, modeling the supply of assets in the same way as Simsek (2013) does not change the equilibrium, except for banks' date-0 consumption, which is immaterial. With this interpretation, the two models become comparable.

[21] In the original model of Simsek (2013), agents only consume at date 1 with the consumption good (a dollar) being storable. But we can equivalently assume that agents consume at both dates and consider the consumption good as perishable in his model due to the risk-neutral nature of the agents and the absence of discounting.

There are two forces that pin down the equilibrium bond face value ω^* and the risky asset price p. First, banks maximize their expected leveraged return on equity

$$R_b^L(\omega^*) \equiv \frac{\mathbb{E}_b[\omega] - \mathbb{E}_b[\min(\omega, \omega^*)]}{p - \mathbb{E}_d[\min(\omega, \omega^*)]}. \tag{26}$$

The optimality implies

$$p = \underbrace{\mathbb{E}_d[\min(\omega, \omega^*)]}_{\text{debt component } q} + \underbrace{\frac{1 - F_d(\omega^*)}{1 - F_b(\omega^*)}\mathbb{E}_b[\max(\omega - \omega^*, 0)]}_{\text{equity component } E}, \tag{27}$$

which is different from the decomposition in my model in (17) in two aspects.[22] Depositors' $MRS^d(\omega)$ is endogenous in my paper, while it is constant in Simsek's model due to universal risk-neutrality. Moreover, belief heterogeneity in Simsek's model leads to the extra term $\frac{1-F_d(\omega^*)}{1-F_b(\omega^*)}$ in the decomposition (27). As a result, the asset price p is decreasing in the bond face value ω^* in (27) but is increasing in ω^* in (17). Second, the asset market clears (banks' date-0 budget constraint) with

$$p = \underbrace{\mathbb{E}_d[\min(\omega, \omega^*)]}_{\text{debt component } q} + \underbrace{\frac{e_0^b}{K}}_{\text{equity component } E}. \tag{28}$$

Substituting (27) into (28), we have the bond face value ω^* determined by

$$\frac{1 - F_d(\omega^*)}{1 - F_b(\omega^*)}\mathbb{E}_b[\max(\omega - \omega^*, 0)] = \frac{e_0^b}{K}, \tag{29}$$

which is banks' budget constraint in holding the equity component of the risky asset. In contrast, the condition (18) of my model is depositors' budget constraint in holding the debt component of the risky asset. This implies that banks' date-0 consumption good endowment e_0^b matters in Simsek's model, while depositors' date-0 consumption good endowment e_0^d matters in my model. Note that the equity component on the left-hand side of (29) is decreasing in ω^* given the assumed hazard-rate order, while the debt component on the left-hand side of (18) is increasing in ω^*. This implies that the collateral supply K has the opposite effects on the bond face value ω^* in these two models. Finally, (29) can also be interpreted as the equality of the two equity components in

[22] Equation (27) is a rearrangement of equation (11) in Simsek (2013). Equation (28) below is equation (16) in Simsek (2013) after the supply of the risky asset is generalized from one unit to K units.

(27) and (28). It implies that, unlike in my model, the value of the equity component E of the risky asset in equilibrium, $\frac{e_0^b}{K}$, is not affected by agents' beliefs about collateral quality in Simsek's model.

The discount factor (equivalently the interest rate) and the ratio of equity and debt components of the asset price (equivalently the haircut) are given by

$$\frac{1}{1+R} = \frac{q}{\omega^*} = \mathbb{E}_d\left[\min\left(\frac{\omega}{\omega^*}, 1\right)\right] \quad \text{and} \quad \frac{1}{1-H} - 1 = \frac{E}{q} = \frac{e_0^b/K}{\mathbb{E}_d[\min(\omega, \omega^*)]}, \tag{30}$$

respectively.

3 COLLATERAL QUALITY

The collateral quality in my model is described by the distribution of the risky asset payoff at date 1. I define the upside and downside quality of collateral as the distribution of the asset payoff over the intervals $(\omega^*, \bar{\omega})$ and $(\underline{\omega}, \omega^*)$, respectively.[23] There are two direct impacts of collateral quality: the impact of downside quality on the debt component of the risky asset held by depositors and the impact of upside quality on the equity component of the risky asset held by banks. Moreover, the first impact has important indirect implications through an income effect of the associated changes in the interest rate on depositors. Since the upside and downside quality of collateral have asymmetric impacts, I study them separately. I first examine the impacts of an improvement in the upside quality of collateral in the sense of first-order stochastic dominance (FOSD). Suppose the bond face value is ω^* before the shock.

Proposition 1. *If there is an FOSD improvement in the upside quality of collateral over $(\omega^*, \bar{\omega})$, while keeping the downside quality unchanged over $(\underline{\omega}, \omega^*)$, the bond face value ω^* and the interest rate R remain the same; however, the asset price p and the haircut H increase.*

If the downside quality of collateral over $(\underline{\omega}, \omega^*)$ does not change, an FOSD improvement in the upside quality over $(\omega^*, \bar{\omega})$ does not change the distribution of depositors' consumption in bad

[23]Note that the threshold, the equilibrium bond face value ω^*, is endogenous. To be precise, in what follows, a shock on the downside quality refers to a shift of the asset payoff distribution over a subset $(\underline{\omega}, \omega')$ of $(\underline{\omega}, \omega^*]$; similarly for a shock on the upside quality.

states $\omega \in (\underline{\omega}, \omega^*)$ at date 1, where the collateral constraint is binding. This improvement over the distribution of the asset payoff in good states $\omega \in (\omega^*, \bar{\omega})$ at date 1 is not valued by depositors since they have already achieved consumption smoothing in these states. Therefore this improvement in the upside quality of collateral does not matter for depositors. Specifically, depositors' date-0 budget constraint (18), which characterizes the equilibrium bond face value ω^*, remains unaffected. Consequently, the bond price q and the interest rate R do not change.

However, an FOSD improvement in the upside quality matters for banks. This improvement increases the value of the equity component of the risky asset held by banks. Therefore it also increases the risky asset price p given that the value of the debt component of the risky asset is unaffected. As a result, a higher haircut H follows.

Remark 1. *In the benchmark model with quasilinear preferences for depositors, the upside quality of collateral does not affect the bond face value ω^*, as in my model. But the reasons behind this phenomenon differ, as evident from (22) and (18). This means that an improvement in the upside quality of collateral in this benchmark model has the same effects as in my model.*

Remark 2. *Simsek (2013) conducts the comparative statics of how disagreements between depositors and banks regarding the upside or downside quality of collateral impact asset prices and haircuts. This involves simultaneously rendering banks more optimistic and depositors more pessimistic. However, I improve the objective quality of collateral in the comparative statics analysis of my model. To make the predictions more comparable, I also conduct comparative statics analysis within his model, where I improve the upside or downside quality of collateral under the subjective beliefs of both depositors and banks.*

In Simsek's model, when banks become more optimistic about the upside quality of collateral over $(\omega^*, \bar{\omega})$, the bond face value ω^* increases; the asset price p increases due to a higher debt component of the asset price (the equity component does not change), resulting in a lower haircut H; the interest rate R increases due to a higher default risk. However, depositors' belief about the upside quality of collateral over $(\omega^*, \bar{\omega})$ has no effect on the equilibrium.

In contrast, in my model, when the upside quality of collateral improves, the bond face value ω^*

and the interest rate R do not change; the asset price p increases due to a higher equity component of the asset price (the debt component does not change), resulting in a higher haircut H.

I next consider the impacts of an improvement in the downside quality of collateral.[24]

Proposition 2. *If there is an FOSD improvement in the downside quality of collateral over $(\underline{\omega}, \omega^*)$, while keeping the upside quality unchanged over $(\omega^*, \bar{\omega})$, the bond face value ω^* and the interest rate R decrease; the asset price p increases, and the haircut H decreases if and only if the upside quality of collateral is high or the downside quality of collateral is low (which implies a high ω^*) such that*

$$\mathbb{E}[\omega|\omega > \omega^*] > \frac{e_0^d - e_1^d}{K}. \tag{31}$$

By Assumption 3, the product of depositors' marginal utility and bond payoff at date 1, $u'(e_1^d + K\omega)\omega$, is a strictly increasing function of the asset payoff ω over the lower tail $(\underline{\omega}, \omega^*)$. An FOSD improvement in the quality of collateral in this region increases the bond price q in (16). A decrease in the interest rate R has a negative income effect on depositors, reducing their demand for bonds. Banks respond by reducing the bond face value ω^*, which reduces the default risk of bonds and decreases the interest rate R further, as per Lemma 3.

For the asset price, the lower bond face value ω^* reduces the asset price by Lemma 2: it increases the equity component E of the asset price but reduces the debt component q of the asset price even more. However, this negative effect on the debt component is dominated by the positive effect on the debt component due to the improvement in the downside quality of collateral. To see this, note that a lower bond face value ω^* implies a higher bond price q in the equilibrium condition (18). Overall, the asset price p increases due to both higher debt and equity components in (17).

However, both higher debt and equity components of the asset price make the impact on the haircut less straightforward. Although the improvement of the downside quality of collateral directly reduces the haircut by raising the bond price q in (16), a lower bond face value ω^* tends

[24] In Section B in the appendix, I show that an increase in depositors' risk aversion has the same impacts on the variables of interest as an improvement in the downside quality of collateral. The intuition is that depositors would value the bonds more when they are more risk averse.

to indirectly increase the haircut by Lemma 2 for a fixed quality of collateral. It turns out that the direct effect dominates if and only if condition (31) is satisfied.

To obtain intuition for condition (31), I decompose the comparative statics using the chain rule. Instead of taking derivatives with respect to a distribution, I consider an improvement in the downside quality of collateral that results in an infinitesimal increase in the bond price, $dq(\omega^*)$, where the ω^* in parentheses indicates that the bond face value ω^* is fixed, and this change in the bond price is purely due to the change in the downside quality of collateral. We first obtain the sensitivity of ω^* in equilibrium to the change in the downside quality of collateral as represented by $dq(\omega^*)$. Taking derivatives of both sides of the equilibrium condition (18) with respect to the change in the downside quality of collateral, we have:

$$\frac{d\omega^*}{dq(\omega^*)} = -\left(1 + \frac{\partial q}{\partial \omega^*}\right)^{-1} < 0. \tag{32}$$

Now we can decompose the comparative statics of the haircut in (20), which is positively related to the equity-debt ratio $\frac{E}{q}$, with respect to the change in the downside quality of collateral:

$$\begin{aligned}
\frac{d(E/q)}{dq(\omega^*)} &= \underbrace{\frac{\partial(E/q)}{\partial q(\omega^*)}}_{\text{direct effect}(-)} + \underbrace{\frac{\partial(E/q)}{\partial \omega^*}\frac{d\omega^*}{dq(\omega^*)}}_{\text{indirect effect}(+)} \\
&= -\frac{E}{q^2} + \frac{\frac{\partial E}{\partial \omega^*}q - E\frac{\partial q}{\partial \omega^*}}{q^2}\frac{d\omega^*}{dq(\omega^*)} \\
&\propto -\left(E + \frac{\partial E}{\partial \omega^*}q\right) \\
&= -\left(\int_{\omega^*}^{\bar{\omega}} \omega dF(\omega) - (1 - F(\omega^*))\frac{e_0^d - e_1^d}{K}\right),
\end{aligned} \tag{33}$$

which is negative if and only if condition (31) is satisfied. In (33), I applied the chain rule in the first line, plugged in $\frac{d\omega^*}{dq(\omega^*)}$ from (32) in the third line, and substituted the bond price q in the equilibrium condition (18) in the last line. We can see that when the upside quality is high, the equity component E is big, and thus the direct negative effect of improved downside quality, $-\frac{E}{q^2}$, is relatively larger; this is captured by the term $\int_{\omega^*}^{\bar{\omega}} \omega dF(\omega)$ in the last line. We can also see that when the bond face value ω^* is high, which is the case when the downside quality is low by the first part of Proposition 2, the indirect positive effect of reduced bond face value is relatively smaller because the sensitivity of the equity component of the asset price to the bond face value, $\frac{\partial E}{\partial \omega^*}$, is lower; this is captured by the term $1 - F(\omega^*)$ in the last line.[25]

Remark 3. *In the benchmark model with quasilinear preferences for depositors, the downside quality of collateral does not affect the bond face value ω^*, which is determined by (22). This means that when the downside quality of collateral is improved, the bond face value channel in my model is absent in this benchmark model, which would always predict a decrease in the haircut H. The asset price p increases due to a higher debt component (the equity component does not change), and the interest rate falls accordingly.*

Remark 4. *In Simsek's model, banks' beliefs about the downside quality of collateral over $(\underline{\omega}, \omega^*)$ do not impact the equilibrium. However, when depositors become less pessimistic about the downside quality of collateral over $(\underline{\omega}, \omega^*)$, it raises the bond price, despite no effect on the bond face value ω^*. Consequently, it has the same effects as an improvement in the downside quality of collateral in the benchmark model with quasilinear preferences for depositors.*

In contrast, as the downside quality of collateral improves in my model, the bond face value ω^* decreases, both reducing the interest rate R; the asset price p increases due to higher both debt and equity components of the asset price; the haircut H decreases if and only if the upside quality is high or the downside quality is low.

4 SAVING NEEDS AND COLLATERAL SUPPLY

In this section, I investigate how haircuts and interest rates adjust in response to shocks on depositors' saving needs or banks' collateral supply. In reality, depositors' saving needs in shadow banking may change for various reasons. In my model, an increase in depositors' consumption good endowment at date 0 raises their saving needs due to a greater consumption smoothing motive. Consequently, there is a higher demand for collateralized bonds. Given the limited collateral supply, banks respond by increasing the bond face value, ω^*, per unit of collateral asset. As per Lemma 3,

[25]Note that a higher bond face value ω^* also implies a lower equity component E of the asset price and thus a smaller direct effect of improved downside quality. However, this is of second order as $\frac{e_0^d - e_1^d}{K} > \omega^*$ by (18).

the interest rate R increases due to higher default risk. Furthermore, Lemma 2 indicates that the haircut decreases as the asset price p increases while its equity component E shrinks.

Overall, depositors' consumption at date 0 and in good states ($\omega > \omega^*$) at date 1 increases, while their consumption in bad states ($\omega < \omega^*$) at date 1 remains unchanged. Depositors achieve consumption smoothing in a smaller set of good states at date 1 and value the consumption in bad states even more.

Proposition 3. *When depositors' date-0 consumption good endowment e_0^d increases, the bond face value ω^* rises; the asset price p increases; the haircut H decreases, and the interest rate R increases.*

As a scarce resource, the market supply of collateral assets can change over time in the real world. In my model, banks always pursue an expected return as high as possible. With more assets in hand, they want to leverage up, creating an excess supply of collateralized bonds relative to the depositors' saving needs at the equilibrium prices. This tension is resolved with banks reducing the bond face value, ω^*, per unit of collateral asset. According to Lemma 3, the interest rate R decreases due to lower default risk. Additionally, Lemma 2 suggests that the haircut increases as the asset price p decreases while its equity component E rises.[26]

Overall, depositors' consumption at date 0 and in good states ($\omega > \omega^*$) at date 1 decreases, while their consumption in bad states ($\omega < \omega^*$) at date 1 rises.[27] Depositors achieve consumption smoothing in a larger set of good states at date 1 and value the consumption in bad states less.

Proposition 4. *When banks' time-0 endowment of the risky asset K increases, the bond face value ω^* falls; the risky asset price p decreases; the haircut H increases, and the interest rate R decreases.*

Note that these predictions are exactly in the opposite directions of those in Proposition 3 when depositors' saving needs increase. This is not surprising since the scarcity of collateral is about the market supply of collateral relative to the depositors' saving needs. As can be seen from

[26] The asset supply K also directly affects depositors' consumption, but this effect is of second order compared to the effect of the change in the bond face value ω^*.

[27] To see this, note that $K\omega^* + Kq = e_0^d - e_1^d$ is a constant by equation (18). A lower interest rate implies that ω^* decreases relative to q.

the right-hand side of equation (18), what matters for the equilibrium bond face value ω^* is the ratio $\frac{e_0^d - e_1^d}{K}$.

Remark 5. *Unlike in my model, depositors' date-0 consumption good endowment e_0^d has no effect on the equilibrium except for depositors' consumption at date 0 in the benchmark model with quasi-linear preferences for depositors. The reason is that the bond face value ω^* is determined by (22). Nevertheless, an increased supply of the risky asset K has the same effects on variables of interest in this benchmark model as in my model. The benchmark model predicts a decrease in both the bond face value ω^* and the asset price p, but through a different mechanism. First, the bond face value ω^* decreases because $K\omega^*$ must be a constant as implied by (22). Second, in this benchmark model, the asset price p decreases because a larger supply of the risky asset K reduces depositors' marginal utility at date 1 in states $\omega < \omega^*$, while in my model, it is because a lower ω^* increases depositors' marginal utility at date 0. This benchmark model also predicts an increase in the haircut and a decrease in the interest rate.*[28]

Remark 6. *As the supply of the risky asset K increases, Simsek's (2013) model generates opposite predictions for the variables ω^*, H, and R compared to my model. In his model, to finance the purchase of additional risky assets, cash-constrained banks must borrow more by raising the bond face value ω^*. The interest rate R increases due to higher default risk, and the haircut H decreases as a result of both reduced equity contributions by banks and larger loans. Nevertheless, Simsek's (2013) model generates the same prediction for the asset price p as my model. This is because the tightened date-0 budget constraint of banks depresses the asset price. Technically, this occurs because, unlike in my model, a higher bond face value ω^* reduces the risky asset price in his model.*

For the effects of agents' consumption good endowments at date 0, although depositors' consumption good endowment e_0^d matters in my model, it has no effect on the equilibrium in Simsek's model. However, although a larger banks' consumption good endowment e_0^b has no effect on the equilibrium in my model, it relaxes banks' date-0 budget constraint in Simsek's model, having exactly

[28] A lower ω^* increases the equity component E of the asset price, so a higher haircut H follows. Although a higher K reduces $\frac{q}{\omega^*}$, this effect is dominated (by Assumption 3) by a lower ω^*, which increases $\frac{q}{\omega^*}$. A lower interest rate R follows.

the opposite effects of an increased supply of the risky asset K in his model.

5 EMPIRICAL IMPLICATIONS

In this section, I discuss the empirical implications of the model and provide guidance regarding the implementation of the predictions. First, I demonstrate how these implications help explain the three main empirical observations stated at the beginning of this paper. Then, I suggest empirical contexts in which the novel predictions can be tested.

5.1 Collateral Quality and Haircuts

The impact of collateral quality on haircuts is of great interest, and it is commonly expected that higher collateral quality is associated with lower haircuts. However, the findings from the data are mixed. Baklanova et al. (2019) find a robust negative relationship between "extreme price falls" of collateral and repo haircuts across different asset classes. However, Hu et al. (2019) find that haircuts in the high-risk segment of tri-party repos, where collateral is more concentrated, are not sensitive to collateral concentration.[29] They also find that stock volatility is not a significant determinant of repo haircuts. Both Gorton and Metrick (2012) and Auh and Landoni (2016) find that volatility is significant in explaining repo haircuts for some collateral classes but not for others.

However, Propositions 1 and 2 of this paper demonstrate that higher collateral quality does not necessarily result in reduced haircuts. First of all, these results imply that the upside and downside quality of collateral should be measured separately since they have asymmetric effects on haircuts.[30] As a result, the model favors measures such as tail risk (Baklanova et al., 2019) or Value at Risk (VaR) (Julliard et al., 2019) instead of volatility (Gorton and Metrick, 2012; Auh and Landoni, 2016; Hu et al., 2019) or concentration of collateral portfolios (Hu et al., 2019). In particular, it is inappropriate to measure the quality of collateral based on its expected payoff when studying its effects on haircuts.

[29]They do find that collateral concentration affects haircut decisions across the low-risk segment, where collateral is well diversified, and the high-risk segment, as well as within the low-risk segment.

[30]Fostel and Geanakoplos (2015) show that in binomial economies, the leverage of riskless debt contracts in no-default equilibria is determined by down risk rather than volatility. In their study, down risk refers to the worst-case equilibrium return of the collateral asset, while the upside and downside quality discussed in this paper pertain to the distribution of asset payoffs in good and bad states, respectively.

Secondly, even when collateral quality is appropriately measured, Proposition 1 implies that higher upside quality always increases haircuts, while Proposition 2 indicates that the impact of downside quality depends on both downside and upside quality. For the empirical implementation of these predictions, the model suggests the following specification:

$$H = \beta_0 + \beta_1 Q_u + (\beta_2 + \beta_{2u} Q_u + \beta_{2d} Q_d) Q_d + \epsilon, \tag{34}$$

where Q_u and Q_d represent the upside and downside quality of collateral, respectively, measured as tail risk or VaR. The impact of downside quality is specified by $\beta_2 + \beta_{2u} Q_u + \beta_{2d} Q_d$ to capture the dependence of this impact on both upside and downside quality. Proposition 1 predicts that the coefficient on upside quality is positive, $\beta_1 > 0$. Proposition 2 predicts that $\beta_{2u} < 0$ and $\beta_{2d} > 0$, which results in a negative coefficient on downside quality, $\beta_2 + \beta_{2u} Q_u + \beta_{2d} Q_d < 0$, when the upside quality is high or the downside quality is low.

Nevertheless, Proposition 2 does imply that when an asset exhibits heavy tails on both sides of the distribution of its payoff, its haircut is more likely to increase when the left tail of the distribution becomes even heavier. This aligns with the significant increase in haircuts (and interest rates) observed for private-sector collateral, including stocks, corporate bonds, and private-label asset-backed securities (ABS), in the repo market during the 2007-2008 financial crisis (Gorton and Metrick, 2012; Krishnamurthy et al., 2014; Copeland et al., 2014). The significantly worsened downside quality of these collateral assets makes condition (31) in the model more likely to be satisfied, thereby increasing the haircut (and interest rate).

5.2 Insensitivity of Interest Rates

The model also provides insights into the empirical observation that interest rates tend to adjust less than haircuts to shocks. Krishnamurthy et al. (2014) note that "The lender can protect against collateral risk by raising the haircut on the repo contract... The lender can also raise the repo rate to compensate for all... risks, although in practice this appears to be a less significant margin." At the macro level, Geanakoplos (2003) points out that during the liquidity crises in the fixed-income markets in 1994 and 1998, the interest rates charged remained virtually the same, despite the increase in margin requirements on borrowing. Krishnamurthy et al. (2014) find that despite the turmoil during the 2007-2008 financial crisis, repo rates reverted to near pre-crisis levels as financial markets normalized in 2009 and 2010, while haircuts on certain asset classes continued to increase. At the transaction level, Hu et al. (2019) do not find evidence that collateral concentration affects interest rate decisions within segments of the tri-party repos between MMFs (lenders) and dealer banks (borrowers). Auh and Landoni (2016) find that some of their measures of collateral quality affect bilateral repo haircuts, but none of them affect interest rates.

The model in this paper presents two reasons why interest rates are less sensitive to shocks than haircuts.[31] First, Proposition 1 demonstrates that when shocks solely affect the upside quality of collateral, haircuts adjust while interest rates do not.

Second, when depositors approximately have logarithmic preferences, $u(c) = \log(c - e_1^d)$, interest rates would not change even as bond face values and haircuts endogenously adjust due to shocks, such as changes in depositors' saving needs or banks' collateral supply. Note that Assumption 3 was made to ensure that the bond interest rate R increases with credit risk as captured by the bond face value ω^*. However, if condition (21) in Assumption 3 holds as equality, indicating log utility, the interest rate would not change as the bond face value ω^* adjusts endogenously. In this case, the bond price q and face value ω^* would change proportionally. For example, an empirical case might arise where both the bond price and face value decrease by 3% due to an increase in collateral supply, resulting in an unchanged interest rate, while the collateral asset price only decreases by 1%, leading to an approximate 2% increase in the haircut.

The intuition behind this implication can be derived from the explicit expression for the ratio of bond price and face value:

$$\frac{q}{\omega^*} = \int_{\underline{\omega}}^{\omega^*} \frac{u'(e_1^d + K\omega)\omega}{u'(e_1^d + K\omega^*)\omega^*} dF(\omega) + \frac{\omega^*(1 - F(\omega^*))}{\omega^*}. \tag{35}$$

First, note that the bond value derived from payoffs in non-defaulting states $\omega > \omega^*$ changes proportionally with the face value (the second term on the right-hand side). Additionally, depositors'

[31] Ozdenoren et al. (2023) provide an alternative explanation that the severity of adverse selection in the collateral posted is mostly reflected in haircuts and less so via interest rates.

marginal rate of substitution in defaulting states $\omega < \omega^*$ increases with the bond face value ω^*—the higher the bond face value, the lower the depositors' marginal utility at date 0. When depositors have log utility, $u'(e_1^d + K\omega^*)\omega^*$ in the denominator of the first term on the right-hand side becomes a constant. Consequently, the depositors' marginal rate of substitution in defaulting states $\omega < \omega^*$ and, thus, the bond value derived from payoffs in these states also change proportionally with the face value.

5.3 The Trade-off between Haircuts and Interest Rates

The findings of this paper provide clarity on whether a trade-off exists between haircuts and interest rates for the same collateral. A common presumption in the empirical literature is that a negative relationship should be expected, as a higher haircut implies lower default risk for depositors, who would consequently demand lower interest rates. However, this paper demonstrates that haircuts and interest rates are uniquely determined. There is no menu offering pairs of haircut and interest rate for market participants to trade off one for the other. The predictions regarding the impact of depositors' saving needs and banks' collateral supply suggest that evidence of a trade-off between haircut and interest rate can be spurious, arising from omitted variable problems in empirical designs. This occurs when other determinants of haircut and interest rate are not appropriately controlled for. Specifically, shocks over time or heterogeneity across borrowers and lenders in saving needs and collateral supply can all contribute to a spurious trade-off between haircut and interest rate.[32] Auh and Landoni (2016) find that, with collateral held constant, a one-point higher spread substitutes for an approximately nine-point lower haircut. However, the trade-off observed in successive loans within a lender can be driven by daily fluctuations in saving needs and collateral supply, while the trade-off observed across lenders concurrently may simply be evidence of lender heterogeneity. Baklanova et al. (2019) do not find strong evidence of a negative relationship (with a correlation of -0.04) between the haircut and interest rate for the same Treasury collateral after controlling for a variety of observable repo characteristics. Although this finding may seem

[32] Kuong (2021) shows that a similar trade-off can also be driven by heterogeneity in the severity of moral hazard friction, although his interpretation of the margin corresponds to $\frac{1}{\omega^*}$ (the number of units of the risky asset used as collateral for a bond with a face value of 1) rather than the percentage haircut in (20).

surprising, this paper suggests that it should not be.

5.4 Total Assets of Money Market Funds and the Repo Market

Proposition 3 predicts the impact of depositors' saving needs on haircuts and interest rates. As MMFs and SLs are the primary creditors to the shadow banking system, this prediction naturally establishes a connection between shocks to the total assets of MMFs or the cash collateral of SLs and changes in repo haircuts and interest rates.[33] For example, if the regulation regarding the supplementary liquidity ratio of commercial banks tightens, resulting in a shift of money from deposits at these banks to MMFs, it would reduce repo haircuts and increase repo interest rates. Conversely, the Federal Reserve's monetary policy operations through the Overnight Reverse Repurchase Agreement Facility (ON RRP), where the Federal Reserve borrows from eligible cash investors in the tri-party repo market, would decrease the amount of cash that MMFs can invest in the repo market,[34] thus having the opposite effects.

This prediction also offers a potential explanation for two surprising patterns identified by Krishnamurthy et al. (2014) in the tri-party repos during and after the 2007-2008 financial crisis. Firstly, they report that haircuts in tri-party repos of MMFs increased to a lesser extent *during* the crisis compared to the haircuts in bilateral repos documented by Gorton and Metrick (2012).[35] Additionally, they document that the total assets of MMFs increased by approximately 50% from 2007Q1 to 2009Q1, and the cash collateral of SLs remained relatively stable from 2007Q1 to 2008Q3. These findings are consistent with increases in the tri-party repo haircuts during the crisis that were mitigated by a positive shock to saving needs.[36]

Secondly, Krishnamurthy et al. (2014) observe that during the *post-crisis* period of 2009–2010, interest rates reverted following the peak of the financial crisis in 2008. However, haircuts for

[33] When MMFs and SLs rebalance their portfolios between short-term secured loans and other assets, their needs of saving in shadow banks change as well.

[34] See Han and Nikolaou (2016) and Anderson and Kandrac (2018).

[35] Gorton and Metrick (2012) report average haircuts exceeding 50% for several categories of corporate debt and securitized products, whereas Krishnamurthy et al. (2014) report increases in haircuts from around 3-4% to about 5-7% for corporate debt and private-label asset-backed securities (ABS).

[36] Krishnamurthy et al. (2014) point out that bilateral repos between dealer banks and hedge funds, as well as between dealer banks, primarily focus on liquidity allocation within the shadow banking system. Consequently, they are less influenced by repo funding obtained from external sources such as MMFs and SLs.

private-sector collateral in 2010 remained as high as, or even higher than, those at the end of 2008. Furthermore, they document that the total assets of MMFs declined by around 22% from 2009Q1 to 2010Q1, and the cash collateral of SLs decreased by about 45% from 2008Q3 to 2010Q1. Indeed, the total amounts of outstanding repos for these two main types of cash investors also decreased proportionally during the same post-crisis period. In particular, the volumes of repos involving private-label ABS remained consistently low throughout this period (as also observed by Copeland et al., 2014). These empirical facts align with higher haircuts and lower interest rates after the crisis in response to a negative shock to saving needs.[37]

5.5 Collateral Asset Holdings of Dealer Banks and the Repo Market

Proposition 4 provides insights into the impact of banks' collateral supply on haircuts and interest rates.[38] This establishes a connection between the collateral asset holdings of dealer banks and the repo haircuts as well as interest rates. Firstly, it suggests that the creation of more securitized assets within the shadow banking system *before* the 2007–2008 financial crisis would have led to higher repo haircuts on those assets. Secondly, it indicates that changes in the collateral supply could have played a significant role in determining the haircuts during the crisis, although the primary driver might have been the increased risk and illiquidity associated with low-quality collateral. According to He et al. (2010), hedge funds and dealer banks reduced their holdings of securitized assets by approximately \$800 billion from the fourth quarter of 2007 to the first quarter of 2009. Additionally, the Federal Reserve introduced the Term Securities Lending Facility (TSLF) in March 2008, where they lent dealers Treasury securities against non-Treasury collateral. These actions

[37] The possible explanation proposed by Krishnamurthy et al. (2014) is that market participants' risk assessments of private debt instruments changed permanently due to the financial crisis.

[38] Since the collateral assets earn a collateral premium, the financial institutions have incentives to create new collateral assets (Fostel and Geanakoplos, 2016). For example, before the 2007–2008 subprime debt crisis, the banks issued a huge amount of subprime mortgage loans and packaged them into mortgage-backed securities, which were then used as collateral in many transactions. As financial innovations, other types of securities backed by assets such as credit cards, student loans, and auto loans have also been created and used as collateral. Moreover, although the banks can purchase the collateral assets from the financial markets outside shadow banking, it is subject to their changing capital constraints (the e_0^b in our model) and to the availability of even the most common type of collateral, the U.S. Treasury securities, because of their issuance cycles and the demand from other market participants. Finally, changes in the collateral eligibility criteria, including the acceptance of additional asset classes or rejection of existing asset classes, also affect the supply of collateral. Infante and Vardoulakis (2020) analyze the collateral run on the asset side of a dealer's balance sheet.

reduced the supply of low-quality collateral and helped stabilize the haircuts, particularly observed in the tri-party repo markets (Krishnamurthy et al., 2014; Copeland et al., 2014). The Federal Reserve's Large-Scale Asset Purchase Programs, conducted from the end of 2008 through October 2014, are also predicted to have had a similar effect.

6 CONCLUSION

This paper examines in a general equilibrium model how the two alternative tools of haircuts and interest rates are employed to address the credit risk associated with collateralized loans in the shadow banking system. Contrary to conventional belief, the model uncovers a nuanced relationship between collateral quality and haircuts. The findings suggest a novel specification for empirical studies examining the impact of collateral quality on haircuts. Moreover, the model demonstrates two novel reasons underlying the relative insensitivity of interest rates compared to haircuts in practice. The model further demystifies a presumed trade-off between haircuts and interest rates, as pursued in the literature, offering guidance for empirical research designs.

This paper also generates new testable predictions. The model connects the cash holdings of shadow banking system creditors (e.g., MMFs and SLs) and the collateral asset holdings of dealer banks to the behavior of repo haircuts and interest rates. These predictions hold significant policy implications. Specifically, when the Federal Reserve conducts monetary policy operations that change investors' cash holdings or the circulation of collateral assets in financial markets, it also influences haircuts and interest rates within private repo markets.

For future research, the model developed in this paper can be extended to encompass multiple assets, allowing for an examination of differential haircuts applied to various assets and the bundling of collateral assets. Furthermore, the model can be generalized to investigate the interactions between traditional and shadow banks. Finally, the model's scope can be expanded to incorporate additional critical frictions and features of the collateralized loan market, such as counterparty risk, intermediation, and rehypothecation.

APPENDIX

A. The competitive equilibrium allocation is constrained efficient

In this section, I show that the equilibrium allocation in my model is necessarily a solution to the social planner's problem of maximizing a particularly weighted social welfare function subject to the incentive or collateral constraint. This allows me to characterize the equilibrium allocation by solving the social planner's maximization problem. This approach is in the spirit of Negishi (1960). However, Negishi's result is derived for perfect competitive markets. In order to apply his approach to my model with a constraint, a generalization is necessary. Moreover, what is shown here is slightly different from Negishi's perspective, who shows that the solution to the social planner's problem with a particularly weighted social welfare function is an equilibrium allocation. This not only proves the existence of equilibrium but also provides a way of finding an equilibrium. My result does not indicate the existence of equilibrium but allows for finding all equilibria if there are multiple ones. Nevertheless, the existence of equilibrium in my specific model is not an issue as we can easily see that the allocation characterized is supported by a price system. Moreover, the uniqueness of equilibrium will be shown in the proof of Theorem 1.

We first consider the following social planner's problem,

$$\max_{\{c_0^b, c_1^b(\omega)\}} \alpha \mathbb{E}\big[u(e_0^d + e_0^b - c_0^b) + u\big(e_1^d + e_1^b + K\omega - c_1^b(\omega)\big)\big] + \mathbb{E}\big[c_0^b + c_1^b(\omega)\big] \tag{A.1}$$

$$s.t. \quad c_1^b(\omega) \geq e_1^b, \tag{A.2}$$

where I have substituted the binding resource constraints.

Definition 1. *The solution $\{c_0^d, c_1^d(\omega), c_0^b, c_1^b(\omega)\}$ to the social planner's problem is called a welfare maximum point. The necessary and sufficient condition for it is as follows: there exists $\lambda(\omega) \geq 0$ such that*

$$u'(c_0^d) = \frac{1}{\alpha}, \quad u'(c_1^d(\omega)) = \frac{1}{\alpha} + \frac{\lambda(\omega)}{\alpha}, \tag{A.3}$$

$$c_1^b(\omega) \geq e_1^b, \quad \lambda(\omega)(c_1^b(\omega) - e_1^b) = 0, \tag{A.4}$$

$$c_0^d + c_0^b = e_0^d + e_0^b, \quad c_1^d(\omega) + c_1^b(\omega) = e_1^d + e_1^b + K\omega. \tag{A.5}$$

We next consider the competitive equilibrium allocation. Given the Arrow security prices $\{q(\omega)\}$, the depositors' problem is

$$\max_{\{c_0^d, c_1^d(\omega)\}} u(c_0^d) + \mathbb{E}\big[u(c_1^d(\omega))\big]$$
$$s.t. \quad c_0^d + \int_\Omega q(\omega)c_1^d(\omega)d\omega \le e_0^d + \int_\Omega q(\omega)e_1^d d\omega. \tag{A.6}$$

and the banks' problem is

$$\max_{\{c_0^b, c_1^b(\omega)\}} c_0^b + \mathbb{E}\big[c_1^b(\omega)\big]$$
$$s.t. \quad c_0^b + \int_\Omega q(\omega)c_1^b(\omega)d\omega \le e_0^b + \int_\Omega q(\omega)(e_1^b + K\omega)d\omega, \tag{A.7}$$
$$c_1^b(\omega) \ge e_1^b. \tag{A.8}$$

Definition 2. *A competitive equilibrium of the economy is a price system $\{q(\omega)\}$ and an allocation $\{c_0^d, c_1^d(\omega), c_0^b, c_1^b(\omega)\}$ such that*

1. *Given the prices, $\{c_0^d, c_1^d(\omega)\}$ solve the depositors' problem. The necessary and sufficient condition for it is as follows:*

$$f(\omega)\frac{u'(c_1^d(\omega))}{u'(c_0^d)} = q(\omega), \tag{A.9}$$
$$c_0^d + \int_\Omega q(\omega)c_1^d(\omega)d\omega = e_0^d + \int_\Omega q(\omega)e_1^d d\omega. \tag{A.10}$$

2. *Given the prices, $\{c_0^b, c_1^b(\omega)\}$ solve the banks' problem. The necessary and sufficient condition for it is as follows: there exists $\lambda(\omega) \ge 0$ such that*

$$q(\omega) = f(\omega) + \lambda(\omega)f(\omega), \tag{A.11}$$
$$c_1^b(\omega) \ge e_1^b, \quad \lambda(\omega)(c_1^b(\omega) - e_1^b) = 0, \tag{A.12}$$
$$c_0^b + \int_\Omega q(\omega)c_1^b(\omega)d\omega = e_0^b + \int_\Omega q(\omega)(e_1^b + K\omega)d\omega. \tag{A.13}$$

3. *The consumption good markets clear: $c_0^d + c_0^b = e_0^d + e_0^b$ and $c_1^d(\omega) + c_1^b(\omega) = e_1^d + e_1^b + K\omega$.*

Proposition 5. *The allocation $\{c_0^d, c_1^d(\omega), c_0^b, c_1^b(\omega)\}$ of a competitive equilibrium is a welfare maximum point and, therefore, is constrained efficient.*

Proof. Given the allocation, define the coefficient on depositors' welfare in a social welfare function as

$$\alpha = \frac{1}{u'(c_0^d)}. \tag{A.14}$$

The coefficient on banks' welfare is still normalized to 1. By (A.9) and (A.11), we have

$$u'(c_1^d(\omega)) = \frac{1}{\alpha} + \frac{\lambda(\omega)}{\alpha}. \tag{A.15}$$

So the condition (A.3) of a welfare maximum point is satisfied. The condition (A.4) is the same as (A.12), and the condition (A.5) is the same as the market clearing condition 3 in Definition 2. Hence all conditions of a welfare maximum point are satisfied. Then note that the allocation of a welfare maximum point is necessarily constrained Pareto optimal. □

B. Depositors' risk aversion

In this section, I examine the effects of depositors' risk aversion, which may be useful for the empirical studies examining variations in loan terms over time and across lenders and borrowers. In order to capture changes in risk aversion, I consider the following parametric utility function of depositors.

Assumption 4. *Depositors have a CRRA utility function* $u(c) = \frac{c^{1-\eta}}{1-\eta}$.

Note that η governs both the risk aversion and the elasticity of intertemporal substitution of depositors. But since depositors and banks have the same time discount factor of 1, we can isolate the effects of risk aversion.

Proposition 6. *When the depositors' risk aversion* η *increases, the bond face value* ω^* *and the interest rate* R *decreases, the asset price* p *increases, and the haircut* H *decreases if and only if the upside quality of collateral is high or the downside quality is low (which implies a high* ω^**) such that*

$$\mathbb{E}[\omega|\omega > \omega^*] > \frac{e_0^d}{K}. \tag{B.16}$$

A higher depositors' risk aversion increases the price they are willing to pay for consumption good in bad states, hence a higher bond price. The lower interest rate has a negative income effect and decreases the depositors' demand for bonds, leading to a lower bond face value, ω^*, per unit of collateral asset. While a decrease in ω^* tends to increase the haircut by Lemma 2, the depositors' higher valuation of bonds has the opposite effect. Overall, the latter dominates and the haircut H decreases if and only if the upside quality of collateral is high or the downside quality is low. Note that (B.16) is identical to (31), so a higher depositors' risk aversion has the same effects as an improved downside quality of collateral.

Remark 7. *In the benchmark model with quasilinear preferences for depositors, the effects of a higher depositors' risk aversion are also the same as those of an improved downside quality of collateral. Under Assumption 4, (22) is reduced to* $e_1^d + K\omega^* = 1$*, so the bond face value* ω^* *is unaffected by the risk aversion. A higher risk aversion increases depositors' marginal rate of substitution and thus the bond price* q*, leading to a higher asset price, a lower interest rate, and a lower haircut.*

Remark 8. *In Simsek's (2013) model, as both types of agent are risk neutral, there is no corresponding comparative statics with respect to risk aversion. Geanakoplos (2003, 2009) notes that heterogeneity in beliefs may be regarded as a reduced-form version of heterogeneity in risk aversion in terms of determining who are the natural buyers of collateral assets because differences in risk aversion mean different risk-adjusted probabilities. Then one may tend to think that an increase in depositors' risk aversion in my model is corresponding to an increase in the belief disagreements in Simsek's model. However, this is not exactly the case. In my model, the equilibrium admits a representation in terms of a risk-neutral measure for the risk-averse depositors.*[39] *The endogenously determined* $MRS^d(\omega)$ *of risk-averse depositors is equal to that of the risk-neutral banks in high states but is higher in low states. So the associated risk-adjusted (risk-neutral) belief of risk-averse*

[39] To be precise, the equilibrium allocations and prices in the original economy can be supported in a new economy where the originally risk-averse depositors become risk-neural and their belief and the time preference are replaced with their risk-adjusted probabilities and mean pricing kernel, respectively. But note that the equilibrium concept in the original economy is stronger since the equilibrium allocations must be consistent with the pricing kernel, while in the new economy the risk-neutral belief is fixed.

depositors adjusts probabilities of low states upward and probabilities of high states downward. But importantly, their implicitly associated risk-free discount factor (the mean $MRS^d(\omega)$), which can be represented by a modified time discount factor, is higher than the risk-neutral banks. Therefore, within a risk-neutral representation, the equilibrium features both heterogeneity in beliefs and heterogeneity in time preferences, and these heterogeneities are also endogenous rather than exogenously fixed. This is why the charaterizations of equilibria in these two models are different. Not surprisingly, the model predictions are also different. In Simsek's model, when depositors become more pessimistic overall, the bond face value ω^* decreases, the asset price p decreases, and the haircut H increases.

C. Proofs

C.1. Proof of Theorem 1

First, I show that there exists a unique solution $\omega^* \in (\underline{\omega}, \bar{\omega})$ to equation (18). To see this, first note that $\frac{\partial q}{\partial \omega^*} > 0$, which is established as Lemma 1 below. So the left-hand side of (18) is strictly increasing in ω^*, while the right-hand side is a constant. Then note that Assumption 2 guarantees that left-hand side is lower than the right-hand side at $\underline{\omega}$ but is greater than the right-hand side at $\bar{\omega}$.

Second, it is straightforward to verify that the equilibrium conditions of Definition 2 in Section A of this appendix are satisfied for the specified allocation and Arrow security prices in Theorem 1.

Finally, I now show that the equilibrium is unique. In Section A of the appendix, I showed that every equilibrium allocation is constrained efficient, that is, it will be of one of three types of allocations characterized in Section 2.2 of the main text. In the first type of allocation, depositors' consumption plan is $c_0^d = c_1^d(\omega) \leq e_1^d + K\underline{\omega}$ for every $\omega \in \Omega$. However, with depositors' date-0 budget constraint, this implies

$$e_0^d = c_0^d + (c_1^d - e_1^d) \leq 2K\underline{\omega} + e_1^d, \tag{C.1}$$

which contradicts equation (11) of Assumption 2. In the second type of allocation, depositors' consumption plan is $c_0^d \geq c_1^d(\omega) = e_1^d + K\omega$ for every $\omega \in \Omega$. However, with depositors' date-0 budget constraint, this implies

$$e_0^d = c_0^d + \mathbb{E}\left[\frac{u'(e_1^d + K\omega)}{u'(c_0^d)} K\omega\right] \geq e_1^d + K\bar{\omega} + \mathbb{E}\left[\frac{u'(e_1^d + K\omega)}{u'(e_1^d + K\bar{\omega})} K\omega\right], \tag{C.2}$$

which contradicts equation (12) of Assumption 2. Hence, the equilibrium allocation must be of the third type that this paper focuses on and is unique.

C.2. Proof of Lemma 1

Note that the debt component of the asset price, or the bond price q, in (17) is given by

$$q = \mathbb{E}\left[\frac{u'(e_1^d + K\min(\omega, \omega^*))}{u'(e_1^d + K\omega^*)} \min(\omega, \omega^*)\right]. \tag{C.3}$$

Then we have

$$\frac{\partial q}{\partial \omega^*} = \int_{\underline{\omega}}^{\omega^*} \frac{-u'(e_1^d + K\omega)u''(e_1^d + K\omega^*)}{(u'(e_1^d + K\omega^*))^2} K\omega dF(\omega) + 1 - F(\omega^*) > 0 \tag{C.4}$$

given that $u'(\cdot) > 0$, $u''(\cdot) < 0$, and $\omega > 0$.

C.3. Proof of Lemma 2

Note that the equity component E of the asset price in (17) is given by

$$E = \mathbb{E}[\max(\omega - \omega^*, 0)],$$

which is decreasing in ω^*.

However, we still have

$$\frac{\partial p}{\partial \omega^*} = \int_{\underline{\omega}}^{\omega^*} \frac{-u'(e_1^d + K\omega)u''(e_1^d + K\omega^*)}{(u'(e_1^d + K\omega^*))^2} K\omega dF(\omega) > 0. \tag{C.5}$$

C.4. Proof of Lemma 3

The ratio of bond price over face value is given by

$$\frac{q}{\omega^*} = \int_{\underline{\omega}}^{\omega^*} \frac{u'(e_1^d + K\omega)\omega}{u'(e_1^d + K\omega^*)\omega^*} dF(\omega) + 1 - F(\omega^*).$$

Then we have

$$\frac{\partial(q/\omega^*)}{\partial\omega^*} = \int_{\underline{\omega}}^{\omega^*} \frac{-u'(e_1^d + K\omega)w(u''(e_1^d + K\omega^*)K\omega^* + u'(e_1^d + K\omega^*))}{(u'(e_1^d + K\omega^*)\omega^*)^2} dF(\omega) < 0 \tag{C.6}$$

given that $u'(\cdot) > 0$, $u''(e_1^d + K\omega^*)K\omega^* + u'(e_1^d + K\omega^*) > 0$ by Assumption 3, and $\omega > 0$. Therefore, the interest rate $R = \frac{\omega^*}{q} - 1$ is increasing in the bond face value ω^*.

C.5. Proof of Proposition 1

Since the downside quality of collateral over $(\underline{\omega}, \omega^*)$ remains the same for either of the two types of shocks, the equation (18), which characterizes the equilibrium bond face value ω^*, is unaffected, so ω^* is remains the same. Then the bond price q in (16) and its interest rate R do not change. However, an FOSD or a mean-increasing SOSD improvement of the upside quality of collateral over $(\omega^*, \bar{\omega})$ increases the equity component E of the asset price p in (17), so the asset price p and the haircut H increase. A mean-preserving SOSD improvement of the upside quality of collateral over $(\omega^*, \bar{\omega})$ does not change the equity component E of the asset price p in (17), so the asset price p and the haircut H are unaffected.

C.6. Proof of Proposition 2

Assumption 3 implies that $u'(e_1^d + c)c$ is a strictly increasing function of c. Hence, $u'(e_1^d + K\omega)\omega$ is strictly increasing in ω over $(\underline{\omega}, \omega^*)$. Therefore, an FOSD improvement of the downside quality over $(\underline{\omega}, \omega^*)$ increases the bond price q in (C.3) and reduces the interest rate R. Then the left-hand side of (18), which is strictly increasing in ω^*, moves upward. So the bond face value ω^* adjusts downward, which further reduces the interest rate R by Lemma 3.

For the asset price, a lower ω^* increases its equity component. Although a lower ω^* tends to reduce its debt component by Lemma 1, this effect is dominated by the improvement of the downside quality of collateral. This can be seen from (18), where a lower ω^* implies a higher bond price q. Overall, the asset price p increases.

For the haircut, it is less obvious since both the equity and debt components of the asset price increase. Nevertheless, substituting the expression of bond price q in (18) into (20), we have

$$\frac{E}{q} = \frac{\mathbb{E}[\max(\omega - \omega^*, 0)]}{(e_0^d - e_1^d)/K - \omega^*}, \tag{C.7}$$

with

$$\begin{aligned} \frac{\partial (E/q)}{\partial \omega^*} &= \frac{-\int_{\omega^*}^{\bar{\omega}} dF(\omega)((e_0^d - e_1^d)/K - \omega^*) + \int_{\omega^*}^{\bar{\omega}} (\omega - \omega^*) dF(\omega)}{((e_0^d - e_1^d)/K - \omega^*)^2} \\ &= \frac{-(1 - F(\omega^*))(e_0^d - e_1^d)/K + \int_{\omega^*}^{\bar{\omega}} \omega dF(\omega)}{((e_0^d - e_1^d)/K - \omega^*)^2}, \end{aligned} \tag{C.8}$$

which is positive so that the haircut decreases if and only if

$$\mathbb{E}[\omega | \omega > \omega^*] > \frac{e_0^d - e_1^d}{K}. \tag{C.9}$$

C.7. Proof of Proposition 3

When e_0^d increases, the right-hand side of (18) shifts upward. As the left-hand side is increasing in ω^*, the bond face value ω^* increases. By Lemma 2, the haircut H decreases. By Lemma 3, the interest rate R increases.

C.8. Proof of Proposition 4

We first examine the effect on the equilibrium bond face value ω^*. Totally differentiating (18) with respect to K, we have

$$\left(1 + \frac{\partial q}{\partial \omega^*}\right) \frac{d\omega^*}{dK} = -\frac{e_0^d - e_1^d}{K^2} - \frac{\partial q}{\partial K}. \tag{C.10}$$

Since $\frac{\partial q}{\partial \omega^*} > 0$ by Lemma 1, the sign of $\frac{d\omega^*}{dK}$ is the same as $-\frac{e_0^d - e_1^d}{K^2} - \frac{\partial q}{\partial K}$. We have

$$\begin{aligned} -\frac{e_0^d - e_1^d}{K^2} - \frac{\partial q}{\partial K} &= -\frac{\omega^* + q}{K} - \frac{\partial q}{\partial K} \\ &= -\frac{\omega^*}{K} - \frac{1}{K}\left(\int_{\underline{\omega}}^{\omega^*} \frac{u'(e_1^d + K\omega)}{u'(e_1^d + K\omega^*)} \omega dF(\omega) + \omega^*(1 - F(\omega^*))\right) \\ &\quad - \int_{\underline{\omega}}^{\omega^*} \frac{u'(e_1^d + K\omega)}{u'(e_1^d + K\omega^*)} \left(\frac{u''(e_1^d + K\omega)\omega}{u'(e_1^d + K\omega)} - \frac{u''(e_1^d + K\omega^*)\omega^*}{u'(e_1^d + K\omega^*)}\right) \omega dF(\omega) \\ &= -\frac{\omega^*}{K}(2 - F(\omega^*)) + \int_{\underline{\omega}}^{\omega^*} \frac{u'(e_1^d + K\omega) u''(e_1^d + K\omega^*)\omega^*}{(u'(e_1^d + K\omega^*))^2} \omega dF(\omega) \\ &\quad - \frac{1}{K} \int_{\underline{\omega}}^{\omega^*} \frac{u'(e_1^d + K\omega)}{u'(e_1^d + K\omega^*)} \left(1 + \frac{u''(e_1^d + K\omega) K\omega}{u'(e_1^d + K\omega)}\right) \omega dF(\omega) \\ &< 0 \end{aligned} \tag{C.11}$$

given that $\omega > 0, u'(\cdot) > 0, u''(\cdot) < 0$, and $1+\frac{u''(e_1^d+K\omega)K\omega}{u'(e_1^d+K\omega)} > 0$ by Assumption 3. So the equilibrium bond face value ω^* decreases.

For the bond price q, we first have

$$\frac{dq}{dK} = \frac{\partial q}{\partial K} + \frac{\partial q}{\partial \omega^*}\frac{d\omega^*}{dK} = \left(1+\frac{\partial q}{\partial \omega^*}\right)^{-1}\left(\frac{\partial q}{\partial K} - \frac{\partial q}{\partial \omega^*}\frac{e_0^d - e_1^d}{K^2}\right), \tag{C.12}$$

where I have plugged in $\frac{d\omega^*}{dK}$ in (C.10). Then note that

$$\begin{aligned}\frac{\partial q}{\partial K} - \frac{\partial q}{\partial \omega^*}\frac{e_0^d - e_1^d}{K^2} =& \int_{\underline{\omega}}^{\omega^*} \frac{u'(e_1^d+K\omega)}{u'(e_1^d+K\omega^*)}\left(\frac{u''(e_1^d+K\omega)\omega}{u'(e_1^d+K\omega)} - \frac{u''(e_1^d+K\omega^*)\omega^*}{u'(e_1^d+K\omega^*)}\right)\omega dF(\omega) \\ &- \frac{e_0^d - e_1^d}{K^2}\left(\int_{\underline{\omega}}^{\omega^*} \frac{-u'(e_1^d+K\omega)u''(e_1^d+K\omega^*)}{(u'(e_1^d+K\omega^*))^2}K\omega dF(\omega) + 1 - F(\omega^*)\right) \\ =& \int_{\underline{\omega}}^{\omega^*} \frac{u'(e_1^d+K\omega)}{u'(e_1^d+K\omega^*)}\left(\frac{u''(e_1^d+K\omega)\omega}{u'(e_1^d+K\omega)} - \frac{u''(e_1^d+K\omega^*)}{u'(e_1^d+K\omega^*)}\left(\omega^* - \frac{e_0^d-e_1^d}{K}\right)\right)\omega dF(\omega) \\ &- \frac{e_0^d - e_1^d}{K^2}(1-F(\omega^*)) \\ <&0\end{aligned} \tag{C.13}$$

given that $\omega > 0, u'(\cdot) > 0, u''(\cdot) < 0$, and $\omega^* - \frac{e_0^d - e_1^d}{K} = -q < 0$ by (18). So the bond price q decreases.

As the equity component E of the asset price is not directly affected by K, it increases as the bond face value ω^* decreases by Lemma 2. Given that the bond price q decreases, the haircut H increases.

For the asset price p, we have

$$\begin{aligned}\frac{dp}{dK} =& \frac{dq}{dK} + \frac{\partial E}{\partial \omega^*}\frac{d\omega^*}{dK} \\ =& \left(1+\frac{\partial q}{\partial \omega^*}\right)^{-1}\int_{\underline{\omega}}^{\omega^*} \frac{u'(e_1^d+K\omega)}{u'(e_1^d+K\omega^*)}\left(\frac{u''(e_1^d+K\omega)\omega}{u'(e_1^d+K\omega)}(2-F(\omega^*)) \right. \\ &\left. - \frac{u''(e_1^d+K\omega^*)}{u'(e_1^d+K\omega^*)}\big((1-F(\omega^*))\omega^* - q\big)\right)\omega dF(\omega) \\ <&0\end{aligned} \tag{C.14}$$

given that $(1-F(\omega^*))\omega^* - q < 0$. In the second line, I substituted $\frac{d\omega^*}{dK}$ in (C.10), $\frac{dq}{dK}$ in (C.12), and $\frac{\partial q}{\partial K}$ in (C.11). Note that $\frac{\partial E}{\partial \omega^*} = -(1-F(\omega^*))$. So the asset price p decreases.

For the interest rate R, we have

$$\begin{aligned}\frac{d(q/\omega^*)}{dK} =& \frac{1}{\omega^{*2}}\left(\frac{\partial q}{\partial K}\omega^* + \frac{d\omega^*}{dK}\left(\frac{\partial q}{\partial \omega^*}\omega^* - q\right)\right)\\ =& \frac{\omega^* + q}{\omega^{*2}(1+\frac{\partial q}{\partial \omega^*})}\left(\frac{\partial q}{\partial K} - \frac{1}{K}\left(\frac{\partial q}{\partial \omega^*}\omega^* - q\right)\right)\\ =& \frac{\omega^* + q}{K\omega^{*2}(1+\frac{\partial q}{\partial \omega^*})}\int_{\underline{\omega}}^{\omega^*}\frac{u'(e_1^d + K\omega)}{u'(e_1^d + K\omega^*)}\left(1 + \frac{u''(e_1^d + K\omega)K\omega}{u'(e_1^d + K\omega)}\right)\omega dF(\omega)\\ >& 0\end{aligned} \tag{C.15}$$

given that $1 + \frac{u''(e_1^d+K\omega)K\omega}{u'(e_1^d+K\omega)} > 0$ by Assumption 3. In the second line, I substituted $\frac{d\omega^*}{dK}$ in (C.10) and used (18). In the third line, we substituted $\frac{\partial q}{\partial K}$ in (C.11), $\frac{\partial q}{\partial \omega^*}$ in (C.4), and q in (C.3). So the interest rate R decreases.

C.9. Proof of Proposition 6

Under Assumption 4, the bond price q in (16) is given by

$$q = \mathbb{E}\left[\left(\frac{e_1^d + K\min(\omega,\omega^*)}{e_1^d + K\omega^*}\right)^{-\eta}\min(\omega,\omega^*)\right]. \tag{C.16}$$

A higher relative risk aversion coefficient η increases the bond price q in (C.16) and reduces the interest rate R. Then the left-hand side of (18) shifts upward. As the left-hand side is increasing in ω^*, the bond face value ω^* decreases, which further reduces the interest rate R by Lemma 3.

For the asset price, a lower ω^* increases its equity component. Although a lower ω^* tends to reduce its debt component by Lemma 1, this effect is dominated by the increase in the risk aversion. This can be seen from (18), where a lower ω^* implies a higher bond price q. Overall, the asset price p increases.

For the haircut, based on the expression of equity/debt ratio in (C.7) and the comparative statics thereafter, we can see that as the bond face value ω^* decreases, the haircut H decreases if and only if

$$\mathbb{E}[\omega|\omega > \omega^*] > \frac{e_0^d - e_1^d}{K}.$$

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