Reverse Mortgages, Housing, and Consumption: An Equilibrium Approach *

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Abstract

Reverse Mortgages (RMs) allow homeowners 62 and older to access their home's liquidity without moving out or repaying until loan termination when they die or relocate to long-term care. We incorporate RMs into a quantitative equilibrium life-cycle model to assess their impacts on household decisions, the mortgage and the housing market. We demonstrate that retired RM borrowers have significantly enhanced consumption smoothing. Additionally, The presence of RMs in the mortgage market enhances the perceived home value to households, making homeownership more financially desirable and driving housing demand. Finally, RMs lower social security costs for the government and working-age population. These effects show that RMs improve overall household welfare, highlighting their positive benefits.

JEL Classification: G21, E21, J14

Key Words: RM, Mortgage, Housing, Retirement, Welfare

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1 Introduction

Financial stability in later life is a global challenge: rising life expectancy, socioeconomic disparity, and insufficient public resources for social security funding. Most US household wealth is accumulated as illiquid housing assets.[1] Pension freedom allows retirees to withdraw tax-free. These lead to decreased retirement incomes and "equity-rich and cash-poor" households (see Caplin (2002)).

Reverse Mortgages (RMs hereafter) have emerged as a popular financial tool for older households to finance their retirement expenses. These products have gained popularity in developed economies such as the US, UK, Australia, Canada, Hong Kong, and South Africa.[2,3] They allow homeowners to extract the value of their homes to supplement their retirement income without having to move out or downsize. This feature is also known as *ageing-in-place*, termed by Cocco and Lopes (2020), which refers to the benefits of retiring in a familiar home and community environment. Several emerging economies, such as Brazil, Chile, South Africa, and Thailand, have also adopted RMs. However, RMs' structural complexity makes it difficult to analyze their impacts on household decisions, balance sheet dynamics, mortgage-housing markets, and social welfare.

In this paper, we measure RM's economic impacts through various channels using US loan-level data from CoreLogic and Black Knight. We set up a quantitative equilibrium model with heterogeneous households, competitive lenders, and endogenous house prices, with the presence of conventional mortgages and RMs, accounting for the transition of macroeconomic states among booms, recessions, and crises with cyclical interest rates. Mortgage and RM spreads are set to break even by lenders. Households derive utility from housing, non-durable consumption, and leaving a bequest. Households' decisions influence the equilibrium house prices and spreads of mortgages and RMs, which then feed back to them. We consider multiple risks that households

[1]According to Chen et al. (2020), on the household balance sheet, housing estates are 10 times the size of liquid assets, such as cash and equivalents (financial securities (equities, and bonds). Similarly, Nakajima and Telyukova (2020) states that excluding housing wealth, retirees' net worth would be 28% - 44% lower, depending on age groups. Iacoviello (2011) also estimates that housing wealth is about one-half of total household net worth with the U.S. data.

[2]In the US, RMs are commonly Federal Housing Administration (FHA) - insured Home Equity Conversion Mortgages (HECMs), available to individuals aged 62 and older. They are primarily used to finance retirement consumption. Home equity financing solutions encompass various instruments, including non-age-restricted home equity loans (HELs), Home Equity Lines of Credit (HELOCs), and second mortgages that utilize housing equity as collateral.

[3]See Nakajima and Telvukova (2017, 2020), and Chen et al. (2020).

are exposed to, such as untradeable labour income risk, health risk, longevity risk, interest rate risk, uncertainty in house prices and macroeconomic volatility. Based on their expectations about equilibrium house prices, mortgage and RM rates, households make decisions on consumption, saving, homeownership, default, and (re)financing, maintenance and RMs.

At a high level, our general equilibrium model makes three important contributions. To our best knowledge, our model is one of the first to clear the housing and credit markets simultaneously, with presence of conventional mortgages and RMs. In contrast, Nakajima and Telyukova (2020) and Nakajima and Telyukova (2017) model retirement home equity using RMs, assuming deterministic house prices, while Cocco and Lopes (2020) model RMs' demand and incorporate a random walk process of house prices. Most papers examining the RM's low demand despite the evident benefits for eligible households employ a partial equilibrium life-cycle model that considers only the household balance sheet, not interactions between the financial (mortgage) market and real economy (housing market).[4][5] Meanwhile, our model allows homeowners endogenously choose a lump sum or line of credit, in contrast to the common assumption in existing literature that homeowners only take a lump sum. This endogenous choice adds a realistic dimension to our analysis, reflecting the actual decision-making process faced by homeowners considering RMs.

Second, realistically, RMs can be used to repay conventional mortgages within loan-to-value (LTV) limits. This feature makes them an attractive alternative to refinancing because RMs relieve borrowers from regular repayments, thereby reducing liquidity concerns, which can be observed from proprietary datasets such as CoreLogic and Black Knight, with granular loan-level details with substantial coverage. Thus, calibrating our model using such datasets enables us to better capture RMs' impacts in a realistic setting.[6] Most papers modelling RMs used public datasets such as the Panel Study of Income Dynamics (PSID) and Health and Retirement Study (HRS), which are survey-based datasets dependent on estimation and weighting methodologies. As RM participation is relatively small compared to the eligible households, the loan-level granular

[4]Low RM take-up rates may be due to bequest motives, conservative savings, longevity risk, and medical expenses, etc.

[5]See Cocco and Lopes (2020) for the impact of maintenance costs, and Nakajima and Telyukova (2017) for the impact of bequest motives and initial setup costs on RM demand.

[6]For example, both CoreLogic and Black Knight can be linked to credit information and American Community Survey (ACS) that provides household demographics, socioeconomic characteristics, household consumption and investment behaviors, alongside their housing condition, house prices, first liens (conventional mortgages and RMs), and second or junior lien mortgage (if any) information.

datasets add significant value to the accuracy of our analysis.

The third important contribution is that our calibrated model generates a homeownership rate, household net wealth, and LTV ratio close to empirical evidence from the Survey of Consumer Finances (SCF). In the presence of RM, our model result matches the Payment-to-Income (PTI) ratio with the empirical evidence very well. In comparison, Guren et al. (2021) generates a PTI ratio that grows exponentially throughout retirement, a finding that contradicts empirical evidence. Furthermore, the refinancing rate of conventional mortgages in our model remains relatively stable and moderate during the retirement period. These findings differ from the study by Guren et al. (2021), whose findings indicate a low refinancing rate during the working period followed by a sudden spike after retirement. The difference may result from introducing RMs into our model, such that households can use RMs as a substitution for refinancing with the additional benefit of no monthly repayments and an embedded put option of housing price; thus, the refinancing rate becomes moderate after retirement.

Our findings indicate that RMs effectively smooth household consumption over the life cycle by enabling eligible households to access home equity and address income reduction post-retirement. The consumption growth volatility of households borrowing RMs is reduced by 17.04% (working period) and 8.36% (retirement). In addition, RMs provide liquidation for senior households: In the earlier years of the loans, these households save slightly less or extend their mortgage borrowing period. This enhanced flexibility smooths their consumption throughout their working and retirement periods. RMs lower the cost of social security borne by the government and the working-aged population who are contributing to the public retirement system in the US.

Moreover, Using HECM guidelines' insurance premium and cost data, our model predicts a low RM take-up rate of 2.3% for eligible homeowners, with a hump-shaped pattern across the life cycle. Our investigation reveals that eliminating initial RM setup and product expenses can boost take-up rates by up to 10% for eligible homeowners aged 62-65, significantly higher than empirical evidence. Considering the set-up and product costs reduces the RM take-up rate considerably, aligning with prior research by Cocco and Lopes (2020) and Nakajima and Telyukova (2017). In addition, our findings on RM take-up rates using households' age profiles align with actual data, providing a modelling advantage. One potential limitation is that the service cost used in our model is relatively high compared to published values. Our equilibrium setting assumes households' perfect access to housing and credit markets without friction and information asymmetry, such that they make rational decisions on savings, consumption, borrowing decisions of mortgages & RMs, and homeownership. To target the RM take-up rate, the service cost in our model needs to be higher than those reported, as the current RM market is far from saturated.

To assess the impacts of RMs on the housing market and household welfare, we evaluate an alternative economy (AE) without RMs in the credit market, keeping all other parameters constant as in the original economy (OE) with RMs. Keeping the housing supply fixed increases the house prices significantly (from 0.66% to 1.90%) in OE. Meanwhile, homeowners attain substantial welfare gains measured by the equivalent consumption variation as 1.29% (aged <62), 4.77% (aged 62-75), and 7.47% (aged >75). Renters, on the other hand, enjoy marginal welfare improvement. Moreover, households in OE experience enhanced consumption smoothing. Therefore, these findings suggest that introducing RMs as a financial product for trading liquidations enhances the efficiency of the housing market and benefits households.

Our paper first draws on existing RM literature. Case and Schnare (1994) investigate RM's growth in mid-2000s, including loan & property characteristics; borrowers' socioeconomic and demographic aspects. Rasmussen et al. (1995) and Merrill et al. (1994) demonstrate that RMs potentially benefit households. Recently, Nakajima and Telyukova (2017) suggest that older households must balance bequest motives, *aging-in-place*, retirement cash shortfall, longevity, health, interest and inflation uncertainty, while RM borrowers enjoy ex-ante and ex-post welfare improvements. In comparison, we show that RMs have an asymmetric yet positive impact on retired and working households, resulting in overall societal welfare improvement. Both Nakajima and Telyukova (2017) and Cocco and Lopes (2020) predict RMs' low demand, aligned with empirical data. Cocco and Lopes (2020) derives the household utility from *aging-in-place*, but high product and maintenance costs dampened the demand. [7,8] In contrast to Davidoff (2005), assuming exogenous home prices, mortgage and RM rates, we endogenizes housing-credit markets, capturing feedback loops between financial and real markets, improving the assessment of RMs' impacts.

[7]Davidoff (2005) yields similar results that find RM's low demand caused by high costs, despite clear benefits.

[8]Campbell and Cocco (2003) estimate only 2%–3% senior homeowners among eligible older homeowners borrow RM, as the US Census Bureau reports. On the other hand, Nakajima and Telyukova (2017) estimate this ratio to be 1.9% in 2013, down from 2.1% in 2011.

Further, Our article highlights the impact of housing market dynamics, macroeconomic conditions, household income, and home equity on household decisions about refinancing and default. Empirical studies Campbell (1983) and Deng et al. (2000) suggest that borrowers delay default until it is irreversible and their option to default is deep in the money. Studies conducted following the 2008 mortgage market collapse find an inconsistent correlation between mortgage default, unemployment (which affects household income negatively) and precautionary savings, depending on the research period and location (see Elul et al. (2010)); Quercia et al. (2012); Ghent and Kudlyak (2011); Archer and Smith (2013); Tian et al. (2015)). Chen et al. (2020) follows Campbell and Cocco (2003) partial equilibrium approach, integrating mortgage refinancing and home equity-based borrowing (HELOC). They show that during the recession, liquidity-constraint households refinance even with heightened borrowing costs due to negative income shocks, contradicting the conventional belief that refinancing is driven by low-interest rates. Our study supports this observation by endogenizing the housing and credit market at equilibrium, household utility maximization, and refinancing decisions are independent of interest rate fluctuation.

Last but not least, our paper contributes to the mortgage-housing literature. Real estate's unique characteristics, illiquidity, borrowing constraints for low-wealth households, heterogeneous preferences, and inaccessible rental yield as implicit dividends, hinder immediate demand-supply equilibrium. Early attempts by Case and Shiller (1989) and Mankiw et al. (1989) demonstrate increased mortgage default rates, foreclosures, and vacancies, defying hedonic pricing and general equilibrium models assuming instant demand and supply adjustment. DiPasquale and Wheaton (1994), Riddel (2004), and Hwang and Quigley (2006) analyze housing market disequilibrium with supply and demand disdurbances. Fluctuations in the housing market affect housing prices, which feed back on all types of mortgages, including RMs. Existing methodologies have often simplified or treated mortgages as exogenous, deviating from empirical evidence (Himmelberg et al. (2005); Atif and Sufi (2009); Glaeser et al. (2010); Favara and Jean (2010); Adelino et al. (2014); Kung (2014a)). For example, Ortalo-Magné et al. (2006), Favilukis et al. (2010), and Tim et al. (2014) study equilibrium housing markets, treating mortgages as exogenous or stylized; Campbell and Cocco (2015) and Corbae and Quintin (2014) consider equilibrium mortgage market, taking housing market as given. Closer to our model, Kung (2014b) creates a general equilibrium housing-mortgage model, endogenizing house prices, mortgage rates and leverage ratios. In contrast, we introduce RMs into the general equilibrium framework, better reflecting mortgage market dynamics and complexities by incorporating the interplay between mortgages and RMs.

The rest of the paper is organized as follows. Section 2 highlights the importance of RMs from the data. Section 3 introduces the model setup. Section 4 shows the calibration, and Section 5 presents the quantitative analysis. Section 6 concludes.

2 Empirical Motivation

Using proprietary data from CoreLogic and Black Knight, we compile a complete and granular dataset on household mortgage open liens (up to four liens), including first liens, which may include conventional mortgages. RMs are identified as "Mortgage Subordinate Type" loans.

The advantage of our dataset is that it covers actual mortgage and RM borrowers' property information, such as their historical and market value, housing type and condition, and all mortgage and RMs (initial balance, interest rate, terms, etc.) secured on the properties in the entire country during our research period, which covers almost all of the properties secured on such mortgages. Other datasets used in previous literature, such as Panel Study Income Dynamics (PSID) and Health Retirement Study (HRS), cover only a subset of the population. These types of surveys use estimation and various weighting methodologies that can sometimes bias the model results because RM is a relatively small market compared to conventional mortgages and other more developed financial products. This makes our contribution distinct and essential.

As depicted in Figure 1, we first notice that the RM take-up rate in the US has increased almost ten-fold, from approximately 0.3% in 1997 to 2.8% in 2021. Over the course of its development, RMs have experienced several major milestones. The first FHA-insured Home Equity Conversion Mortgage (HECM) was issued in 1989, followed by the first major news of HECM becoming a permanent product as established by the Housing and Urban Department (HUD) Appropriations Act in 1994. During the 2000s' housing market boom, demand for RM surged, such that homeowners took advantage of the housing value insurance embedded in the RM contract. At the same time, more lenders joined the RM game as HUD increased origination fees and engaged with the American Association of Retired Persons (AARP) to improve counselling services to boost the

RM market. More rules were relaxed so that refinancing existing HECM became possible. RMs have become ever more important for older adults in boosting their retirement incomes, and proprietary RMs specifically designed for those who are not eligible for HECM schemes have emerged in the market.

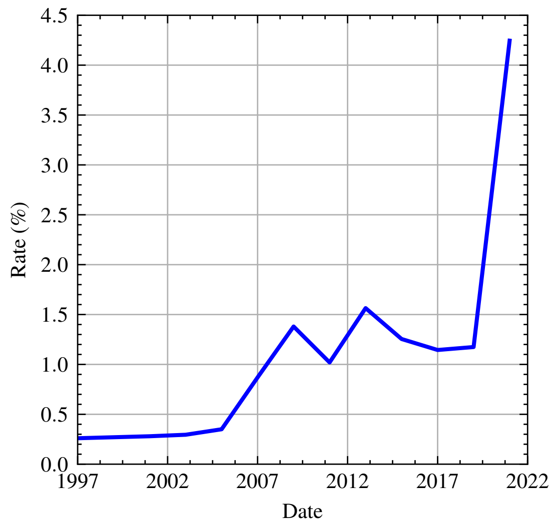

Figure 1: Take-up Rate of RM in the US

Who takes up RMs in the US? Figure 2 shows the number of RMs by borrower age and marital status in the US. The figure shows that younger retirees (those aged 62-70) and single retirees (unmarried) are more likely to take up RMs. Younger retirees, who have more years of retirement ahead of them, may opt for RMs to secure additional funds for their extended retirement period and their increased consumption needs. On the other hand, single retirees, who lack a spouse to share financial risks and typically have lower average wealth and retirement income compared to married retirees, may also seek RMs to supplement their available funds and enhance their consumption capacity.

A second observation in Figure 1 is the obvious drop (from 1.4% to 1%) from 2008 to 2011. Since the GFC in 2008, some lenders have left the market, and proprietary RM has disappeared for a while. This results in RM plan issuance in the US being reduced following the GFC (see Cocco and Lopes (2020)). A similar effect in the UK market is evident in Figure 3, Panel A, from 2007 to 2011, during which the market contracted by almost two-thirds. This is due to the correction of the US and international housing market following the previous artificial housing boom from

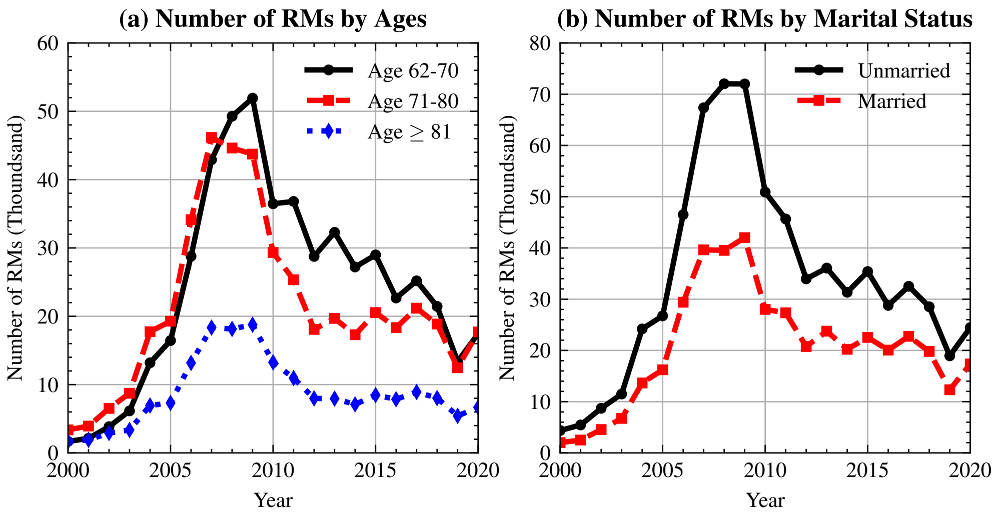

Figure 2: Number of RMs by Borrower Age and Marital Status in the US

the 1990s to 2006. RMs were hit harder because the lending criteria are mainly based on LTV, which heavily relies on housing market performance. The US RM market gradually recovered in 2011 and has significantly increased over the past decade. Moreover, the 2015 legislation allowing RM to be used in comprehensive cases such as a down payment for a property of an eligible senior homeowners' offspring boosts this market.

A third observation in Figure 1 is the uninterrupted growth in the US RM market following the COVID-19 crisis, in contrast to the market contraction following the GFC. In comparison, the UK RM market initially suffered a minor contraction (2019-2020) but quickly recovered from 2020 onwards, as seen in Figure 3, Panel A. The post-COVID-19 period saw high interest rates as the central banks increased the base rate multiple times in a short period of time in order to cool inflation caused by the prolonged ultra-low interest rate environment following the GFC. The empirical experience of international RM market growth following the COVID-19 crisis is in contraction to some prediction that usually heightened interest rates cool down the housing market and mortgage market due to higher (re)financing costs. However, the surge of RMs in the post-COVID-19 crisis supports the conclusion by Hurst and Stafford (2004) that enabling households to convert home equity into liquid assets re-adjusts households' decision to re-finance, even at the costs of higher borrowing costs. This is one of the reasons that RMs have become increasingly important in the household finance and credit literature, such that RMs provide a direct way to borrow against home value as an alternative to mortgage refinancing. For example, Chen et al. (2020) overshoot the average refinancing rate (11% by the model vs. 7% in the empirical data) and the size of cashouts conditional on refinancing (by nearly a factor of three). Such overestimation can be explained by the existence of RMs in the credit market that boost household refinancing activities. Another direct factor that boosts the UK RM market post-COVID-19 is the combination of a negative labor income shock, inflated living costs, and the energy crisis in Europe exacerbated by the war between Russia and Ukraine.

In summary, the two crisis periods (GFC in 2008 and COVID-19) have caused opposite RM market movements because of complex compounding effects of central bank base rates, housing market performances, and labor income instability. These dynamics on the household balance sheet and choices integrate LTV and LTI constraints and economic states, which are essential and highlight our contribution to this end.

At the same time, the recent boom (over the past decade) in international RM markets is captured in Figure 3 (all panels). The consistent growth pattern has already been observed in several developed countries, such as the UK, Australia, and Singapore (see details in the Appendix). All these countries share similar socio-economic, demographic shifting trend, pension systems, as well as financial market advancement and consumer financial literacy level.[9] As a high-level snapshot, Figure 3 Panels A and C show the number of new RM plans (including lump sums, draw-downs, withdrawals, and home reversion plans) in the UK and Hong Kong, respectively, during the 2005 to recent period. In comparison, Panel B shows the active number of RM plans in any given year in Australia for the same period, allowing expired plans to drop out from the dataset. In addition, according to Securities et al. (2018), from 2013 to 2017, lenders issued more than 17,000 RM plans. Hence, the market has the potential to reach up to 58,000 active plans in 2017, as represented by the dashed line from 2013 to 2017. All panels show significant growth in our studies during the past two decades. This signals strong and growing demand and supply of the RM products in these countries, although with some fluctuations (caused by crises), which shows potential for this product to be popular in other countries with a growing ageing population, housing market and homeownership rate, and the (lack of) social security system. This motivates our interest

[9] Singapore and Hong Kong do not have public pension systems, only private employers sponsored pension schemes. However, this does not affect our analysis.

in quantitatively assessing the impacts of RM on household decisions, the housing market, and credit market. Moreover, while the take-up rate is small in the US, the size of the RM market that it implies is nontrivial.

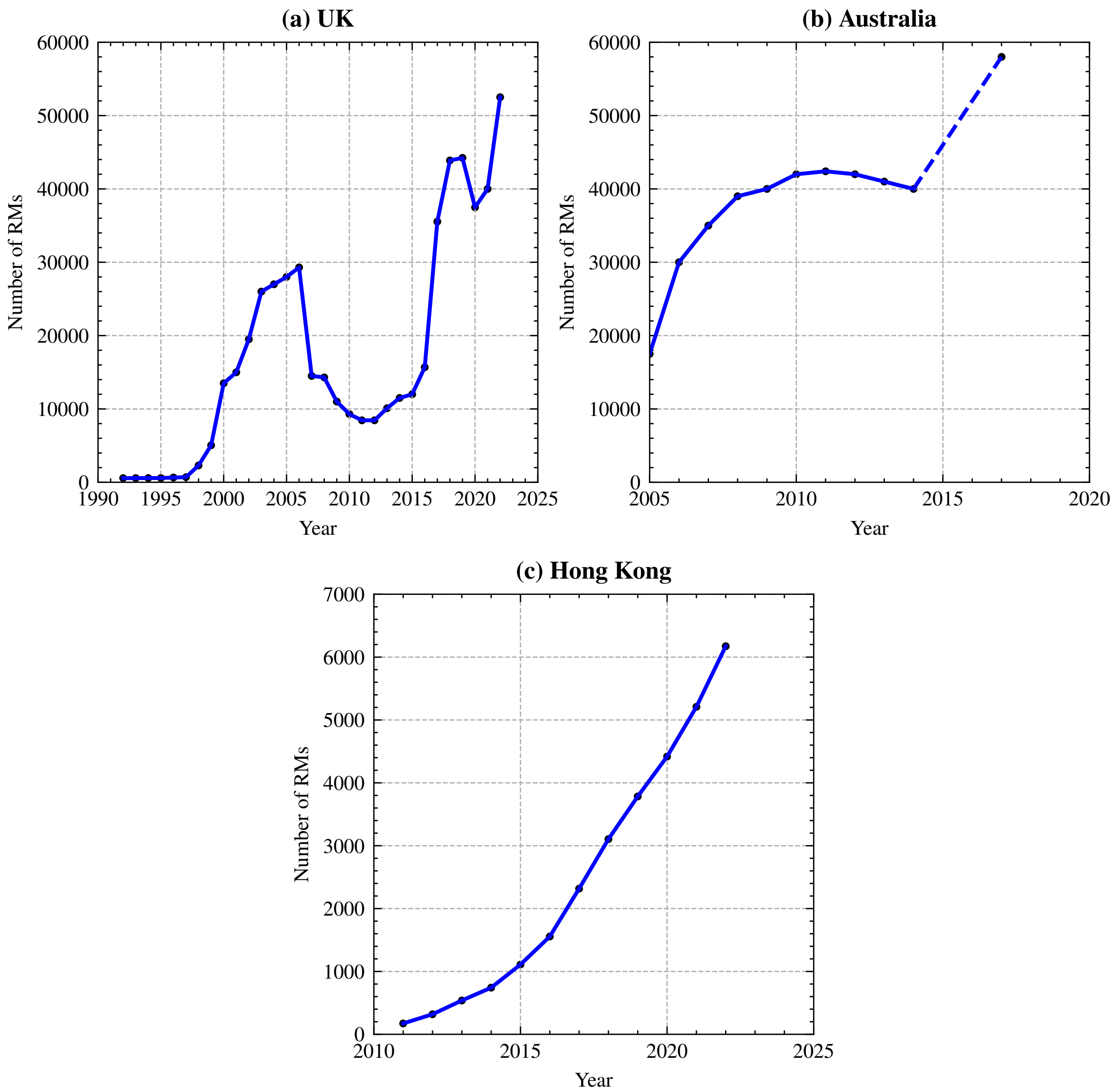

Figure 3: Number of RMs in Different Markets

Finally, Figure 4 shows the growing trends of the conventional mortgage rates, RM rates, and the housing price index (HPI) in the US from 2005 to 2025. It reveals that, as anticipated, RM rates generally exceed conventional mortgage rates to compensate for the riskiness of such instruments. Although fluctuating, declines have been notable during the past two decades, mainly because of increased lenders' appetite to lend and improved regulation and consumer knowledge

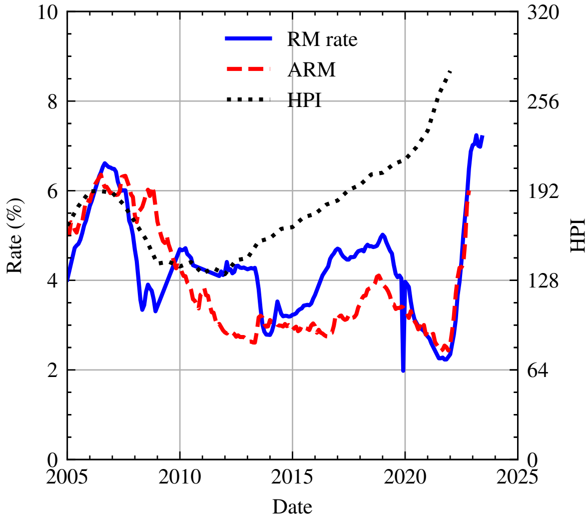

Figure 4: Mortgage Rates, RM Rates, and Housing Price Index in the US

of the product. In some periods, RM rates fall below conventional mortgage rates during housing and credit market frictions, for example, the period following the GFC in 2008, during which HPI dropped significantly, coupled with extra low-interest rates. The inherent reason for the lowered RM rate is driven by the low central bank base rate and the lack of appetite by consumers for borrowing and lenders for lending. RM rates recovered as the HPI rose in 2010 and stayed above conventional mortgage rates. Another period when RM rates experienced a sharp drop below conventional mortgage rates was the immediate aftermath of the COVID-19 crisis in late 2019., but it has quickly picked up and followed the pattern of ARM since then. This is a short-term negative shock on the RM rates, possibly because of the unstable market activities due to the COVID-19 crisis.

In recent years, a remarkable increase in the HPI has been accompanied by significant mortgage and RM rate decreases, shown in figure 4. This observation emphasizes the dynamic nature of these variables and calls for an integrated framework when considering RMs. Such a framework should consider the interplay between the mortgage market and the housing market, recognizing their simultaneous influence on RM dynamics.

3 The Model

We consider a general equilibrium model that includes heterogeneous households and competitive mortgage lenders, within which house prices and mortgage spreads are determined at equilibrium. Households have a finite living span divided into two phases: the working period and the retirement period. During the working periods, households earn labour income and pay taxes. During retirement, households receive social security. Households decide to rent or buy homes. Renters make decisions on consumption and saving. We allow renters to buy houses and finance housing through conventional mortgages during their working years. Homeowners choose how much to consume and save and whether to stay in their house or sell and become a renter. Additionally, they can borrow against their home equity using RMs during retirement. However, homeowners who use RMs cannot simultaneously borrow using a conventional mortgage.

Mortgage lenders price conventional mortgages and RMs. They receive the mortgage payments until households decide to default or to refinance. In the case of RMs, lenders recover the collateral when households die or move out of the property to long-term care facilities. Since our model abstracts from inflation dynamics, we consider a real economy in which both mortgages and RMs are inflation-indexed.

3.1 Economy

Our model is discrete, and each period corresponds to 1 year. In each time period t, the economy (θ_t) can be in crisis, recession, and expansion. We employ an exogenous transition probability matrix (P_θ) to describe the evolution between these macroeconomic conditions. The risk-free real interest rate ($r^f_{\theta_t}$) is also exogenous and depends on the economy. In expansions, the risk-free real interest rate is higher, while it hits the lowest point during crises.

3.2 Households

Following the convention in life-cycle models, we assume households start working at age 20, retire at 62, and can live up to 100 ($T = 80$). The probability that households are alive at age ($a + 1$) conditional on being alive at age a is denoted by p_a (p_{80} equal to 0).

Preferences Following Campbell and Cocco (2015), we assume household i derive utility from housing H_{it}, from nondurable consumption C_{it} and from leaving a bequest motive. Since house size remains fixed over the time, we can drop housing from the preference specification. On the other hand, household i is penalized for default. The recursive utility function of household i in period t can be written as[10]

$$U_{it} = \frac{C_{it}^{1-\gamma_i}}{1-\gamma_i} + u_o D_{it}^o - u_d D_{it}^d + \beta_i \left[p_{a_{it}} \mathbb{E}_t[U_{it+1}] + (1 - p_{a_{it}}) \psi^{bq} \frac{(Q_{it+1} + \xi)^{1-\gamma_i}}{1-\gamma_i} \right] \tag{1}$$

where γ_i is the coefficient of relative risk aversion, β_i is the discount factor, u_o is the utility benefit from owning a house, u_d is the utility penalty from defaulting on mortgages, D_{it}^o and D_{it}^d are indicator functions for owning a house and default on mortgages, respectively. a_{it} is the age of household i in period t, Q_{it} is the net worth of household i at the beginning of period t, ξ is a bequest motive shift parameter, and ψ^{bq} is the bequest motive weight.

Labor Income Before retirement, households' labor income is exposed to both aggregate shocks from the economic state and idiosyncratic shocks. Specifically, before the retirement period $R = 42$ (corresponding to the retirement age 62), the labor income at age t (Y_{it}) is given in logs ($y_{it} = log(Y_{it})$) by

$$y_{it} = v_t + \epsilon_{it}, \quad t < R, \tag{2}$$

where v_t is the aggregate labor income as a function of economics state θ_t and the idiosyncratic labor income shock ϵ_{it} follows a Markov process with transition matrix $P_y(\theta_t)$ as a function of the economic state θ_t.

For simplicity, we assume a linear income taxation rule. Working households' labor income is taxed at the constant tax rate τ_i.

Social Security Benefits We incorporate Social Security benefits in our model. Seniors receive these benefits after retirement, which amount to a fraction λ of their last working year's labor

[10]Since housing size H_{it} remains fixed in our model, we drop housing from the preference specification.

income:

$$Y_{it} = \lambda Y_{iR}. \quad t \geq R, \tag{3}$$

The benefits are not subject to income tax.In addition, Social Security follows a pay-as-you-go system. The benefits are financed by a payroll tax τ_s on labor income of working households. Thus, we have the following Social Security balance equation:

$$\int_{\Omega^W} \tau_s Y_{it} dt = \int_{\Omega^R} \lambda \tau_s Y_{iR} dt, \tag{4}$$

where Ω^W and Ω^R are the distributions of working and retired households, respectively. In our model, the labor income is exogenously given; thus, we can solve for the payroll tax rate τ_s from equation (4) and a given λ. We will discuss the effect of reverse mortgages on the Social Security system in the following sections.

Housing and Rental Markets The date t real housing price P_t^h is endogenously determined by the equilibrium of each period and depends on the economic state. In each time period, households can decide to be homeowners (denoted as $D_{it}^o = 1$) or renters (denoted as $D_{it}^o = 0$), and housing size H_{it} is fixed for all homeowners. We assume that homeowners must pay a per-period maintenance cost $C_{ma}(H_{it} \mid P_t^h)$, proportional to the value of the property

$$C_{ma}(H_{it} \mid P_t^h) = c_{ma} H_{it} P_t^h, \tag{5}$$

where c_{ma} measures the maintenance cost per unit of total property value. Households unable to pay the maintenance cost will be forced to sell their houses and become renters.

We assume that homeowners face moving shocks, which forces the homeowners to move out from their houses, capturing the probability p_m of moving involuntarily with moving cost $C_{ms}(H_{it} \mid P_t^h)$

$$C_{ms}(H_{it} \mid P_t^h) = k_{ms} + c_{ms} H_{it} P_t^h, \tag{6}$$

where k_{ms} captures the fixed cost and c_{ms} quantifies the variable moving cost per unit of the total property value.

Existing literature has shown that the rental market and owner-occupied markets are segmented. Rental housing can be produced and destroyed at a variable cost q, so in equilibrium, renters face a renting cost $C_r = q$. We do not differentiate the owner-occupied house price and the rental property price and only consider the equilibrium of the owner-occupied housing stock. This implies that in our model, the price-to-rent ratio varies with the housing price.

Mortgages, Refinancing and Default During the working period, households pay a downpayment (deposit) to buy houses. Let ζ_t^m denote the downpayment as a proportion of the house value. It is subject to the maximum LTV constraint (ϕ_t^m). The initial balance M_{it}^m for the household i at the beginning of the period t satisfies

$$M_{it}^m = (1 - \zeta_t^m) P_t H_{it}. \tag{7}$$

A lower proportional downpayment leads to a higher LTV, but it can not exceed ϕ_t^m:

$$\zeta_t^m \geq 1 - \phi_t^m. \tag{8}$$

We assume that ϕ_t^m varies exogenously and depends on the macroeconomic states.

Two main types of mortgage contracts are available on the conventional mortgage market: fixed-rate mortgages (FRM) and adjustable-rate mortgages (ARM). For simplicity, we only model one type of the main residential mortgage: the long-term FRM. Households with an FRM bear the same mortgage rate $r^m(\theta_t)$ determined by the economic state θ_t at origination. We assume that FRM rates stay constant for all households in the same economic states.

With the timing assumption that households pay their interest between periods t and $t+1$ in advance at time t, the minimum payment on a mortgage for an agent who does not move or refinance at age a_{it} is given by:

$$M_{it-1}^m - M_{it}^m(1 - r^m(\theta_t)) \geq M_{it-1}^m \frac{r^m(\theta_t)\left(1 + \left(\frac{r^m(\theta_t)}{1 - r^m(\theta_t)}\right)^{T-a_{it}+1}\right)}{\left(\frac{r^m(\theta_t)}{1 - r^m(\theta_t)}\right)^{T-a_{it}+1} - 1}. \tag{9}$$

We assume the mortgage's maturity is $T = 80$ for all households.

When initial interest rates are high and later drop, households can refinance. If households decide to refinance, they face LTV constraint (8) and pay for refinancing costs

$$C_{re}(M_{it+1}^m) = k_{re} + c_{re}M_{it+1}^m, \tag{10}$$

where k_{re} is the fixed refinancing cost and c_{re} is the variable refinancing cost per unit of the mortgage balance. Households can refinance their mortgage at the beginning of each period, and the refinancing cost is paid at the time of refinancing.

If households choose to default, they are guaranteed a lower bound per-period cash-on-hand, which can be viewed as a subsistence level. This assumption is motivated by the existence of social welfare programs such as means-tested income support. Defaulting households are temporarily barred from the credit markets. However, their default status may be lifted, enabling them to re-enter the housing market using mortgages.

Reverse Mortgages RMs allow older homeowners to borrow against their housing wealth without moving out of the house. This option also offers insurance against substantial declines in house prices. In our model, we incorporate this feature, following the guidelines established by the Home Equity Conversion Mortgage (HECM) program, the most popular RM program in the US.

Although HECM provides several types of repayment options for borrowers to choose from, we only focus on the two most common types of RMs: 1) the lump sum RM, which is a single large payout at the start of the loan, and 2) the drawdown RM, which is a stream of unscheduled payments at the borrower's request until the line of credit is exhausted. Households can also combine the two types of RMs, retaining a line of credit for future use while initially taking a lump sum. In addition, households can choose from fixed rate RMs (FRRM) or adjustable rate RMs (ARRM). Only lump sum RM is available for FRRM.

To borrow against the home value, households must be at least 62 years old and have significant home equity. We allow households to borrow RMs with an outstanding mortgage balance and must pay off the existing mortgage when closing the origination of RMs. Specifically, households must satisfy the following conditions when initiating an RM:

$$1 - \frac{M_{it}^{m}}{P_t^h H_{it}} \geq \phi_{own}^{rm}(\theta_t), \tag{11}$$

where $\phi_{own}^{rm}(\theta_t)$ is the minimum equity requirement for RMs determined by the economic state θ_t.

Households are also constrained by the maximum amount of RM they can borrow, which is determined by the lower value between a constant maximum claim amount (Q^{rm}) and the principal limit based on the home value. According to HECM, the principal limit factor ϕ^{rm} is determined by the age of the borrower a_{it} and the risk-free rate. Let M_{it}^{rm} denote the amount a household chooses to claim, including the initial lump sum and credit. Therefore, M_{it}^{rm} should satisfy

$$M_{it}^{rm} \leq \min\left\{Q^{rm}, \phi^{rm}(a_{it}, r^f(\theta_t))P_t^h H_{it}\right\}. \tag{12}$$

If households decide to take an ARRM, they can determine the proportion of the total amount they wish to receive as a lump sum, denoted by $\alpha_{it}^{rm} M_{it}^{rm}$ at origination, while the remaining amount will be available in the form of a line of credit. Given that the amount M_{it}^{rm} includes the cost c_0^{rm}, which comprises the initial mortgage insurance cost and origination fee, households will receive $(\alpha_{it}^{rm} - c_0^{rm})M_{it}^{rm}$ at closing of the origination. If the household chooses to take an FRRM, α_{it}^{rm} is fixed to be 1, indicating that the entire amount, M_{it}^{rm}, will be received as a lump sum.

Upon origination, households make decisions whether to draw down the credit line at each period if the credit line is not exhausted ($\alpha_{it}^{rm} < 1$). They are also allowed to prepay the RM balance. The remaining RM balance and credit line both accumulate with the RM rate $r^{rm}(\theta_t)$ and the annual loan fee rate c_a^{rm}, including annual insurance cost and annual service cost.[11] Specifically,

[11] We calibrate the RM rates for FRRM and ARRM separately.

after origination, households face the following constraints:

$$M_{it}^{rm} \leq M_{it-1}^{rm}(1 + r^{rm}(\theta_{t-1}) + c_a^{rm}), \tag{13}$$

$$(1-\alpha_{it}^{rm})M_{it}^{rm} \leq (1-\alpha_{it-1}^{rm})M_{it-1}^{rm}(1 + r^{rm}(\theta_{t-1}) + c_a^{rm}). \tag{14}$$

The second constraint suggests that households can not adjust the remaining credit threshold according to their preferences.

RMs terminate whenever the borrowers move out to long-term care facilities or decrease, and RM lenders get paid for the roll-up RM balance or the home value, whichever is lower. Usually, lenders will sell the house to recover the loan balance, and the remaining residue home value is inherited by the borrowers' heirs. There is a possibility that a housing market slump could result in a decrease in the property portfolio value of lenders, leading to potential losses on RMs upon termination. As HUD insures the potential loss, the mortgage rate R_t^{rm} includes both RM spread and insurance premiums.

Health Risk and Medical Cost During retirement, households are exposed to health risks. We introduce a health status indicator, D_{it}^{med} to capture this risk. It takes a value of 1 if households are in good health and a value of 0 if they are sick. Once households become sick, they will remain in that state for the rest of their lives. We assume that the health state D_{it}^{med} of household i in period t follows a Markov process independent of the economic and idiosyncratic states. The probability of transitioning from a good to a bad state is assumed to be 0.2 in each period.

A senior household i pays medical expenses of $c_{med}(a_{it}, D_{it}^{med})Y_{it}$, where c_{med} represents the proportion of income and is determined by the health state D_{it}^{med} and the age a_{it}.

Households Optimization Problem Households are price takers who maximize their lifetime utility, given their expectations about future economic state and housing prices. We denote the aggregate state as a vector $\Theta_t = \{\theta_t, r^f(\theta_t), P_t^h, r^m(\theta_t), r^{rm}(\theta_t), \Pi_t\}$, where Π_t is the distribution of households.[12] Household i's idiosyncratic state at the start of each period t before making decisions is $S_{it} = (a_{it}, Y_{it}^{id}, W_{it}, D_{it}^o, D_{it}^{mov}, D_{it}^{med}, M_{it}^m, r_{it}^m, D_{it}^d, D_{it}^{rm}, M_{it}^{rm}, D_{it}^{fix}, r_{it}^{rm}, \alpha_{it}^{rm})'$, a vector

[12]The distribution of households affects the equilibrium and, thus, should be included in the aggregate state. The distribution is over the idiosyncratic states of households.

of age, idiosyncratic labor income, saving, homeownership status, moving shocks, health state, mortgage balance, the mortgage accumulated rate determined by the economic state when the mortgage originated, default status, RM availability, and the RM claim amount,[13] indicators for FRRM, the RM accumulated rate (r_{it}^{rm}),[14] and the share of the RM claim amount retrieved.

In each period t, household i makes decisions $A_{it} = (D_{it}^H, W_{it+1}, M_{it+1}^m, M_{it+1}^{rm}, D_{it+1}^{fix}, \alpha_{it+1}^{rm})'$ of housing (D_{it}^H), saving (W_{it+1}), mortgage and RM (M_{it+1}^m, M_{it+1}^{rm}, $D_{it+1}^{fix}, \alpha_{it+1}^{rm}$), to maximize the lifetime utility. The housing decision, represented as D_{it}^H, is characterized by a range of values from -2 to 4, each representing a distinct decision. Renters have the option to continue renting ($D_{it}^H = 0$), while those without a default flag can choose to purchase a house ($D_{it}^H = 3$). On the other hand, homeowners have the choice to retain ownership ($D_{it}^H = 1$), default on their mortgage ($D_{it}^H = -1$), refinance the mortgage ($D_{it}^H = 4$), or sell the house and become renters ($D_{it}^H = -2$),[15] or move to a new house ($D_{it}^H = 2$). However, homeowners who face a moving shock have only two options: either move or default.

The optimization problem is then

$$V(S_{it}, \Theta_t) = \max_{A_{it}} U_{it}(S_{it}, A_{it}, \Theta_t), \tag{15}$$

subject to

$$Q_{it}(S_{it}, \Theta_t) = W_{it} - M_{it}^m + D_{it}^o \max(H_{it}P_t^h - \alpha_{it}^{rm} M_{it}^{rm}, 0), \tag{16}$$

$$\begin{aligned} C_{it}(S_{it}, A_{it}, \Theta_t) = & \left[Y_{it} - \tau(Y_{it}) - \phi_c(S_{it}, A_{it}, \Theta_t) - \left(M_{it}^m - (1 - r_{it}^m)M_{it+1}^m\right)\right] \\ & + \left[\frac{W_{it} - W_{it+1}}{1 + r^f(\theta_t)}\right] + (D_{it+1}^o - D_{it}^o)P_t^h \\ & + \left[(\alpha_{it+1}^{rm} - c_0^{rm}\mathbb{1}(M_{it}^{rm} = 0))M_{it+1}^{rm} - \alpha_{it}^{rm}(1 + r_t^{rm} + c_a^{rm})M_{it}^{rm}\right], \end{aligned} \tag{17}$$

$$\phi_c(S_{it}, A_{it}, \Theta_t) = D_{it+1}^o C_{ma} + (1 - D_{it+1}^o)q + D_{it}^m C_{ms} + \mathbb{1}(D_{it}^H = 4)C_{re}, \tag{18}$$

$$C_{it}(S_{it}, A_{it}, \Theta_t) > 0, \tag{19}$$

[13] It is worth noting that if a household is unaware of RMs, they will never consider them.

[14] For FRRM, r_{it}^{rm} depends on the economic state when the RM originated, and for ARRM, r_{it}^{rm} is the current rate.

[15] In this case, households will need to pay off remaining mortgage.

$$D^o_{it+1} = \begin{cases} 1, & \text{if} \quad D^H_{it} > 0 \\ 0, & \text{if} \quad D^H_{it} \leq 0 \end{cases}, \tag{20}$$

$$D^d_{it+1} = \begin{cases} 1, & \text{if} \quad D^H_{it} = -1 \\ 0, & \text{if} \quad D^H_{it} \neq -1, D^d_{it} = 0 \\ 0 \quad \text{with probability } p_{md}, & \text{if} \quad D^H_{it} \neq -1, D^d_{it} = 1 \end{cases}, \tag{21}$$

$$M^m_{it+1} = 0, \quad \text{if} \quad D^H_{it} \leq 0, \tag{22}$$

$$\text{LTV constraint} \quad (8), \quad \text{if} \quad D^H_{it} = 2, 3, 4, \tag{23}$$

$$\text{Minimum payment} \quad (9), \quad \text{if} \quad D^H_{it} = 1, \tag{24}$$

$$M^{rm}_{it+1} = 0, \quad \text{if} \quad D^H_{it} \neq 1, \tag{25}$$

$$\text{RM maximum claim} \quad (12), \quad \text{if} \quad D^H_{it} = 1, a_{it} \geq R, M^{rm}_{it} = 0 \text{ and } (11) \tag{26}$$

$$\alpha^{rm}_{t+1} = 1, \quad \text{if} \quad D^{fix}_{it} = 1 \text{ or } D^{fix}_{it+1} = 1 \tag{27}$$

$$\text{RM balance accumulation} \quad (13), \quad \text{if} \quad D^H_{it} = 1, M^{rm}_{it} > 0 \tag{28}$$

$$\text{RM credit accumulation} \quad (14), \quad \text{if} \quad D^H_{it} = 1, M^{rm}_{it} > 0. \tag{29}$$

where $\phi_c(S_{it}, A_{it}, \Theta_t)$ is the total cost of housing (maintenance), moving and financial services (refinance),[16] and we impose the budget constraint and positive consumption constraint. The existence of social welfare programs guarantees that households can maintain a minimum level of consumption C_m through feasible decisions. However, to qualify for such welfare programs, households must have no savings and either be renters or decide to become renters.

3.3 Mortgage Lenders

Lenders are perfectly competitive and discount loan pay-offs using a stochastic discount factor. We assume that this stochastic discount factor depends on the current macroeconomic state,

$$d(\theta_t) = \frac{1}{1 + r^f(\theta_t) + \kappa(\theta_t)}. \tag{30}$$

[16] The cost of RMs is included in the balance and loan rate.

where $\kappa(\theta_t)$ is the expected risk premium of lenders.

Lenders hold the mortgages and RMs portfolio and receive the payments from borrowers in each period t. These payments depend on the borrowers' decisions (A_{it}) and the aggregate state (Θ_t). Given the multi-period nature of the loan, lenders need to compute the net present value of loans recursively. The net present value of the expected payments from the mortgage originated to household i is given by

$$\begin{aligned}\Pi^m(S_{it}, A_{it}, \Theta_t) =&(1 - p_{a_{it}}) \min(M^m_{it-1}, W_{it} + H_{it}P^h_t) + p_{a_{it}} \Big[\mathbb{1}(D^H_{it} = -2)M^m_{t-1} \\ &+ \mathbb{1}(D^H_{it} = -1)\Upsilon H_{it}P^h_t + \mathbb{1}(D^H_{it} > 0) \left(M^m_{it-1} - M^m_{it}(1 - r^m_{it})\right) \\ &+ \mathbb{E}_t \left[d(\theta_{t+1})\Pi^m(S_{it+1}, A_{it+1}, \Theta_{t+1})\right]\Big] \end{aligned} \tag{31}$$

where Υ is the foreclosure sale recovery rate in case the household defaults on the mortgage. Equation (31) consists of two terms that represent the expected benefits in different scenarios. The first term represents the expected payoff if the household passes away in period t, while the second term represents the payoff if the household is alive until the end of the period, including the present payoff of moving, defaulting, or staying in the house and the expected payoff in the next period.It is important to note that if households move, default or reach the maximum age of 100, the net value for the next period $\Pi^m(S_{it+1}, \Theta_{t+1})$ will be zero.

Besides mortgages, lenders also issue RMs. Similarly, lenders calculate the net present value $\Pi^{rm}(S_{it}, A_{it}, \Theta_t)$ of the expected payments from the RM originated to household i,

$$\begin{aligned}\Pi^{rm}(S_{it}, A_{it}, \Theta_t) =&(1 - p_{a_{it}}) \min(M^{rm}_{it-1}, H_{it}P^h_t) + p_{a_{it}} \Big[\mathbb{1}(D^H_{it} = -2, 2) \min(M^{rm}_{it-1}, H_{it}P^h_t) \\ &+ \mathbb{1}(D^H_{it} = 1) \left(\alpha^{rm}_{it-1}M^{rm}_{it-1}(1 + r^{rm}_{it} + c^{rm}_a) - (\alpha^{rm}_{it} - c^{rm}_0\mathbb{1}(M^{rm}_{it-1} = 0))M^{rm}_{it}\right) \\ &+ \mathbb{E}_t \left[d(\theta_t)\Pi^{rm}(S_{it+1} \mid \Theta_{t+1})\right]\Big]. \end{aligned} \tag{32}$$

Equation 32 considers two scenarios: one where households have a probability of $p_{a_{it}}$ of passing away, and the other where households continue to live. If households move or reach the maximum age of 100, the next period's net value $\Pi^{rm}(S_{it+1}, A_{it+1}, \Theta_{t+1})$ will be zero.

3.4 Equilibrium

The economy is characterized by the law of motion for the aggregate state $\Theta_t = \{\theta_t, r^f(\theta_t), P_t^h, r^m(\theta_t), r^{rm}(\theta_t), \Pi_t\}$. Households choose their consumption and housing decisions (A_{it}) to maximize their expected life-time utility, given their expectations about future housing prices.

Solving the equilibrium requires households to forecast the law of motion for Θ_t, which includes an infinite-dimensional object Π_t. The law of motion for the other components of Θ_t all depend on θ_t, which is a Markov process and transition probabilities are exogenous. To address the problem, we follow Krusell and Smith (1998), as is standard in the literature. We focus directly on the law of motion for house prices and consider a forecasting model where households use a simple AR(1) forecast rule depending on the current and next-period economic states:

$$\ln P_{t+1}^h = f_{(\theta_t,\theta_{t+1})}(\ln P_t^h). \tag{33}$$

We parameterize f as a linear spline:

$$f_{(\theta_t,\theta_{t+1})}(x) = b_h + \sum_{i=1}^{n_h} k_{ih}(x - x_i), \tag{34}$$

where b_h, k_{ih} depends on (θ_t, θ_{t+1}), and x_i are the knots of the spline.

Housing market clear:

$$\int D_{it}^o d\Pi_t = \int D_{it+1}^o d\Pi_{t+1}. \tag{35}$$

Household expectations for housing prices can be verified in equilibrium.

Meanwhile, mortgage lenders reach a break-even point with the mortgage and RM rates:

$$\mathbb{E}_{\theta_t=\theta_i}\left[\mathbb{E}_{S_{jt}\sim\Pi_{m,t}^{orig}}\left[d(\theta_t)\Pi^m(S_{jt+1}, A_{jt+1}, \Theta_{t+1}) - (1 - r^m(\theta_t))M_{jt+1}^m\right]\right] = 0, \forall\theta_i, \tag{36}$$

$$\mathbb{E}_{\theta_t=\theta_i}\left[\mathbb{E}_{S_{jt}\sim\Pi_{rm,t}^{orig}}\left[d(\theta_t)\Pi^{rm}(S_{jt+1}, A_{jt+1}, \Theta_{t+1}) - \frac{(1-\alpha_{jt+1}^{rm})M_{jt+1}^{rm}}{1 + r^{rm}(\theta_t)}\right]\right] = 0, \forall\theta_i, \tag{37}$$

where $\Pi_{m,t}^{orig}$ and $\Pi_{rm,t}^{orig}$ are the distribution of newly originated mortgages and RMs in period t, respectively.

Denote the endogenous parameters of the equilibrium as $\Xi = (b_h, k_{ih}, r^m, r^{rm})^T$.[17] We further solve the equilibrium numerically. First Initialize Ξ as Ξ_0, and for $n = 1, 2, \cdots$

i) Solve the household's optimization problem under Ξ_{n-1}.

ii) Simulate an economy with a population size of 100,000 for 19,000 periods with the home price decided by (35) each period.[18]

iii) Estimate Ξ as Ξ_n by (34), (36) and (37) based on the simulation results.

iv) Terminate if:

$$||\Xi_n - \Xi_{n-1}||_\infty < \epsilon. \tag{38}$$

4 Calibration

We calibrate our baseline model with RMs using several data sources, including mainly macroeconomics and health & retirement statistics. We adopt a two-stage strategy to estimate the model, each with a distinct purpose. Within our models, there are sets of variables related to macroeconomic states, human mortality and survival functions, which are exogenous variables, as well as the initial mortgages and RMs interest rates. There is another set of variables inherently estimated from our model setting, such as mortgage and RMs interest rates at equilibrium states, as well as household labour incomes and their decisions on consumption, financial and housing conditions, etc.

In the first stage of estimation, we calibrate all the stand-alone parameters that can be clearly identified without explicitly referring to our model setting. For example, mortality and survival probabilities are exogenous variables that do not depend on our model. Therefore, we adopt such statistics published by the National Center for Health Statistics to parameterize the conditional survival probabilities in our transition matrix. In the second stage, we estimate the remaining parameters to align with specific moments of the empirical data, ensuring an overall goodness-of-

[17] There are n_h and different r^m, r^{rm} for three economic states; thus, the dimension of Ξ is $n_h + 4$.

[18] If a household dies, a new household aged 20 with no assets will enter the economy.

fit between the model and empirical observations. Table 1 summarizes the values of the calibrated parameters.

Aggregate State Variables The first-stage calibration process aims to simulate macroeconomic environments such as peaks, troughs, recoveries and recessions in our model. In this process, we first choose aggregate and idiosyncratic shocks that mimic the dynamics of modern business cycles in the US. subsequently, we exogenously calibrate a set of parameters to widely accepted values in the macroeconomic and housing literature.

Following Guren et al. (2021), we calibrate the Markov transition matrix between macroeconomic states such as crisis, recession, and expansion based on the frequency and duration of such events defined by NBER. Accordingly, crises happen every 75 years, and all other NBER recessions are regular, cyclical recessions. Moreover, the economy switches to expansion following a crisis or recession. Another assumption is that if the NBER peak of the previous expansion takes place in the first half of a given year, that year is classified as the first year of the new recession, whereas if the peak happens during the second half of a year, the recession follows in the subsequent year. The ending date of a recession is defined as the year following the start year of the expansion announced by the NBER because the unemployment rate is a lagging variable and does not immediately fall after NBER troughs. According to this definition, recessions occurred during 1991-1992, 2001-2002, 2008-2010, and 2020.

Subsequently, we calibrate the lender's Stochastic Discount Factor (SDF) to match the spread between loan rates and the interest rate. The lenders charge a premium (κ) depending on the macroeconomic state:

$$m(\theta_t) = \frac{1}{1 + r^f(\theta_t) + \kappa(\theta_t)} \tag{39}$$

where r_t^f is the riskless rate in state θ_t. Lenders impose a requirement that guarantees a certain return on investment for a certain payoff of one unit at date $t+1$. This return is determined by adding the cost of making mortgage loans, denoted as κ, to the risk-free rate R_t^f. In this case, we have set κ to 125 basis points (bps) to ensure that the average difference between the FRM rate and a 10-year bond is 1.65%.

Table 1: Model Parameters in Baseline Parameterization

The table shows parameters for the baseline calibration. Average income is normalized to one. There are five aggregate states, $\theta_t \in$ {Crisis, recession, expansion}, but we assume that income and monetary policy are the same in a recession with loose or tight credit and in an expansion with loose or tight credit. The tuples of interest rates reflect the interest rate in a crisis, recession, and expansion, respectively.

Description	Parameter value
Panel A. Life-cycle parameters	
Years in life (T)	80
Retirement age (R)	62
Income tax rate (τ_i)	0.06
Social Security ratio (λ)	0.35
Social Security tax rate (τ_i)	0.04
Prob. of moving, working (p_{ms})	1/9
Prob. of moving, retiring (p_{ms})	0.02
Medical cost rate (c_{med}) (Age $[65, 85)$)	0.15(good), 0.2(bad)
Medical cost rate (c_{med}) (Age $[85, 100)$)	0.25(good), 0.35(bad)
Panel B. Preference parameters	
Risk aversion (γ)	3.0
Discount factor (β)	0.98
Bequest motive shifter (ξ)	0.5
Bequest motive multiplier (ψ^{bq})	250
Utility from homeownership (u_o)	10
Default penalty (u_d)	10
Panel C. Mortgages	
Max LTV in crisis (ϕ^m)	0.8
Max LTV in recession (ϕ^m)	0.85
Max LTV in expansion (ϕ^m)	0.90
Prob. of default flag removed (p_{md})	0.1
Foreclosure sale recovery (Υ)	0.654
Panel D. Reverse mortgages	
Max LTV, RM (ϕ^{rm})	0.50
Initial fee of RM (c_0^{rm})	0.02
Annual fee of RM (c_a^{rm})	0.01
Variable moving cost rate (c_{ms})	3.0%
Fixed moving cost (k_{ms})	0.1
Variable refinance cost rate (c_{re})	1.0%
Fixed refinance cost (k_{re})	0.04
Maintenance cost rate (c_{ma})	0.025
Panel E. Economy parameters	
Short rate (r)	$[1.26\%, 2.32\%, 4.26\%]$
Aggregate income ($Y^{(agg)}$)	$[0.0976, 0.1426, 0.1776]$
Rent cost(q)	0.20
Homeownership rate (H_r)	0.65

To determine the risk-free rate, we calibrate it based on the historical real interest rate observed between 1985 and 2022. Specifically, during recessions, the risk-free rate is set at 2.32%, reflecting the lower economic activity and market conditions. During expansions, when the economy is performing well, the risk-free rate is set at 4.26%. In the case of a crisis, we assume a significantly lower risk-free rate of 1.26% to account for the heightened uncertainty and economic instability.

Labor Income Process Another condition in our model that needs calibration is the labor income process, in which the replacement ratio during retirement is set to 0.68, and the deterministic component of the labor income process is set to be the same as that in Cocco et al. (2005). We use 0.1^2 for the transitory variance, similar to the value used in Gourinchas and Parker (2002). For permanent income shocks, we rely on the estimates in Guvenen et al. (2014) and Shen (2022), who estimate a quantitative labor income model using a large and confidential US dataset. We allow skewness to depend not only on the business cycle but also on expected growth rates. The moments of permanent income shocks can be easily calculated based on these estimates, and therefore, the parameters with respect to the mixture of normal distributions during expansions and recessions can be calibrated. We then estimate the remaining moments to match the first four moments during expansions and the first four moments during recessions. We adjust the income process to ensure that the average aggregate income equals 1, as per the normalization convention. The tax system is calibrated following the method proposed in Heathcote et al. (2017).

RMs We calibrate all the RM parameters and age-dependent collateral constraints based on the existing RM contracts. For example, we use 2% for the insurance premium and 1% for the annual fee. Both are taken from the HECM. Table 2 shows the principal limit factor for RMs from HECM. For example, for the age group 60-65 and RM interest rate of 4.26%, homeowners can only borrow 45.4% of their home equity. Our primary source of data on mortgage performance is CoreLogic and Black Knights. The RM interest premiums are endogenized, depending on the aggregate states and conventional mortgage rates. Unlike conventional mortgages, the amounts of loans given depend on the ages of the borrowers, which increase as they age. The intuition is that as the borrowers' ages increase, the anticipated interest costs decrease, and the expected future house price appreciation diminishes, lowering the overall risk for the lenders. In general, the collateral constraint related to conventional mortgages becomes more stringent as the borrowers age. In contrast, RMs provide a more relaxed collateral constraint that loosens with age, resulting in a more favourable borrowing environment at all stages of life.

Table 2: Principal Limit Factor for RMs

	Age						
RM rate	60-65	65-70	70-75	75-80	80-85	85-90	90-95
4.26%	0.454	0.474	0.507	0.533	0.572	0.624	0.681
4.32%	0.454	0.474	0.507	0.533	0.572	0.624	0.681
5.26%	0.396	0.417	0.452	0.479	0.522	0.58	0.644

Preferences, Housing and Mortgages In the second-stage estimation, we estimate the rest of the parameters related to households' consumption finance and housing decisions using the method of simulated moments (MSM). In particular, we find the set of parameters that yield the simulated life-cycle decision profiles that best match the profiles from the empirical data. The mechanics of our MSM approach are fairly standard. We compute life-cycle histories for a large number of artificial households. Each of these households is endowed with a specific value for the state vector, and a series of idiosyncratic shocks are assigned to them in a manner consistent with the stochastic processes explicated in Section 3.

To facilitate the computational process, we discretize the state space and employ value function iteration to solve the model numerically. This iterative procedure enables us to derive a set of decision rules that govern the choices made by the households. By combining these decision rules with the simulated endowments and shocks, we are able to simulate various aspects of household behavior, including wealth accumulation, labor income, housing decisions, refinancing, and the decision to take up RMs. Subsequently, we compute age profiles based on the artificial household histories, employing the same methodology used to analyse the actual data. We adjust until the difference between the data and simulated profiles is minimized as follows

$$(\hat{m} - m)' W (\hat{m} - m), \tag{40}$$

where $\hat{m}$ refers to the simulated moments, m refers to the targets, and W is the inverse of the covariance matrix of the empirical moments, which is estimated by bootstrapping the true data.

The empirical evidence we are trying to match our model parameters is from the American Housing Survey (AHS) and the Survey of Consumer Finances (SCF). We assume that households start working at age 20, retire at age 65, may live up to age 100, and have CRRA type of utility. The estimated discount factor β is 0.98, which is within the accepted range of estimates in models of this kind. The coefficient of relative risk aversion is 3, which is in the middle of the spectrum in the literature.[19] We choose the bequest motive parameters to match the ratio of total net worth at age 60 to net worth at age 45 in the SCF.

Household moving and refinancing processes are associated with fixed and variable costs. Specifically, we have established the fixed cost of moving to be equivalent to 10% of the average household annual income, approximately amounting to $5,000. Additionally, proportional costs, incurred by both buyers and sellers, are set at 3% of the house value to account for expenses such as closing costs and realtor fees. Refinancing entails a fixed cost equal to 4% of the average annual income, roughly equating to $2,000, along with a variable cost equivalent to 1% of the remaining mortgage balance. This variable cost aligns with the average refinancing costs reported by the Federal Reserve.

For renters, a monthly rent payment is set at 20% of their income to maintain a rent-to-income ratio of 20%. On the other hand, homeowners are required to contribute a maintenance cost of 2.5% of the house value annually. To ensure a realistic moving frequency, we calibrate the moving shock parameter in a manner that results in homeowners moving on average every nine years, as observed in the AHS. Finally, the homeownership rate is adjusted to match the long-term average homeownership rate of 65% in the United States. To be consistent with the average homeownership in the AHS, we assign a utility benefit of 10 to owning a home.

Following a foreclosure event, we assume that the bank recovers 65% of the property's market value. Additionally, we consider different maximum LTV in different macroeconomic states. Under crisis, the maximum LTV at origination is set at 80%, meaning borrowers can obtain a loan up to 80% of the property's value. When the economy is in a better state, the maximum LTV is

[19] Yogo (2016) uses Epstein-Zin preferences to study the portfolio choice in retirement with health risk and derives the risk aversion coefficient to be 5. Nakajima and Telyukova (2020) only focus on retirees aged 65 and above and calibrate the risk aversion to be 2 to fit the age profiles in the Health and Retirement Study (HRS).

increased to 85% (90%) in recession (expansion). To replicate the observed foreclosure rate of 8% in the housing stock between 2006 and 2013, we set the default penalty to 10.

5 Quantitative Analysis

In this section, we first investigate whether the model can quantitatively match homeownership, wealth accumulation (decumulation), and other life-cycle patterns and compare the model with and without RMs. Then, we study the take-up rate and welfare.

5.1 Life-cycle Profiles

We simulate the decisions of saving, consumption, housing, and RMs of 100,000 households over the life cycle and present the average profiles in Figure 5. For comparison, we also report the average profiles in the SCF from 2007 to 2019. The model matches the life-cycle patterns in the SCF data well.

In Panel A of Figure 5, we present the homeownership rate patterns over the life cycle. The model's simulations yield an average homeownership rate of 65%, which aligns with the data. However, we observe that the model tends to underestimate the homeownership rate for very young households and overestimate it for middle-aged households.

Panels B and C of Figure 5 show the LTV and PTI ratio by age. Our model effectively captures the LTV ratios for young homeowners; however, it tends to overpredict the LTV ratios for older homeowners and increases steadily during retirement. This discrepancy can be primarily attributed to including RMs in our analysis. RMs allow older homeowners to borrow against their home equity, which increases their available funds without the need to sell the property. When older homeowners opt for RMs, they receive funds based on the accumulated value of their homes. This influx of borrowed funds increases the overall value of available resources for these homeowners, resulting in a higher loan amount in the LTV ratio calculation. At the same time, since RMs do not require regular repayments like traditional mortgages, the property value remains relatively unchanged. As a result, the LTV ratio for older homeowners increases, indicating a higher proportion of borrowing relative to the value of their homes. Moreover, we

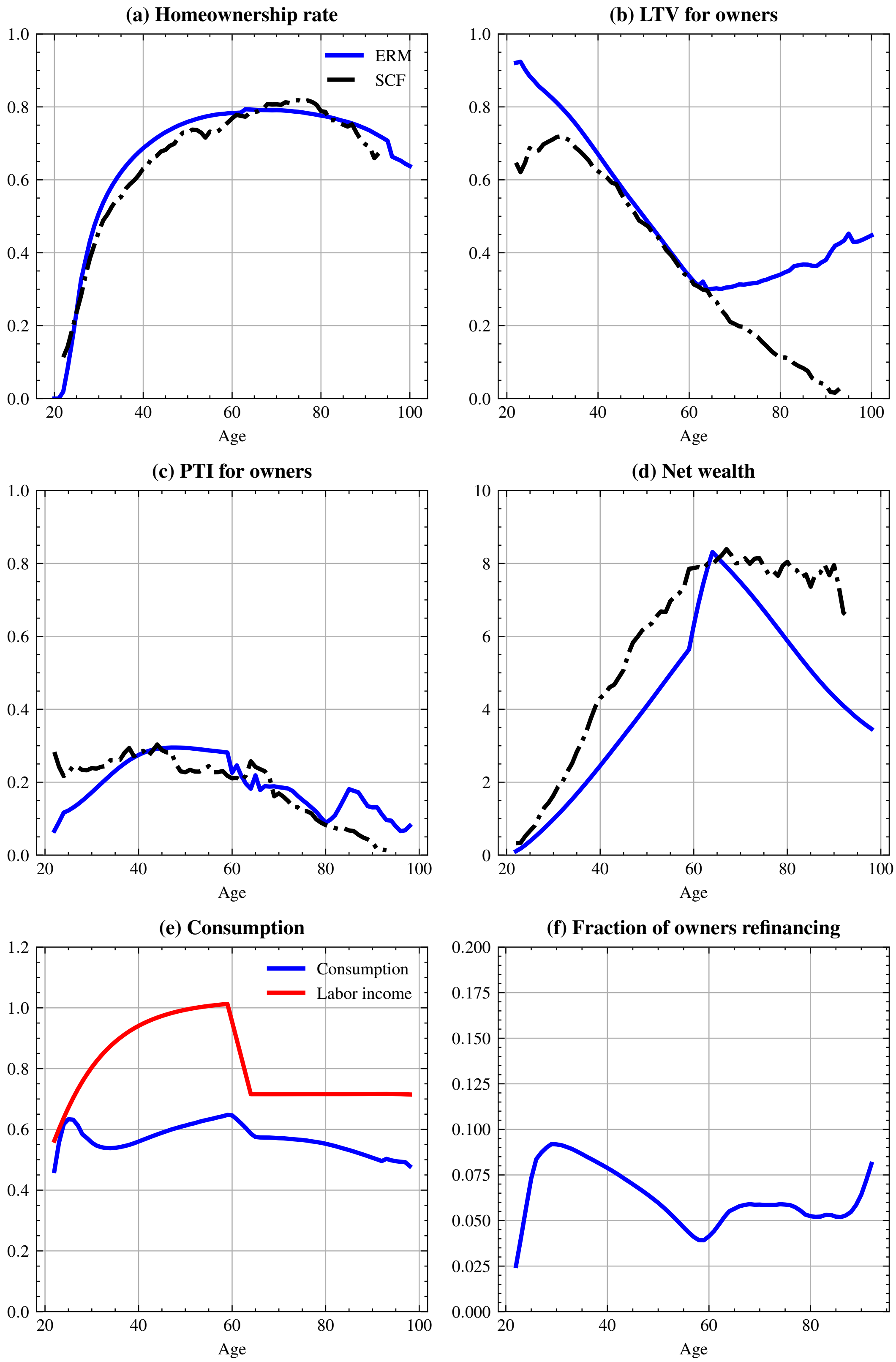

Figure 5: Life-cycle Profiles

observe a close alignment between the PTI ratios and the data for the elderly population, in contrast to the results presented in the study by Guren et al. (2021). This is because of the fact that older homeowners have access to RMs, which serve as an additional source of funds for mortgage repayment. Consequently, their PTI ratios decrease as this supplementary income from RMs helps to alleviate the burden of mortgage repayments. This contrast in findings highlights the significance of considering the impact of RMs on the PTI ratios of older homeowners.

Figure 5, Panel D, shows the net wealth accumulation over the life cycle. We find that households are liquidity-constrained during the first 15 years of their working lives. However, as time progresses, households start accumulating wealth at an accelerated pace. This wealth accumulation serves as a crucial form of insurance, providing a cushion against adverse labor income shocks and uncertainties in the macroeconomic environment. During retirement, we observe a rapid decumulation of wealth. This decumulation reflects the utilization of accumulated assets to support living expenses and maintain the desired standard of living during the retirement phase. Overall, the dynamic patterns of wealth accumulation and decumulation throughout the life cycle emphasize the significance of building a robust financial foundation and managing resources effectively to address income fluctuations and retirement needs.

Figure 5, Panel E, shows the profiles of consumption and income. Consumption closely tracks income over the life cycle. During the early working years, households' income levels are relatively lower as they face liquidity constraints. Consequently, their consumption levels also remain restrained. However, as households progress in their careers and accumulate wealth, their income increases, leading to a corresponding rise in consumption. As households continue to age, the impact of liquidity constraints on consumption diminishes. This is reflected in the consumption profile reaching a point where further increases cease, despite income still rising. This pattern suggests that as individuals advance in age, they become less constrained by liquidity concerns. During retirement, we observe a downward slope in the consumption path. This decline indicates a decrease in consumption as households rely on their accumulated wealth and retirement income sources to sustain their desired lifestyle. The reduced income flow during retirement necessitates a more prudent approach to consumption.

Figure 5, Panel F, shows the fraction of owners refinancing. Refinancing is relatively low across all ages. Compared with Guren et al. (2021), our model does not generate jumps at retirement, as homeowners have additional options to smooth their consumption and manage their financial obligations. The availability of RMs serves as a supplementary avenue for homeowners to access their home equity and alleviate liquidity constraints, reducing the need for traditional refinancing. By considering the impact of RMs on homeowners' financial strategies, our model provides insights into a more nuanced understanding of refinancing behavior, highlighting the role of alternative mechanisms in managing liquidity.

Figure 6 displays the proportion of RM borrowers opting for the FRRMs over age. On average, 24.34% choose lump sum, while 75.66% select the line of credit option. However, based on the 2021 data, 89.44% of borrowers choose the line of credit, with only 6.82% opting for the lump sum. The model predicts a higher rate of lump sum selection due to excluding three other payment options: tenure, term and line of credit, and tenure and line of credit. Moreover, we show that when borrowers age, fewer and fewer borrowers will opt for the FRRMs. FRRMs generally have slightly higher interest rates compared to ARRMs. Therefore, older borrowers may choose ARRM to take advantage of potentially lower interest rates. Another consideration could be that FRRM typically has a fixed lump sum payment, which may not be the most suitable option for older borrowers who do not require a large upfront sum of money. In contrast, ARRM offers a line of credit option, which provides more flexibility in accessing funds as needed.

5.2 Consumption Smoothing and Take-up Rate

Table 3, Panel A, reports the cross-sectional standard deviations of consumption growth for workers and retirees. On average, households utilizing RMs exhibit lower volatility in consumption growth throughout their life cycle. Interestingly, for workers, which has often been overlooked in previous literature on RMs, the availability of RMs as an option for accessing home equity leads to a substantial reduction in the volatility of consumption growth. Moreover, the presence of RMs serves the role to narrow the disparity in consumption growth volatility between working households and retirees. These findings underscore the significance of RMs in enhancing consumption smoothing and bridging the gap between different household segments.

Table 3, Panel B, shows the take-up rate of RMs across the age groups. Our model fits well

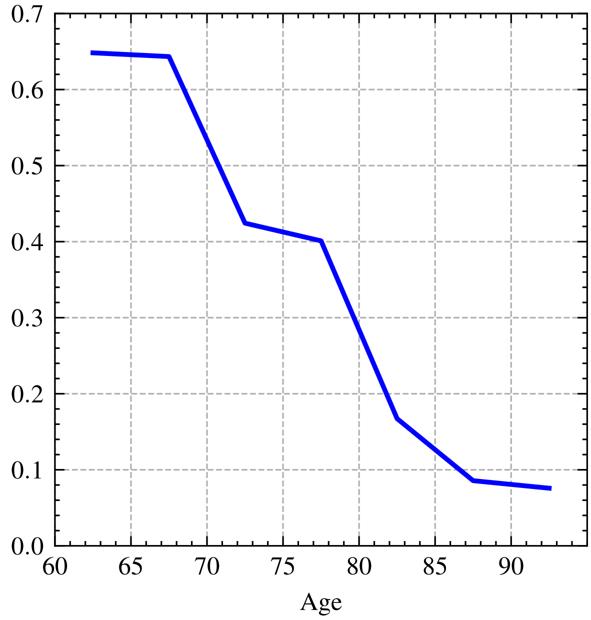

Figure 6: Fraction of Households taking FRRM

Table 3: Consumption Growth Volatility and Take-up Rates

Panel A: Consumption Growth Volatility			
Age Groups	Households without RMs	Households with RMs	Difference
< 62	0.851	0.706	-17.04%
≥ 62	0.323	0.296	-8.36%

Panel B: Take-up Rate of RMs for Owners								
Age groups	[62,65]	[66,70]	[71,75]	[76,80]	[81,85]	[86,90]	[91,95]	>95
Take-up Rate	0.01%	0.04%	0.08%	0.78%	3.70%	1.51%	0.76%	0.00%

with the empirical evidence found in US datasets, which shows a hump-shaped pattern in RM participation rates through life-cycle. When households make the decision to originate RMs, they face a trade-off between an immediate benefit of a cash payout and an increasing principal limit factor (PLF) as they age. Borrowing RMs earlier in life allows households to benefit from it for a longer period during their life cycle; however, they are eligible to extract less home equity due to the lower PLF. These considerations result in the hump-shaped pattern of RM take-up rates across age groups. In comparison, Nakajima and Telyukova (2017) shows a hump-shaped take-up rate, specifically emphasising a spike occurring at age 65, referring to a jump in demand for RM when all households first become eligible for RMs. This is due to their assumption that 65 is the youngest age at which households become eligible for RMs (equivalent to age 62 in our paper). Interestingly, Figure 7 from Shan (2011) suggests that this spike does not exist in all research periods. The spike is observed only between 2003 and 2007 (panel (a)) but disappears from 1989 to 2002 (panel (b)). This finding suggests that the occurrence of the spike is dependent on the specific economic environment, such as interest rates and macroeconomic conditions. For example, following a crisis regime, monetary authorities may implement loose monetary policy, and households with a drop in labour income may have an increased appetite for borrowing. Hence, those eligible for RMs may take advantage of the scheme and the low-interest rate level, which induces a spike of the take-up rates.[20]

[20]When we adjust the parameters in our model, we find that it can also generate the observed spike. This additional validation further confirms that the spike is indeed highly dependent on the specific economic environment.

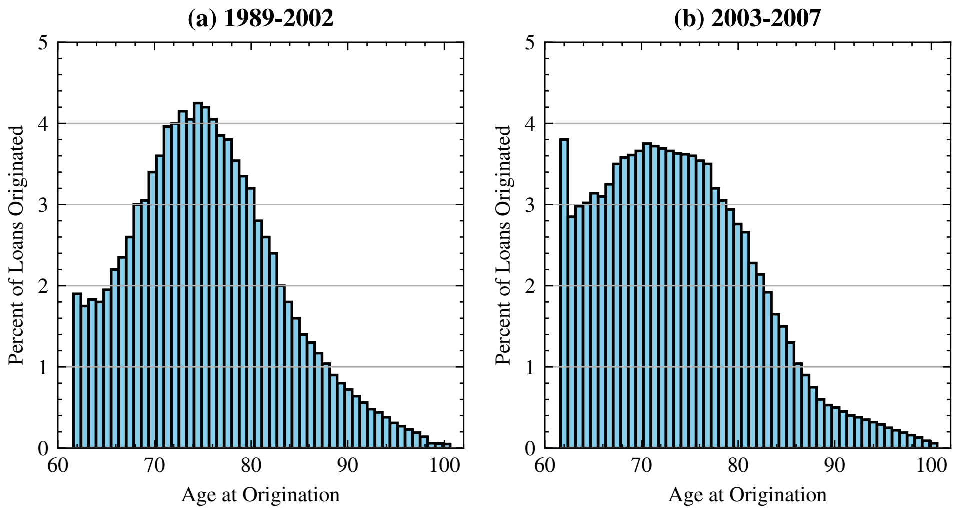

Figure 7: Distribution of HECM Borrower Ages from Shan (2011)

5.3 Mortgage, RM, and House Prices

A significant aspect of our analysis, distinguishing it from previous literature, lies in the equilibrium determination of conventional mortgage spreads, mortgage spreads, and housing prices. By considering the equilibrium determination of these factors, we obtain a holistic understanding of the interplay between mortgage rates, RM rates, credit constraints, and housing market conditions. This approach allows us to capture the intricate relationships and feedback loops within the housing and mortgage markets.

Table 4 shows the conventional mortgage rate, RM rate and corresponding spread under different aggregate states. First, mortgage rates are influenced by the prevailing economic conditions and credit constraints. During the crisis, the mortgage rates touch their lowest point, which aligns with our expectations. This is in line with the policy objectives to stimulate the economy and promote borrowing by keeping interest rates low. In the recession phase with loose credit constraints, the mortgage rate increases to 4.28%. The more relaxed lending standards during this period led to higher mortgage rates as lenders assumed a higher level of risk than normal economic conditions. Continuing into the expansion with loose credit constraints, the mortgage rate further increases. This increase is attributable to heightened demand for loans and increased economic activity, which exert upward pressure on interest rates. Conversely, when credit constraints tighten, the mortgage rate experiences a decline. This decline is more pronounced during the recession with tight credit constraints. Tight credit constraints signify lenders adopting a more cautious and stringent approach to loan approvals, resulting in lower mortgage rates as borrowing becomes more challenging.

Then we find that RM rates consistently remain higher than conventional mortgage rates regardless of the aggregate states. Similar to the conventional mortgage, the RM rates reach their lowest point during the crisis. This aligns with the broader interest rate environment during crises, which aims to facilitate homeowners' access to their home equity at more favourable rates. Moreover, we find that the RM rates only depend on the macroeconomic environment and are irrelevant to credit constraints. This distinction can be attributed to the unique characteristics of RMs. Unlike conventional mortgages, RMs typically do not involve stringent credit checks or income qualifications. Instead, eligibility is primarily determined based on factors such as the homeowner's age, home value, and other specific criteria related to RMs. Furthermore, RMs are structured as non-recourse loans. This means that the borrower's liability is limited to the value of the home. When the loan balance exceeds the home's value upon sale, the insurer (government) absorbs the resulting loss instead of the borrower or their heirs and lenders. This inherent feature of RMs further mitigates the impact of credit constraints on RM rates. Considering these factors, it becomes evident that credit constraints play a minimal role in determining RM rates. Instead, RM rates primarily hinge on the prevailing macroeconomic environment and the specific attributes and regulations associated with RM programs.

Moreover, the difference between conventional and RM rates tends to be larger during tight credit constraints. This is because conventional mortgages are more sensitive to credit conditions. When credit becomes scarce or restricted, lenders adjust their strategies to mitigate risk, often resulting in increased interest rates for conventional mortgages. In contrast, RM rates are not directly influenced by credit constraints. As a result, the gap between conventional mortgage rates and RM rates widens.

In addition to examining the dynamics of RM rates, conventional mortgage rates, and interest rates, it is crucial to explore the implications of RMs on housing prices, as it sheds light on the broader impact of RM programs on the overall housing market. We find that the inclusion of RMs in the mortgage market contributes to an overalll increase in housing prices across all aggregate

Table 4: Mortgage and RM Rates

	Mortgage		RM	
	Rate	Spread	Rate	Spread
Crisis	3.18%	1.92%	4.26%	3.00%
Recession and loose	4.28%	1.96%	4.32%	2.00%
Expansion and loose	5.12%	0.86%	5.26%	1.00%
Recession and tight	4.04%	1.72%	4.32%	2.00%
Expansion and tight	4.59%	0.33%	5.26%	1.00%

states, which enhances the competitiveness of the housing market. This inclusion of RMs in the mortgage market has both direct and indirect effects on housing prices.

One direct effect is that RMs can provide homeowners with additional financial resources, allowing them to invest in home improvements or other expenditures that can increase the value of their property. This increased investment in housing puts upward pressure on housing prices.

Indirectly, RMs influence the supply and demand dynamics of the housing market. By enabling older homeowners to access their home equity without selling their homes, RMs reduce the supply of *available* housing for sale. With a limited supply of housing but consistent or increasing demand, the scarcity of available properties drives up prices.

Additionally, the presence of RMs in the mortgage market can influence the overall dynamics of housing transactions. As more homeowners opt for RMs, it can lead to a decrease in traditional home sales, limiting the turnover of properties in the market. Reduced turnover can contribute to a slower rate of housing entering the market, further exacerbating the scarcity of available homes and potentially pushing prices higher.

However, it is worth noting that there is notable heterogeneity among the aggregate states. Specifically, when credit conditions are tight, the housing price experiences a relatively smaller increase compared to other states. This aligns with the expectation that when credit conditions are tight, potential homebuyers may face greater difficulties in obtaining mortgage financing. This can lead to a decrease in overall demand for homes, including those financed through RMs. With lower demand, the upward pressure on housing prices is alleviated, resulting in a smaller increase in prices.

Meanwhile, lenders tend to tighten their lending standards and criteria during tight credit conditions. This means that borrowers, including those seeking RMs, may find it more challenging to qualify for loans. The reduced availability of credit can limit the number of potential buyers utilizing RMs, thereby mitigating the impact on house prices.

Table 5: House Prices

Econ state	Ignore RM	Recognize RM	Change	No. of periods
Crisis	3.70	3.77	1.90%	60
Recession and loose	4.71	4.80	1.91%	203
Expansion and loose	4.54	4.57	0.66%	1015
Recession and tight	4.43	4.44	0.25%	113
Expansion and tight	4.25	4.27	0.48%	608

5.4 Welfare Gains

When examining the presence of RMs, the assessment of welfare gain becomes important for comprehending the potential benefits and implications of these financial instruments. Table 6 reports the welfare gains associated with the availability of RMs. In our analysis here, we refer to the economy without RMs as Economy 0 and the economy with RMs as Economy 1. To quantify the household welfare gains, we calculate the equivalent consumption variation. This represents the amount by which households would need to increase their consumption in Economy 0 to achieve the same level of satisfaction as in Economy 1. Specifically, we denote the value functions under Economy 0 and Economy 1 as V^0 and V^1, respectively. The equivalent consumption variation $\Delta\left(S_{it}, \Sigma_t\right)$ is defined implicitly as

$$\begin{aligned} V^1\left(S_{it}, \Sigma_t\right) =& E_t \Bigg\{ \sum_{n=t}^{t+T-a_{it}} \beta^{n-t} \prod_{m=t+1}^{n} p_m(a_{im}) \left[\tfrac{\left(C_n^0(1+\Delta(S_n,\Sigma_n))\right)^{1-\gamma_i}}{1-\gamma_i} + \alpha H_n^0 + D_{it}^o u_o - D_{it}^d u_d \right] \\ &+ \beta^{n-t} \left(1 - \prod_{m=t+1}^{n} p_m(a_{im}) \right) \psi \frac{\left(Q_{in}^0 + \xi\right)^{1-\gamma_i}}{1-\gamma_i} \Bigg\} \\ =& V^0\left(S_{it} \mid \Sigma_t\right) + \left[\left(1 + \Delta\left(S_n \mid \Sigma_n\right)\right)^{1-\gamma_i} - 1 \right] E_t \left\{ \sum_{n=t}^{t+T-a_{it}} \beta^{n-t} \prod_{m=t+1}^{n} p_m(a_{im}) \left[\frac{\left(C_n^0\right)^{1-\gamma_i}}{1-\gamma_i} \right] \right\}, \end{aligned} \tag{41}$$

where C_n^0 and D_{it}^h are the corresponding policies, and Q_{in}^0 is the total wealth under Economy 0. Consequently,

$$\Delta\left(S_{it} \mid \Theta_t\right) = \left(1 + \frac{V^1\left(S_{it} \mid \Sigma_t\right) - V^0\left(S_{it} \mid \Sigma_t\right)}{E_t\left\{\sum_{n=t}^{t+T-a_{it}} \beta^{n-t} \prod_{m=t+1}^{n} p_m(a_{im})\left[\frac{(C_n^0)^{1-\gamma_i}}{1-\gamma_i}\right]\right\}}\right)^{\frac{1}{1-\sigma}} - 1 \tag{42}$$

We report the welfare gains for four groups of households: (1) Young, working homeowners, (2) young, working renters, (3) Retired homeowners, (4) Retired renters. We find that the retired homeowners experience significant welfare gains. As homeowners age, welfare gains increase. On the other side, we also observe marginal improvement for the renters. While the rising housing prices may create additional challenges for renters, especially young households who often aspire to become homeowners, an economy equipped with RMs can provide some advantages for renters, albeit modestly. This positive spillover effect on the rental market might come from the fact that renters still have the potential to become homeowners in the future and utilize RMs as an additional option for accessing home equity. Understanding the potential benefits of RMs allows renters who have the intention to become homeowners to incorporate this option into their long-term financial planning and take advantage of the financial stability that RMs offer.

We can obtain additional insights into the differences between the two economies from Table 6. Here we show the cross-sectional standard deviations of consumption growth for workers and retirees under two economies. In the economy where RMs are accessible, homeowners, regardless of whether they choose to take up RMs or not, experience lower volatility in consumption growth compared to the economy where RMs are not available. This suggests that the presence of RMs has a stabilizing effect on consumption patterns for homeowners. On the other hand, it is indeed not surprising to find that the consumption smoothing effect is subtle for renters. The decrease in consumption volatility for both working and retired renters, amounting to 0.004 and 0.003 respectively, suggests a relatively modest impact. While renters may not have direct access to the consumption-smoothing benefits offered by RMs, they might become homeowners later. This contributes to a small but noticeable impact on consumption smoothing. Overall, an economy with RMs offers potential benefits and implications for various segments of the population, including homeowners and renters.

Table 6: Comparison between Two Economies

Panel A: Welfare Gains						
	Homeowners			Renters		
Age groups	<62	[62,75]	>75	<62	[62,75]	>75
Welfare Gains	1.29%	4.77%	7.47%	0.18%	0.31%	0.76%

Panel B: Consumption Growth Volatility						
	Homeowners			Renters		
Age groups	Ignore RM	Recognize RM	Change	Ignore RM	Recognize RM	Change
< 62	0.741	0.695	-6.21%	0.914	0.910	-0.44%
≥ 62	0.295	0.286	-3.05%	0.387	0.384	-0.78%

5.5 The Impact of RM Costs on RM Demand

In this section, we explore how the take-up rate for RM loans is affected by their current terms, in response to the popular claim that the high costs of RMLs are to blame. We consider RM take-up rate in a counterfactual model economy where all the costs related to the RMs are all eliminated. Table 7 shows the take-up rate of RMs across the age groups. We don't target the take-up rate of RMs by homeowners. Instead, we introduce a hypothetical assumption in our analysis, considering a scenario where no upfront costs are imposed on homeowners when they opt for RMs. Consequently, it is not surprising to observe a significantly higher take-up rate under this hypothetical condition. Consistent with previous studies such as Cocco and Lopes (2020) and Nakajima and Telyukova (2017), which have identified the high costs associated with RMs as a contributing factor to the low take-up rate. By considering the hypothetical elimination of upfront costs, our results provide further support to the notion that reducing the financial barriers linked to RMs could lead to an increased adoption rate among homeowners.

Additionally, we observe a gradual decline in the proportion of homeowners opting for RMs as they advance in age. Older homeowners are more hesitant to engage in RM agreements compared to their younger counterparts. Several factors could potentially contribute to this trend. First, homeowners approaching retirement age may have already made substantial progress in paying off their mortgage or accumulating home equity, thereby reducing the immediate need for RM products. Second, older homeowners may have concerns about the long-term financial implications and potential risks associated with RMs, leading them to opt for alternative strategies to access their home equity. The diminishing take-up rate of RMs among older homeowners highlights the significance of considering age-related factors when examining the adoption and utilization patterns of these financial instruments.

Table 7: Take-up Rate of RMs for Owners without Costs

Age groups	[62,65]	[66,70]	[71,75]	[76,80]	[81,85]	[86,90]	[91,95]	>95
Take-up Rate	10.71%	12.44%	10.24%	9.80%	9.64%	9.12%	8.62%	0.00%

6 Conclusion

In this paper, we analyze the RM using a quantitative model where households make decisions of saving, consumption, homeownership and RMs over the life cycle. By doing so, our model incorporates the mortgage market and housing market, enabling a comprehensive understanding of how the presence of RM influences both sectors.

Our analysis reveals that older households who opt for RMs experience enhanced consumption smoothing compared to those who do not participate in RM programs. Additionally, our model demonstrates that the inclusion of RMs has a significant impact on housing prices, particularly during the expansion with loose credit constraints. As a result, the housing market becomes more competitive, making homeownership more appealing. While these dynamics pose challenges for young renters, as they contend with rising house prices, it is important to consider their potential for future homeownership. In an economy with RMs, young renters have the opportunity to aspire to become homeowners and utilize RMs as an additional avenue for accessing home equity. Thus, even though they face some hurdles, young renters still enjoy marginal benefits in an economy that incorporates RMs. However, as expected, older homeowners experience very notable welfare gains. Overall, an economy that includes RMs exhibits superior total welfare benefits.

From a policy perspective, policymakers play a crucial role in promoting the understanding of RMs among households and minimizing the barriers that impede access to them, all while maintaining an effective risk control framework. Encouraging older households to consider RMs carries multiple advantages, benefiting not only the households themselves but also the broader society. Additionally, such encouragement helps alleviate the strain on public pension schemes, reducing the related pressures and challenges.

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Appendix: RM Markets in Global Context

Compared to the case of US and other developed countries of similar socio-economic, demographic conditions and financial market development, **UK** has the most sophisticated and penetrated RM market in the world, defined by the product ranges and optional features such as repayment and withdrawal, number of lenders and consumers, as well as consumer awareness and product reputation and regulation. Since its inception in 1991, the RM market in the UK shows very healthy and significant growth trend, with several turning points. For example, the first downturn of the UK RM market is caused by the Great Financial Crisis (GFC) rooted by the subprime mortgage crisis in late 2007 in the US, causing a global housing market slump. The RM market gradually recovered since 2008 low point, and has remained healthy growth, with a threefold size in 2016, due to major new product introduction. Another major turning point for the UK RM market is the Coronavirus (COVID-19) crisis, which induced the prevailing extra-low interest rate regime during and after the crisis, boosting the housing market, causing a temporary contraction of the new lump sum plans in 2019, and growth of further advance contracts. Lump sum plans pick up the growth trend in 2020 during the living-cost crisis and stays as stable since then.

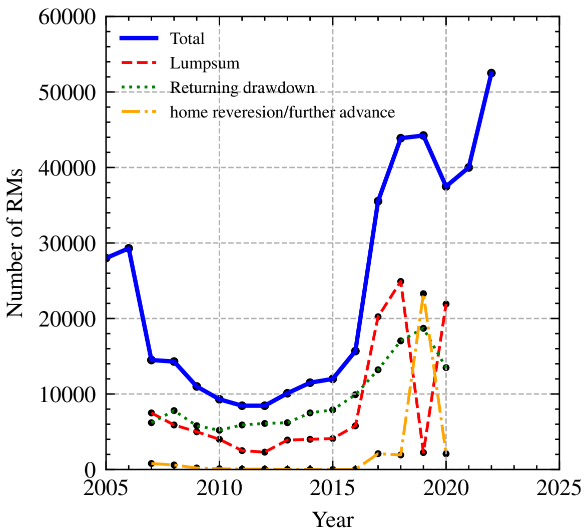

Figure 8: Number of RMs of Different Types in UK Market

Our estimates using the Equity Release Council (ERC) market report and Census data reveals that around 12% of eligible senior households have taken up RM in the UK, which is a relatively high and healthy number for this market and shows strong appetite by consumers and providers, thus a huge potential for this product to succeed in other countries in similar economic and social conditions.[21]

The case of **Hong Kong** is described in the Hong Kong Monetary Authority market reports. Although still small, the RM market has gained significant interest and growth since its inception in 2011. There are multiple factors contributing to this slow but upward growth trend: Firstly, the lack of social security (public pension plans) in Hong Kong forces senior households to consider alternative methods to increase retirement income. Secondly, the housing market boom in Hong Kong makes housing asset a significant part of family's wealth. However, the homeownership rate in Hong Kong is low compared to other developed countries such as US and UK, due to the un-affordability of housing for local citizens, stated by housing price to income ratio of 23.3.[22]

Australian RM regulatory body SEQUAL uses a different estimation methods from the US and UK. They calculate the RM participation rate using the population of suitable senior individuals instead of number of eligible households. This makes the estimated RM take-up rate lower and not comparable to those reported by the US and UK regulators or market participants. MortgageBusiness (2021) estimate about half of collective wealth in Australia is tied up in housing, and it is skewed towards the senior population, that is, around AUD$1 trillion. Out of this housing wealth, AUD$300 billion is being accessed by RM, which amounts to only about 1%-1.5% in 2021, although the market has grown by more than 5 times since its inception. Product ranges, consumer knowledge and brand reputation are not as strong as in the UK and US. The challenges of Australian RM market development is due to major lenders' unwillingness to offer such products,

[21]The recent UK RM take-up rate is estimated by the cumulative number of RM plans issued since its inception (1991-2022), e.g. according to ERC report, there are 592,000 RM plans issued during this period, divided by the number of eligible senior households who own their main house outright. To make this ratio comparable to the case of US and other countries, we use the number of households headed by someone 65+ by report Later Life in UK (2019), 6,500,000, with the homeowner ratios of 78%, and the fraction of those still paying mortgages 6%, this makes the RM eligible households to be 4,765,800. Hence the RM participation ratio is estimated to be 12% in 2022. However, if instead the denominator is the population of 65+, the number becomes significantly large, estimated to be 15,500,000 (p4 ERC report 2022), then the RM take-up rate becomes 3.8%.

[22]According to Secretariat (2021), in 2019, home ownership ratio in Hong Kong fell to the lowest point in the past two decades, as 49.8%, following a nearly four-fold growth in flat prices during that period. Although homeownership rate modestly grows to 51.2% during 2020, it is still far below the prevailing 60% level in developed countries. Ernst and Young (2022) states that the median house price to median income ratio of Hong Kong in a 2021 survey is 23.3, see (Cox (2023)), which compares house to income ratios over 92 major metropolitan areas. Hong Kong was the highest for consecutive 12 years.

and hence only proprietary, niche lenders are willing to take the risk and offer such products. On the consumer side, consumer awareness, financial literacy and product education are not through.