The Optimal Look-back Period for Adequate and Less Procyclical Credit Capital Forecasts

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Abstract

This paper finds the optimal look-back period for adequate and less procyclical credit capital requirement forecast of the aggregated credit portfolio. Our empirical experiments firstly show that credit capital forecast based on shorter look-back period is adequately lower compare to longer look-back period due to stronger homogeneity of historical scenarios during the non-GFC period. On the contrary, shorter look-back period provides larger credit capital forecast as the GFC occurs, since it is more risk sensitive to newly observed credit events due to its smaller sample size for calibration. Thus, capital requirement forecasts based on shorter-term samples are more adequate, but also become more procyclical compare to longer-term samples. Secondly, capital requirement forecasts based on stand-alone method are more adequate than the counterparts reflecting diversification due to strong tail dependence especially during the GFC period. Finally, stand-alone VaR and capital requirement forecasts based on 11-years length of look-back period, the average duration of two business cycles in U.S economy, are optimal on the basis of micro-prudential adequacy and macro-prudential less procyclicality for the aggregated portfolio in U.S. banking system.

Keywords: Unconditional loss distribution, VaR, Procyclicality, Diversification, Copula, Capital requirement.

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1 Introduction

The Basel regulatory capital for credit risk has evolved for the right balance between adequacy, on the one hand, and less procyclicality of capital requirement on the other. The capital adequacy is to ensure individual financial institutions' stability by linking their capital charges to asset risk over a risk horizon at a given confidence level via internal risk models from micro-prudential regulatory perspective. The imposition of this risk sensitive capital requirement may make the financial institutions safer in a timely manner, but amplify business cycle fluctuations and impose more systemic risk on the financial system as a whole. In this regard, macro-prudential regulation focuses on reducing the impact of capital requirement on the procyclicality of the financial system and fostering its stability (see, e.g. Acharya (2009), Behn et al. (2016), Bonfim (2009), Borio (2003), Lee et al. (2011), Marcucci and Quagliariello (2009), and Morrison and White (2005)). This paper aims to investigate the tradeoff between credit capital adequacy and less procyclicality according to different look-back periods and achieve these two conflicting micro- and macro-prudential goals in balance by suggesting the optimal look-back period under the current regulatory framework.

The regulatory credit capital requirement is found on an unconditional loss distribution to smooth the capital requirements and moderate procyclicality. The calibration of unconditional loss distribution requires certain look-back period of historical losses for bank's internal risk models as the advanced Internal-Rating-Based (IRB) approach. Thus, the optimal look-back period is as critical as the risk models themselves, but many regulators and banks are uncertain about how to determine it. Moreover, its related research on credit risk is very limited compare to market risk (Mehta et al. (2012)). In this regard, we find the optimal look-back period by conducting empirical experiments which forecast and back-test VaR and capital requirement of the aggregated charge-offs in U.S. banking system. Our emprical work can contribute to adjust banks' internal risk models for credit capital requirements with right response to future credit events but also minimizing their impacts on the procyclicality on financial system and hence enhance the efficiency of banks' risk capital management and the stability of banking system.

The reliance of unconditional loss distribution on historical sample could be strengthened by banks' subjective selection of particular sample window, for example whether it includes recessions, expansions, and stagnation periods. This reliance on sample is even more aggravated due to the stylized fact of credit losses such as skewed shape and strong fat-tail clustering of losses and their low observation frequency. This is called a sample dependency problem for bank's internal risk models.

Regarding to the sample dependency problem, Dietsch and Petey (2004) suggest using a sample containing at least one complete cycle of the economy including recession and expansion to avoid a downward bias in the estimation of risk measure and economic capital. Lucas (2007) also finds such downward bias in the risk measurement and capital estimation over using short sample periods. Ferrer et al. (2015) analyze the sample dependency problem more systematically using U.S. charge-off data and shows the significant sample dependency of the capital estimation according to the selected different length of data. They find lower capital requirements when using the sample focused only on the Great Financial Crisis (GFC) period compare to using the entire sample period. They explain this lower capital requirement estimates based on the downturn period as the strong homogeneity in economic scenarios within the used sample. The strong homogeneity during the downturn period is due to successive high realization of conditional charge-offs' clustering around a high level of conditional expected loss. Hence VaR and capital requirement decreases eventually. This is also related with the downward bias mentioned by Lucas (2007).[1]

Our first experiment investigates the effect of look-back period length used for calibrating the probability distribution of credit loss on the adequacy of VaR and capital requirement forecasts over the upcoming risk horizon. Shorter look-back period may include similar historical losses. Thus it has stronger homogeneity and this may lead to smaller capital forecasts compare to longer look-back period. On the other hand shorter look-back period could ensure VaR forecasts to be more reactive to new extreme credit events in timely manner according to different phases of economy, improve forecasting accuracy, result in more adequate VaR and capital requirement forecasts and provide early-warning signals especially around the initial phase of a crisis. This is because shorter look-back period includes less observation from the ordinary period than longer look-back period. In this regard we investigate the impact of sample length and in-sample characteristics on the adequacy of VaR and capital forecasts for the aggregated banking system by back-testing and comparing outcomes according to different length of look-back periods.

Our empirical results of the first experiment show that credit capital forecast based on shorter look-back period is less than those based on longer one due to stronger homogeneity of historical scenarios within sample during the non-GFC period. However, it becomes larger as the GFC begins since it has more risk sensitive to newly observed extreme losses. Thus, VaR forecasts based on shorter-term samples become more adequate compare to those based on longer-term samples.

[1]Ferrer et al. (2015) also find higher capital requirements for the last-half of their total sample including the GFC (2001:Q1~2010:Q4) compare to the first-half sample before the GFC (1991:Q1~2000:Q4) due to the higher variance of higher conditional charge-offs within the former sample, which is called the cyclical effect on capital requirement estimation.

The risk sensitivity due to short look-back period causes VaR and capital requirement forecasts to become cyclical and less stable. The procyclicality of VaR and capital requirement forecasts can amplify business cycle fluctuations and impose more systemic risk on the banking system as a whole which are widely recognized short-comings regarding to VaR models for regulatory capital. (see Danielsson et al. (2001), Kashyap et al. (2004), Repullo and Suarez (2012), Behn et al. (2016) and Amel-Zadeh et al. (2017)).

Our second experiment investigates this procyclicality of VaR and capital requirement to determine the optimal look-back period. The optimal look-back period should be selected on the basis of securing the adequacy and mitigating procyclicality of credit capital forecasts as much as possible. However, these two objectives are impossible to achieve at the same time as if type I and II errors in a statistical hypothesis test. In this regard, we aim to find the optimal length of look-back period not only to moderate procyclicality but also to secure the adequacy in capital requirement forecasts.

Our second empirical results show that VaR forecasts based on shorter-term samples are more adequate but more procyclical compare to those based on longer-term samples. The stand-alone VaR forecasts with perfect dependence are more adequate than the diversified VaR forecasts due to strong tail dependence especially during the GFC period. Ultimately, the stand-alone VaR and capital requirement forecasts based on 11-years length of sample window, the average duration of two business cycles in U.S economy, are optimal on the basis of adequacy and less procyclicality for the aggregated portfolio in U.S. banking system.

Our study is similar to Ferrer et al. (2015) in a sense of investigating the impact of credit losses' homogeneity on an unconditional loss distribution, risk measures and economic capital of the charge-offs of U.S. loan portfolios according to different length of samples. However, our study differs from Ferrer et al. (2015) for the following reasons. Firstly, we investigate whether the homogeneity effect decreasing VaR and capital requirement still hold for not only individual loan portfolios but also the banking-system-wide aggregated portfolio after considering diversification across all phases of U.S. economy using a moving window approach.[2] Secondly, using the same but more recent and longer data as in Ferrer et al. (2015) we conduct ex-ante test to the adequacy and procyclicality of VaR and capital requirement using quarterly ahead forecasts of the aggregated U.S. loan portfolio but Ferrer et al. (2015) is based on ex-post model diagnosis. Finally, our approach is built on the regulatory framework, whereas Ferrer et al. (2015) is based on the univariate ARIMA models. Thus, the outcomes of our approach are more consistent with the regulatory purpose and

[2]Ferrer et al. (2015) investigate the effects of homogeneity and cyclicality on individual portfolios at the end of the GFC period only.

hence have benefits for policy makers and practitioners in credit risk.

The remainder of this paper is organized as follows. Section 2 introduces analytical framework for deriving the loss distributions and economic capitals of individual portfolios and their aggregated portfolio. In Section 3, we introduce the data for our study, apply our method and discuss the comparison results. Section 4 concludes with comments.

2 Analytical framework

2.1 Loss distribution and economic capital for individual portfolio

We first approximate the individual portfolio's losses as the loss distribution based on the asymptotic single risk factor (ASRF) model which is the basis for the IRB risk weight formulas for the regulatory capital. The ASRF model assumes that each individual portfolio is homogeneous and infinitely fine-grained and dependences across exposures are determined by single systematic risk factor. The homogeneous portfolio means that all loans in the same portfolio share the identical parameters of the ASRF model such as default threshold and asset correlation. The infinitely granular portfolio implies that the portfolio consists of large enough number of loans and thus its exposure is evenly distributed.

The ASRF model defines the asset return of the obligor i in the portfolio k, X_i, as

$$X_i = \sqrt{\rho_k} Y + \sqrt{1-\rho_k} \epsilon_i$$

where Y denotes the single systematic risk factor, ρ_k is asset correlation and ϵ_i is the idiosyncratic factor. The factors Y and ϵ_i are respectively assumed to follow the standard normal distribution. Thus the asset return X_i also follows the standard normal distribution. The obligor i becomes default if $X_i \leq h_k$ where h_k is the default threshold of the portfolio k. Thus, given on Y the conditional default probability becomes

$$\mathbb{P}(Y) = \mathbb{P}(X_i \leq h_k | Y) = \Phi\left(\frac{h_k - \sqrt{\rho_k} Y}{\sqrt{1-\rho_k}}\right). \tag{1}$$

The conditional default probability of the portfolio k is empirically observed as

$$L_{k,N_k} = \frac{1}{N_k} \sum_{i=1}^{N_k} I_{\{X_{k,i} \leq d_k\}},$$

where $I_{\{X_{k,i} \leq d_k\}}$ is the indicator function which has the value 1 if the obligor i in the portfolio k

defaults, or 0 otherwise. By the assumptions of the homogeneity and infinitely granularity of the portfolio, as $N_k \to \infty$, L_{k,N_k} converges to the conditional default probability in equation (1), i.e.

$$L_k \doteq \lim_{N_k \to \infty} L_{k,N_k} \to \mathbb{P}(Y)$$

by the law of large number. Then as the regulatory model, the unconditional default probability distribution of the portfolio k is obtained as

$$F_k(\ell_k) = \mathbb{P}[L_k \leq \ell_k] = \Phi\left(\frac{1}{\sqrt{\rho_k}}\left(\sqrt{1-\rho_k}\Phi^{-1}(\ell_k) - h_k\right)\right) \tag{2}$$

by substituting L_k as the conditional default probability in equation (1).

The economic capital of portfolio k at the confidence level q is given by

$$\begin{aligned} EC_k &= VaR_k(q) - EL_k \\ &= EaD_k \cdot LGD_k \cdot \left(\Phi\left(\frac{h_k + \sqrt{\rho_k}\Phi^{-1}(q)}{\sqrt{1-\rho_k}}\right) - \Phi(h_k)\right) \end{aligned} \tag{3}$$

where EaD_k and LGD_k are the exposure-at-default and downturn loss-given-default of the portfolio k, $VaR_k(q)$ is the VaR of portfolio k determined by the q-th quantile of the systematic risk factor Y and the expected loss, EL_k, is obtained by $EL_k = \mathbb{P}(X_i \leq h_k)$. The details for the ASRF model and its corresponding loss distribution, VaR and economic capital for the homogeneous and infinitely fine-grained portfolio are available in Vasicek (2002) and Lütkebohmert (2008).

2.2 Loss distribution and economic capital for entire banking system

We aggregate the individual portfolios' loss distributions and estimate the aggregated risk measures and capital requirements for investigating the stability of the entire portfolio of banking system. For this purpose, we consider the stand-alone method and the diversified method based on the top-down approach using copula. The stand-alone method ignores any diversification between sub-portfolios for most conservative VaR and economic capital of the aggregated portfolio, whereas the top-down approach using copula reflects diversification due to dependences across sub-portfolios.

2.2.1 Stand-alone method

The stand-alone VaR and economic capital of the aggregated portfolio are obtained by simply adding up the VaRs and economic capitals of each sub-portfolio, i.e.

$$VaR_{+}^{SA} = \sum_{k=1}^{K} VaR_k \tag{4}$$

$$EC_{+}^{SA} = \sum_{k=1}^{K} EC_k \tag{5}$$

where K denotes total number of sectors.

2.2.2 Diversified method (top-down approach using copula)

The joint distribution of individual portfolios consisting of entire banking system can be written as

$$F(\ell_1, \ell_2, \cdots, \ell_K) = C(F_1(\ell_1), F_2(\ell_2), \cdots, F_K(\ell_K); \theta),$$

where $F_k(\ell_k)$ denotes the unconditional default probability distribution of the portfolio k in equation (2), $C(\cdot)$ denotes a copula such that $C : [0,1]^K \to [0,1]$ and θ is the parameter of $C(\cdot)$. Then the copula C is

$$C(u_1, u_2, \cdots, u_K; \theta) = F\left(F_1^{-1}(u_1), F_2^{-1}(u_2), \cdots, F_K^{-1}(u_K)\right), \tag{6}$$

where $F_k^{-1}(\cdot)$ is the generalized inverse of F_k such that $F_k^{-1}(u_k) = \inf\{\ell_k : F_k(\ell_k) \geq u_k, 0 \leq u_k \leq 1\}$ and the joint density function of individual portfolios is given by

$$f(\ell_1, \cdots, \ell_K) = c(F_1(\ell_1), F_2(\ell_2), \cdots, F_K(\ell_K); \theta) \cdot \prod_{k=1}^{K} f_k(\ell_k), \tag{7}$$

where $c(u_1, u_2, \cdots, u_K) = \frac{C(u_1, u_2, \cdots, u_K)}{\partial u_1 \partial u_2 \cdots \partial u_K}$ is the density function of copula $C(\cdot)$ and $f_k(\cdot)$ is the density function of $F_k(\cdot)$. The parameters in the joint density in equation (7) can be estimated using maximum likelihood method. The details for copula are available in Nelsen (2007) for its mathematical properties and Cherubini et al. (2004) and McNeil et al. (2005) for its financial applications.

Monte Carlo simulation can be used to calculating the economic capital of an entire portfolio loss after reflecting diversification as the following:

- Generate the vector of uniform random numbers from copula: $(u_1^m, \cdots, u_K^m)'$ for $m = 1, 2, \cdots, M$, where M is the total number of simulations.

- Reverse-engineer the individual portfolio's losses, $(\ell_1^m, \cdots, \ell_K^m)'$ such that

$$F_k(\ell_k^m) = 1 - \Phi\left(\frac{h_k - \sqrt{1-\rho_k}\Phi^{-1}(\ell_k^m)}{\sqrt{\rho_k}}\right) = u_k^m$$

  for all k and m.

- Calculate the aggregated loss of the entire portfolio as

$$L^m = \sum_{k=1}^{K} EaD_k \cdot LGD_k \cdot \ell_k^m \tag{8}$$

  for all m.

- Calculate the economic capital of entire banking system as

$$EC_+^{DV} = VaR_+^{DV}(q) - \bar{L}, \tag{9}$$

  where VaR_+^{DV} is the $q\%$ quantile and $\bar{L}$ are the mean of $\{L^m\}_{m=1}^M$, respectively.

The VaR_+^{DV} and EC_+^{DV} in equation (9) are called the diversified VaR and economic capital for the aggregated portfolio.

3 Empirical Analysis

3.1 Data

We download the quarterly charge-off rates complied by the Federal Deposit Insurance Corporation (FDIC) for "Mortgages" (Real Estate Loans Secured by 1-4 Family Residential Properties), "Business" (Commercial & Industrial Loans to U.S. Addressees), "Credit Cards" (Credit Cards), "Individuals" (Other Loans to Individuals), "Rest" (All Other Loans), and "Lease" (Lease Financing Receivables) during 1990:Q1~2018:Q3. Our data is the same as Ferrer et al. (2015) but covers longer period up to the latest one. For our study, the quarterly charge-off rates are annualized as multiplying by factor of 4 since banks typically measure credit risk over one-year time horizon. Thus, our annualized charge-off rates are four times bigger than those in Ferrer et al. (2015) and this would have no effect on the results of our following experiments.

Fig. 1 shows the annualized charge-off rates of six loan sectors and their aggregated portfolio. Banks write off loans after 120 days of delinquency by the Federal regulation. Thus, the peaks of charge-off rates are likely to lag behind three contraction periods of U.S. business cycle: the commercial real estate crisis (CREC) during 1990:Q3~1991:Q1, the dotcom bubble crisis (DBC) during 2001:Q1~2001:Q4, the Great Financial Crisis (GFC) during 2007:Q4~2009:Q2 indicated as the gray bars. The charge-off rates of all sectors significantly increase during the CREC and the GFC, and the DBC also significantly affects all sectors except for "Mortgages". The GFC has most significant and strongest impact across all sectors. The CREC has stronger impact on "Mortgages", "Business", "Rest" and the DBC has stronger impact on the rest of sectors compare to each other.

Table 1 displays main descriptive statistics of the charge-off rates in Panel A and EaDs in Panel B across all sectors and the aggregated portfolio.[3] The unconditional distributions of six sectors' charge-off rates are highly skewed to the left and have fat tail compare to a normal distribution which is typical properties of credit losses. Sorted by the average EaD, "Mortgages" is the largest followed by "Business", Individuals", "Credit Cards", "Rest", and "Lease".

[Figure 1 is here.]

[Table 1 is here.]

3.2 Empirical results

In general shorter look-back period improves the accuracy of expected loss forecasting and provide smaller but more adequate VaR and capital requirements, since it includes strong homogeneous economic scenarios within sample during non-crisis periods but immediately reacts to a crisis and update risk parameters in timely manner due to relatively smaller number of observations for the calibration. Our first experiment in this section confirms these facts by measuring and comparing VaR and capital forecasts based on 3-years and complete length look-back periods. Our second experiment finds the optimal length of look-back period to secure the adequacy and mitigate the procyclicality of VaR and capital requirement forecasts as much as possible.

3.2.1 3-year sample window vs complete sample window

We compare two sample windows for the estimation of unconditional distribution of charge-off rates and prediction of VaR and capital requirement for the aggregated portfolio. At a given point in time, one estimates the regulatory parameters of unconditional distribution in equation (7) using the recent 3-years sample window (3SW) and the other estimates using complete sample window (CSW) which contains all available observations up to the same given point in time. Then based on the estimated parameters of unconditional distribution, the VaR and capital requirement of the aggregated portfolio over the next quarter are forecasted using equations (4) and (5) for the stand-alone method and equation (9) for the diversified method. The Gaussian copula is applied for the

[3]The EaDs of six loan sectors are simply summrized in Panel B of Table 1 due to space constraint but can be provided on request.

diversified method. These estimations and predictions continue until the end of our sample.[4]

For example, two parameter estimations are conducted in 1992:Q4 using the 3SW from 1990:Q1 to 1992:Q4 and the CSW for the same period and then the VaR and capital requirement are forecasted using EaD and LGD of the next quarter, i.e. 1993:Q1.[5] The next two estimations are conducted in 1993:Q1 using the 3SW during 1990:Q2~1993:Q1 and the CSW during 1990:Q1~1993:Q1 and then the VaRs and capital requirements are forecasted using EaD and LGD of 1993:Q2 based on each estimated parameters of 3SW and CSW, respectively. These estimations of unconditional distribution and predictions of VaR and capital requirement are conducted over and over again until the end of our sample, i.e. 2018:Q3.

[Figure 2 is here.]

Fig. 2(a) shows the forecasts of 99.9% EC_{+}^{SA} and EC_{+}^{DV} based on 3SW and CSW from the first forecasting time, i.e. 1993:Q1 to the last forecasting time 2018:Q3. The stand-alone capital requirement forecasts are always more conservative due to assuming the perfect correlations across sectors compare to the diversified ones. Since the homogeneity of economic scenarios is stronger for short-term sample window, the economic capitals of 3SW is generally smaller than those of CSW regardless of the stand-alone and the diversified method. However, the homogeneity of economic scenarios within 3SW becomes weaker than CSW as the GFC occurs, since the portion of newly observed large charge-offs at the initial phase of GFC are bigger for 3SW than CSW. Hence the economic capital of 3SW is more sensitive to new GFC samples and become bigger than that of CSW in 2008:Q2 for the diversified method and 2008:Q3 for the stand-alone method. This reversal of homogeneity lasts until 2010:Q4 for the stand-alone method and 2011:Q1 for the diversified method.[6] Thus, the lower economic capital due to larger homogeneity within shorter-term sample window usually holds during the non-GFC period but not for the transition period from the non-GFC to the GFC and most of the GFC period.

Fig. 2(b) draws the forecasts of 99.9% VaR_{+}^{SA} and VaR_{+}^{DV} based on 3SW and CSW for the corresponding economic capitals in Fig. 2(a) and Table 2 counts the number of VaR exceedances according to different length of look-back periods and two aggregating methods. In Table 2, the charge-off rate of the aggregated portfolio exceeds 0 times and 4 times over the stand-alone and diversified VaRs of 3SW and 2 times and 6 times over the stand-alone and diversified VaRs of CSW.[7] The stand-alone VaR based on 3SW is only acceptable on the basis of the coverage test

[4]We omit all outcomes of two estimations at each quarter from 1992:Q4 to 2018:Q3 based on 3SW and CSW since there are too many estimates to present. This is also same for the estimations based on 6SW, 9SW, 10SW, 11SW, 12SW and 15SW in the next section. All details are available on request.

[5]In our experiments, LGD is assumed to be 100% and EaD is assume to be known as Panel B of Table 1 for simplicity.

[6]Note that 2010:Q4 coincides with the quarter when a sample containing only the GFC data produces lower capital requirements compare to the complete data due to homogeneity effect in Ferrer et al. (2015).

[7]We take a look at the other sample windows in Section 3.2.2.

using Kupiec (1995) and thus most adequate compare to the others. More often exceedances of charge-off rates over the VaRs based on CSW mainly happen during early times of the GFC owing to the late updates of risk parameters. This delayed update is because the CSW consists of too many accumulated samples before the GFC and hence newly observed GFC samples have weaker impact on updating the parameter estimates compare to 3SW.

On the other hand, the procyclicality is more remarkable for the VaRs of 3SW than CSW as shown in the procyclicality measures in Table 2. The measures of procyclicality in Table 2 is calculated as the standard deviation of the differences between two successive forecasted VaRs and correlation between the forecasted VaRs and the aggregated charge-off rates. The standard deviation measures the variation of capital charges over time and the correlation measures the magnitude of linkage between capital charge forecast and credit cycle in the banking system. Thus, the larger standard deviation and correlation, the stronger procyclicality of VaR forecasts.

Adequate VaR should permit VaR exceedances corresponding to a given $q\%$ confidence level over a specified time horizon to limit the default probability of financial institutions and banking system but also be less procyclical for the stability of a banking system. This motivates us to conduct the additional experiments as the next section.

3.2.2 Optimal sample window for adequacy and less procyclicality of VaR

To find the optimal size of sample window for adequate but less procyclical VaRs, we conduct similar experiments as in Section 3.2.1 but with more various terms of sample windows. The additionally considered lengths of sample window are from 4 years to 15 years.

[Figure 3 is here.]

Fig. 3(a) and Fig. 3(c) show the estimated 99.9% diversified VaRs and Fig. 3(b) and Fig. 3(d) show the stand-alone VaRs according to 6SW, 9SW, 10SW, 11SW, 12SW and 15SW.[8] Each VaRs are calculated as the same way described in Section 3.2.1. Table 2 summarizes the number of VaR exceedances, the results of back-tests and the measures of procyclicality for the diversified and stand-alone VaRs. VaR exceedances occurs too often and clustered for the diversified method. Thus, all diversified VaRs are rejected in the coverage and independence back-tests. This may happen due to using the Gaussian copula which ignores tail dependences. Note that we have totally 103 forecasts from 1991:Q1 to 2018:Q3 and hence the number of exceedances over 99.9% VaR forecasts should be less than zero at 95% confidence level and once at 90% confidence level of coverage test.

[8] We only present the VaRs of 3SW, 6SW, 9SW, 10SW, 11SW, 12SW, 15SW and CSW for clarity. The results of the other sample windows can be previded on request.

The coverage and independence tests in Panel B of Table 2 show that the stand-alone VaRs based on the sample windows shorter than 13 years seem to be acceptable. However, the actual p-values of the coverage tests for the stand-alone VaRs based on 12SW and 13SW are 0.0594 and 0.0544 which are almost significant at 95% confidence level. Thus, in terms of adequacy of VaR forecasts, we conclude that the stand-alone VaR forecasts with sample windows less than 11 years are acceptable.

[Table 2 is here.]

Panel B of Table 2 shows that the shorter-term sample window, the larger variation of VaR forecasts. The absolute magnitude of correlation between the VaR forecasts and the aggregated charge-off rates decreases until 11SW and then increases again as the look-back period increases. The stand-alone VaRs based on 11-years sample window provide the smallest procyclicality among the stand-alone VaRs accepted in the coverage test with 95% confidence level.

Therefore, we conclude that the stand-alone VaR forecast based on 11-years sample window is best in terms of adequacy and less procyclicality. This adequate length of sample window, 11 years, corresponds to twice the average duration of business cycles announced by the National Bureau of Economic Research.[9] Our results suggest using the look-back period containing two complete business cycles including recession and expansion to avoid a downward bias in the estimation of risk parameters, secure the adequacy and mitigate the procyclicality of VaR and capital forecasts.

4 Conclusion

Smaller length of sample window contains more homogenous economic scenarios compare to longer one in a sense that the variation of sample values in short-term window is less than long-term window. This stronger homogeneity of shorter-term sample window produces lower economic capital than longer-term sample window in general. This is called the homogeneity effect of sample on VaR and economic capital.

This paper investigates when the homogeneity effect holds for the aggregated U.S. loan sectors and finds that the homogeneity effect usually holds during the non-GFC period but the reversal of homogeneity effect occurs during the GFC period, i.e. the capital requirement based on short-term sample becomes larger than long-term sample during the GFC. This is occurred by the delayed risk parameter updates in long-term sample window.

We also investigate the adequacy and procyclicality of stand-alone and diversified VaRs according to different length of sample windows. VaRs based on longer-term sample window allows

[9] The average duration of all business cycles between 1945 and 2009 (total 11 cycles) is 69.5 months on the basis of trough from previous trough and 68.5 months on the basis of peak from previous peak.

more VaR exceedances during the GFC. This is because shorter-term sample window much immediately reflect newly observed extreme losses during the GFC and hence provide more adequate VaR during the GFC.

The diversified VaRs are not sufficient at all regardless of the length of look-back period. However, stand-alone VaRs are adequate for sample windows whose length is less than 11 years. Moreover, the procyclicality of stand-alone VaR becomes weaker as the length of look-back period is longer. Therefore, we conclude that most optimal length of look-back period for VaR forecasts is 11 years to ensure the adequacy of VaR and capital forecasts and less procyclical stability of banking system.

Our empirical findings may be time-specific relying on available samples at our experiment time. However, our approach for the optimal look-back period contributes banks and regulators to establish and validate the adequate internal credit risk models under the limitation of such short observation period of credit loss data.

Lastly, further research for the optimal length of look-back period may use the weighting of data points over time rather than the equal-weighting approach for the calibration of risk models and various copulas with strong tail dependence for improving the adequacy of diversified methods.

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Tables

Table 1: Main descriptive statistics for the annualized charge-off rates and Exposure-at-Default during 1990:Q1~2018:Q3.

Panel A: Charge-off rate

Sector	Mean	Std. dev.	Skewness	Kurtosis	Min	Median	Max
Mortgages	0.0046	0.0060	1.8434	5.1800	0.0005	0.0018	0.0254
Business	0.0110	0.0072	1.2063	3.6871	0.0033	0.0079	0.0332
CreditCards	0.0549	0.0180	2.1504	9.2225	0.0347	0.0511	0.1444
Individuals	0.0160	0.0059	1.6857	5.9824	0.0086	0.0144	0.0362
Rest	0.0058	0.0055	2.6550	10.4640	0.0015	0.0039	0.0302
Lease	0.0061	0.0036	1.1993	4.0288	0.0017	0.0047	0.0177

Panel B: Exposure-at-Default

Sector	Mean	Std. dev.	Skewness	Kurtosis	Min	Median	Max
Mortgages	1,873,600	679,350	-0.1752	1.3651	914,710	2,098,900	2,844,800
Business	940,720	371,710	0.6615	2.5332	458,100	878,100	1785,200
CreditCards	396,800	226,150	0.5876	1.8332	129,940	326,350	846,760
Individuals	513,660	165,480	0.1469	1.7494	275,600	520,040	827,350
Rest	251,700	202,430	1.4126	3.6633	87,380	153,260	795,760
Lease	106,190	40,719	-0.5296	2.1522	34,638	113,330	166,540

Table 2: Number of VaR exceedances and measure of procyclicality. The number of VaR exceedances counts time when the charge-off rate is larger than the forecast of each sample window. Coverage and independence tests are based on Kupiec (1995) and Christoffersen (1998). ^{a}, ^{b} and ^{c} indicate statistical significance at the 1%, 5%, and 10% significance level, respectively. Standard deviation of procyclicality measures the standard deviation of the difference between successive two forecasts of VaR for each sample window. Correlation of procyclicality denotes correlation between VaR forecasts and the aggregated charge-off rate in banking system.

Panel A: Diversified method

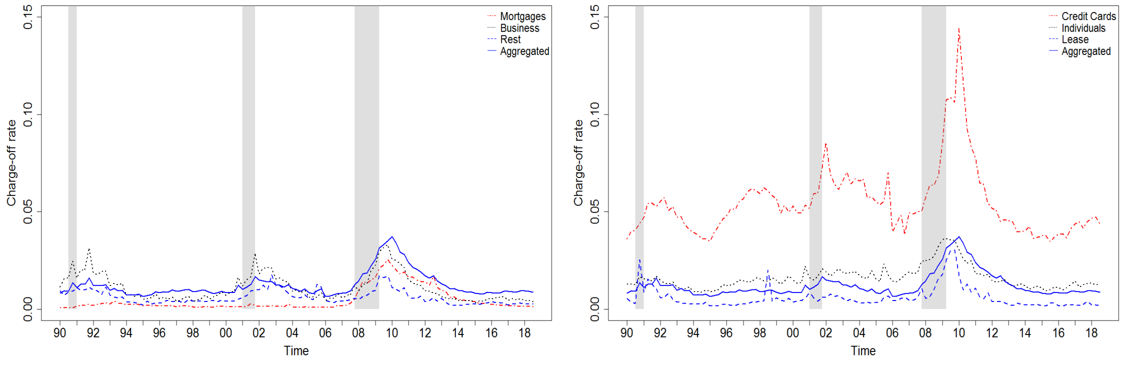
Test / Measure	3SW	6SW	9SW	10SW	11SW	12SW	15SW	CSW
Adequacy								
Number of VaR exceedance	4	3	4	3	4	5	6	6
Coverage test	21.63^a	15.25^a	23.75^a	16.40	24.61^a	33.63^a	45.08^a	37.33^a
Independence test	2.43	3.44^c	7.62^a	10.78^a	15.80^a	11.08^a	13.41^a	29.11^a
Procyclicality								
Standard deviation	0.0037	0.0044	0.0026	0.0023	0.0022	0.0023	0.0019	0.0010
Correlation	0.8496	0.4751	0.1108	0.0323	-0.0137	-0.0365	-0.1400	0.1789

Panel B: Stand-alone method

Test / Measure	3SW	6SW	9SW	10SW	11SW	12SW	15SW	CSW
Adequacy								
Number of VaR exceedance	0	1	0	0	0	1	2	2
Coverage test	0.21	2.98^c	0.16	0.15	0.14	3.55^c	10.55^a	8.11^a
Independence test	n.a.	0.02	n.a.	n.a.	n.a.	0.03	4.45^b	5.71^b
Procyclicality								
Standard deviation	0.0037	0.0042	0.0027	0.0023	0.0022	0.0020	0.0017	0.0010
Correlation	0.8594	0.4838	0.1250	0.0434	-0.0203	-0.0595	-0.1962	0.1539

Figures

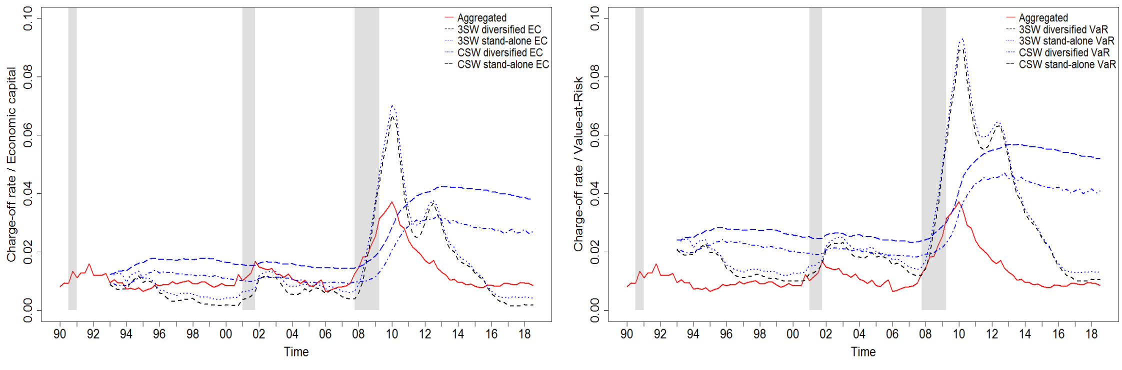
Figure 1: Annualized charge-off rates. The charge-off rates of the aggregated portfolio is calculated as the exposure-at-default (EaD) weighted average of the charge-off rates of six loan sectors at each time across our sample period, i.e. 1990:Q1~2018:Q3. The gray bars show U.S. business cycle contraction periods: commercial real estate crisis from 1990:Q3 to 1991:Q1, dotcom bubble period from 2001:Q1 to 2001:Q4 and the Great Recession from 2007:Q4 to 2009:Q2 defined by the National Bureau of Economic Research.



(a) Mortgages, Business, Rest and Aggregated

(b) Credit Cards, Individual, Lease and Aggregated

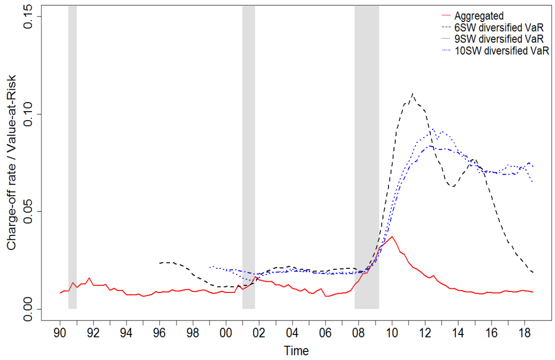
Figure 2: Forecasts of economic captial and VaR for 3SW and CSW. The regulatory parameters of the aggregated portfolio's unconditional distribution in equation (7) are firstly estimated based on 3SW and CSW, respectively. Then given on the estimated parameters, the quarter ahead VaR and capital requirement forecasts of the aggregated portfolio are obtained using equations (4) and (5) for stand-alone method and equation (9) for diversified method. 3SW is rolling over and over again until 2008:Q3.



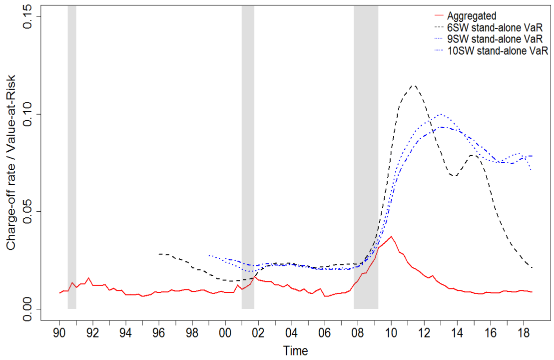
(a) EC forecasts of 3SW and CSW

(b) VaR forecasts of 3SW and CSW

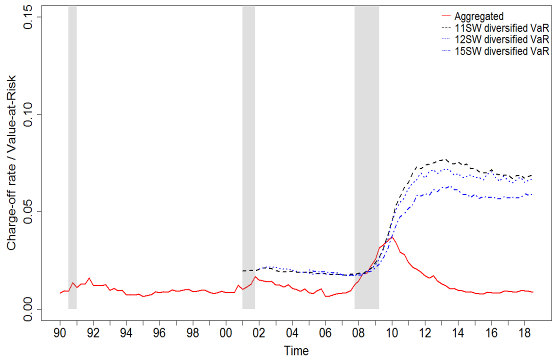
Figure 3: Forecasts of economic capital and VaR for 6SW, 9SW, 10SW, 11SW, 12SW and 15SW. The regulatory parameters of the aggregated portfolio's unconditional distribution in equation (7) are firstly estimated based on 6SW, 9SW, 10SW, 11SW, 12SW and 15SW, respectively. Then given on the estimated parameters, the quarter ahead VaR and capital requirement forecasts of the aggregated portfolio are obtained using equations (4) and (5) for stand-alone method and equation (9) for diversified method. Each estimation is rolling over and over again until 2008:Q3.



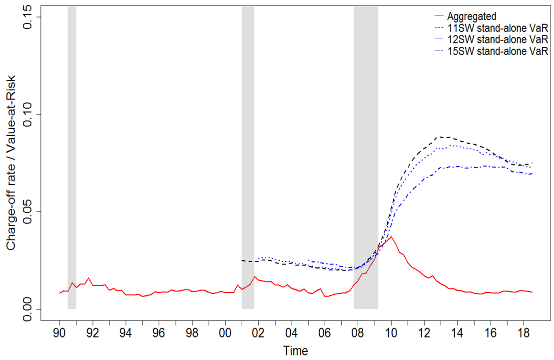
(a) Diversified VaR forecasts of 6SW, 9SW and 10SW



(b) stand-alone VaR forecasts of 6SW, 9SW and 10SW



(c) Diversified VaR forecasts of 11SW, 12SW and 15SW



(d) stand-alone VaR forecasts of 11SW, 12SW and 15SW