Dealer Trading at the Fix

by

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Abstract

We examine dealer conduct at the London 4 pm fix, a major financial market benchmark. Our findings are potentially relevant to benchmarks in Treasury securities, interest rate derivatives, NDFs, and precious metals. We develop a model that identifies the fix dealers' optimal trading strategies in three competitive contexts: independent trading, information sharing, and collusion. The model explains documented fix dealer strategies including front-running, executing client orders before the fix, and banging-the-close. It also explains documented fix-price dynamics including high pre-fix volatility, post-fix retracements, and the persistence of those dynamics after reforms to the benchmark. A statistical test provides further support for the model.

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Late in the afternoon of every London trading day, Thompson-Reuters calculates a critical exchange-rate benchmark known as the WM/Reuters 4 pm Fix. High volatility just before this "London Fix" and quick trend reversals just after it were common by 2008, generating suspicions of dealer misconduct. These suspicions were reinforced in 2013 when *Bloomberg* reported that foreign exchange dealers were using private electronic chatrooms to collude on manipulative strategies (Vaughan et al., 2013). Legal and regulatory investigations ensued and in 2015 five major foreign exchange dealing banks pleaded guilty to collusion and market manipulation (U.S. Department of Justice, 2015). Fines and settlements to date exceed $11 billion and lawsuits continue to move forward (Maton and Gramhir, 2015).

Trading at the fix is poorly understood, despite these legal fireworks, in part because research on benchmarks remains thin. This paper develops a model in which rational risk-averse fix dealers face the incentives and constraints actually imposed by the market. We consider three competitive settings: independent trading among dealers, information-sharing, and outright collusion. The model identifies a single optimal strategy that applies generally to all competitive settings. Under this strategy dealers engage in behaviors documented by regulatory investigations in the UK and the US (FCA, 2014b–f; CFTC, 2015) that include front-running their client orders, executing client trades before the fix, and banging the close. The model also predicts documented fix-price dynamics, specifically high volatility before the fix and partial trend retracements after the fix (Michelberger and White, 2016; Evans, 2017; Ito and Yamada, 2017b). Predictions from other models of dealer behavior at the fix do not conform to most of this evidence (Evans, 2017; Saakvitne, 2017). The paper closes by analyzing the practical implications of our analysis.

The London 4 pm fix was established in 1993 and quickly became an integral and important feature of the forex market. This benchmark is used to value cross-border portfolios and to construct international equity and bond market indexes, including the dominant MSCI indexes, among other functions. Many market participants execute trades at the fix price, most notably institutionally-managed funds that need to adjust hedge positions (Melvin and Prins, 2015) or to minimize tracking risk (Financial Stability Board, 2014). To trade at the fix a client sends the dealer a "fill-at-fix" order indicating how much to buy or sell. Opportunities for misconduct arise because these orders must be received well before the fix.

In the model a finite number of rational risk-averse dealers receive random, positively-correlated client fix orders before trading begins. These dealers then trade in the interbank market against an atomistic fringe of other dealers during three trading periods, two before and one after the fix. The dealers begin and end with the same inventory level; in between they fulfill client fix orders at the fix price, which is set at the period-2 interdealer price. The trades of fix dealers have positive price impact, consistent with the literature on optimal trading strategies (Bertsimas and Lo, 1998; Almgren, 2012; Obizhaeva and Wang, 2013).

In equilibrium each fix dealer adopts the following optimal trading strategy: In period 1 the dealer opens a proprietary position.[1] This generates a rising (falling) trend if he buys (sells) the base currency, other things equal.[2] In period 2 the dealer executes his client-service trades, thereby extending the period-1 trend in expectation. He simultaneously moderates that trend by liquidating part of his proprietary position. The period-2 trend appreciates his proprietary position and is the source of his fix profits. In period 3, after the fix price is set, the pre-fix trend reverses in expectation as the dealer liquidates the rest of his proprietary position. Because fix orders are correlated across dealers, the dealer rationally expects the other fix dealers to trade in parallel with himself, on average, thereby magnifying this and other trends associated with his own trading.

Our analysis introduces to the literature a new form of free-riding. A fix dealer makes two adjustments to his overall optimal strategy in response to the anticipated trades of other fix dealers: he takes a bigger initial proprietary position and he liquidates a larger share of that position before the fix. These adjustments prove critical to an observable feature of fix prices during the period of known collusion: an acceleration of the pre-fix trend as the fix moment approaches. We label this "convexity" and for modeling purposes we measured it as the ratio of the second-period price change to the first-period price change. Free-riding reduces convexity because the additional proprietary trading magnifies the first-period price change and the accelerated liquidation moderates the second-period price change. Convexity is higher with risk-averse dealers than risk-neutral dealers because risk averse dealers take

[1] We associate dealers with male pronouns because it is convenient to pick one gender and most dealers are male.
[2] This paper presents a positive analysis, not a normative analysis, and does not advocate misconduct.

smaller initial proprietary positions; in this case their behavior becomes consistent with common strategy of banging-the-close. Convexity is highest under collusion, when the dealers jointly shut down free-riding, a prediction on which we rely in providing a preliminary test of the model.

Free-riding determines the relative profitability of trading across competitive settings. In equilibrium, each dealer's decision to free-ride on the other dealers reduces expected profits for all dealers. Compared to independent trading, information-sharing leads dealers to free-ride more intensely and thus minimizes expected joint profits. Collusion enables dealers to shut down free-riding and maximize expected joint profits. The finding that profits are reduced by information sharing and magnified by collusion is not unique to this context: it often arises in traditional models of oligopoly with Cournot competition (e.g., Clark, 1983). Nonetheless, the economic forces behind these results are fundamentally different. Fix profits depend on the price path over time while traditional oligopoly profits depend on the price level at a specific time.

We evaluate the model according to its consistency with the evidence. Regulatory reports document four unusual features of dealer behavior: proprietary trading, executing client-service trades before the fix, collusion (FCA, 2014b–f; CFTC, 2015), and banging-the-close (Evans, 2017). Earlier studies document two unusual features of fix-price dynamics: high pre-fix volatility and post-fix retracements (Michelberger and White, 2016; Evans, 2017; Ito and Yamada, 2017b). Section VI documents a third unusual feature of fix-price dynamics: higher convexity during the period of alleged collusion.

Our model is consistent with all this evidence. Proprietary trading, executing client-service trades before the fix, and collusion are unconditionally optimal. Banging the close is optimal when competition is limited, as it was at the London 4 pm fix prior to June 2013. High pre-fix volatility emerges in response to the dealers' proprietary and client-service trades, and post-fix retracements arise when the dealers liquidate their proprietary positions, and the pre-fix price trend will accelerate when risk-averse dealers collude.

These price dynamics are also visible to the eye. Figure 1 plots the average paths of EUR, JPY, GBP and four other liquid currencies vs. USD during 3:30 to 4:15 GMT for the years 1996 through May 2013, just before *Bloomberg* revealed forex dealer misconduct (Vaughan et al., 2013). We include only end-month dates, following Melvin and Prins (2015). The data comprise tick-by-tick best quotes from a

Reuters currency aggregator that covers EBS, Reuters Dealing, and Reuters Matching and other platforms. High pre-fix volatility is visible throughout the sample. Figure 1 also shows that retracements and convexity are more pronounced after 2007, when the banks admit that collusion became commonplace. Both changes are predicted by the model. The prediction that convexity rises under collusion provides a relatively clear statistical test of the model because convexity itself is least sensitive to the implicit assumption that "other things are equal." We develop a rigorous measure of convexity which is indeed higher after 2007 for all seven exchange rates. The null hypothesis that convexity was unchanged after 2007 is rejected statistically.

Note that neither high pre-fix volatility nor post-fix retracements can provide reliable evidence for collusion in an antitrust context because both of them arise even if dealers trade independently. Nonetheless, the model does not imply that fix trading is free from misconduct. The dealers' proprietary trading is essentially front-running, which is universally considered unethical because it worsens prices for client (Comerton-Forde and Putniņš, 2011). Front-running is also illegal in most financial markets. In forex markets front-running is technically discouraged (B.I.S., 2017) but nonetheless legal, because regulators know any prohibition would likely be unhelpful. Limits on forex dealer behavior are difficult to enforce because dis-satisfied dealing banks can simply move to another country. And forex trading it is lucrative and non-polluting and thus an attractive source of local employment.

The paper closes by examining three practical questions. First: Are fix-price dynamics consistent with an efficient market? Our answer is Yes. Second: Why were fix-price dynamics qualitatively unchanged after the fix process was reformed in 2015? We suggest that as fix dealers ceased to exploit the fix non-dealers began to adopt similar strategies. Third: Could an alternative structure for fix trading achieve better outcomes for the market as a whole? We provide reasons for skepticism.

Our analysis is likely relevant to other benchmarks in which collusion and market manipulation are alleged. These include the ECB's foreign exchange reference rate calculated at 1:15 C.E.T., now discontinued (FCA, 2014b); gold; silver; platinum; palladium; Treasury securities; interest rate derivatives; and NDFs.[3] The relevance of our analysis for LIBOR manipulation may be more limited. LIBOR is based on banks' self-reported costs of borrowing from other banks, whereas the London 4 pm fix, like most benchmarks, is based on traded prices. Further, LIBOR manipulation was intended, in part, to support the health of the dealing banks (Abrantes-Metz et al., 2012; Gandhi et al., 2016). Fix dealers, by contrast, are on record as actively disregarding their banks' well-being (e.g., CFTC, 2015; p. 1).

Section I, which follows, relates our work to other studies. Section II presents the model. Sections III and IV develop the model's implications when dealers are risk-neutral, focusing first on independent trading and then on information-sharing and collusion. Section V examines the model with risk-averse dealers. Section VI provides a statistical test of the model's implication that convexity should be higher under collusion. Section VII discusses practical implications of our findings. Section VIII concludes.

I. Literature review

Financial benchmarks are a fairly new topic of research so the relevant literature is limited. This discussion begins with empirical studies and then turn to theoretical models. We evaluate those models based on the same criteria applied in the rest of the paper: whether they are consistent with the findings of investigative reports and empirical research.

Extensive evidence regarding fix-dealer behavior is provided by Investigative reports from the UK and the US. Dealers executed client-service trades before the fix as a matter of course (FCA, 2014b–f; CFTC, 2015). Proprietary trading was also standard practice:

> [Forex t]raders increased the volume traded by them at the fix in the desired direction in excess of the volume necessary to manage the risk associated with the firm's fix position. Traders have referred to this process as "overbuying" or "overselling" (Grabiner, 2014; p. 11).

There is substantial formal and informal evidence for banging-the-close at fixes. Evans (2016) provides evidence that trading volume surges briefly just before the London fix is calculated. FCA presentations available online describe in detail multiple specific episodes of banging-the-close in forex (FCA, 2014b–f).

[3] See FCA (2014a) for gold and silver; Iosebashvili (2014) for platinum and palladium; Stempel (2015) for Treasury securities; Leising and van Voris (2014) for interest rate derivatives; Armstrong (2013) for Asian NDFs; and Ito and Yamada (2017a) for the Tokyo fix.

The CFTC has identified banging-the-close by specific traders in palladium and platinum in 2007 and 2008 (Doering and Rampton, 2010).

Empirical descriptions of pre-reform fix-price dynamics are presented in Michelberger and White (2016) and Evans (2017). Both studies find that fix-price volatility greatly exceeds average volatility at other times of day. Michelberger and White show that this unusual volatility occurs before 4 pm. Evans provides evidence that returns before and after the fix were negatively correlated. Ito and Yamada (2017b) and van der Linden (2017) show that high volatility before the fix and partial retracements after the fix were sustained after the 2015 reforms. Melvin and Prins (2015) provide evidence that month-end fix-price dynamics reflect hedge adjustments by asset managers in response to foreign equity returns.

Evidence presented in Marsh et al. (2017) raises the possibility that positive price impact should not be assumed for period 2. They regress one-minute exchange rate changes on the contemporaneous net of market buy and market sell orders, known in forex research as order flow. The regressions are intended to measure the price impact of aggressive trades and are interpreted as capturing the impact of information, following Kyle (1985). The order flow coefficients are consistently positive but trend downward before 4 pm, trend back upward thereafter, and are significant at all times except 4 pm exactly. Marsh et al. conclude that aggressive fix trading carries information and has positive price impact, consistent with our model and with existing research (e.g., Evans and Lyons, 2002), *except* during the one-minute fix-calculation window. This conclusion would imply that dealers were not rational, or even reasonable, in choosing to bang-the-close (Evans, 2017), which would be surprising. Reassuringly, there are good reasons to believe that the order-flow coefficients in Marsh et al. (2017), and especially the coefficient for 4 pm exactly, underestimate the price impact of aggressive fix trades.

Fix dealers often relied on large marketable limit orders to trade aggressively, rather than market orders. By implication, much of their aggressive trading behavior was not measured by order flow.[4]

[4] A marketable limit order has limit price above (below) or equal to the best ask (bid). An example will help clarify. Suppose the best bid is 1.29, the best ask is 1.30 and depth exists at every penny above 1.30; suppose further that depth at every price is 10. A market-buy order for 15 and a MLO-buy for 15 at 1.31 would be equivalent: in both cases the dealer would buy 10 at 1.30 and 5 at 1.31 and the best ask would rise to 1.31. A market-buy for 50 and a MLO-buy for 50 at 1.31 would differ dramatically. After the market-buy, the best ask would have depth of 10 at 1.35. After the MLO-buy the best ask would have depth of 30 at 1.31, as the unexecuted volume of 30 becomes depth/liquidity at a new best bid of 1.31; meanwhile the new best ask is 1.32.

Marketable limit orders are common: they account for over half of all aggressive orders on the NYSE (Boehmer at al., 2006) and 90% of aggressive orders on the Korean futures exchange (Park and Ryu, 2019). Petersen and Sirri (2003) find that marketable limit orders are most likely to be used under six conditions, of which at least four tend to be satisfied at the fix: the traders are sophisticated; their trades are large; quoted depth is smaller than order size; and a trend is likely.[5]

Large marketable limit orders have important advantages over market orders for dealers intent on moving a fix price. To clarify we compare a large market buy order with a large marketable limit buy order. A large buy market order triggers immediate trading and raises the best ask but not the best bid. This is not ideal for fix manipulation because the London fix is calculated from prices on both sides of the book. In addition, the market buy order provides no support for the new, higher ask price: another trader can easily place a limit sell order at a lower price, thereby lowering the ask.

A large marketable limit buy order likewise triggers immediate trading, but it raises the best bid as well as the best ask. To see why, suppose a marketable limit buy order is large enough to exhaust sell-side depth up through the limit price and still have unexecuted volume. The limit price then becomes the new best bid and the unexecuted volume becomes depth at that price. In addition to raising the bid, this depth forms a wall of liquidity that impedes any price decline. The ask is unlikely to decline right away because a limit sell order below the best ask will be executed against the new best bid rather than resting in the book. The bid is unlikely to decline right away because a market sell order must also first execute against the new bid depth.

This analysis has important implications for empirical estimates of price impact. First: order flow is a distorted measure of aggressive trading. The initially-unexecuted volume of a marketable limit order is clearly intended to be aggressive and should be included in measures of aggressive trading. However, this volume only appears in order flow when it is executed against market sell orders, and is thus associated with a reduction in aggressive trading as measured by order flow. Second: order-flow regressions will underestimate the impact of aggressive orders. As just noted, the unexecuted depth of marketable limit order constrains the price impact of incoming market orders and marketable limit

[5] The other two conditions: depth is relatively low, spreads are narrow.

orders are common. Third: if dealers rely increasingly on marketable limit orders as the fix approaches, which seems plausible, the downward bias in these coefficients will become stronger as the fix approaches and weaker again thereafter. If the true price impact of aggressive trades is constant this pattern of bias could produce the U-shaped pattern in the order-flow coefficients of Marsh et al. (2017).

Figure 2, which shows Citibank's trading before an ECB fix in EUR-USD, demonstrates the power of large marketable limit orders to move prices just before a fix. During the last half-minute before 2:15 CET, Citi placed four large marketable limit buy order orders at roughly five-second intervals. Each order had a limit price two ticks higher than the last and their volumes rose monotonically from EUR 10 mn to EUR 400 mn. The stair-step rise of the bid confirms that each marketable limit order included sufficient depth to sustain the higher bid price and to impede both bid and ask from declining. Overall, Citi's aggressive trading achieved an eight-tick (six-bp) rise in both bid and ask during the last 30 seconds before the fix, and did so in a market that is arguably the most liquid in the world. In combination, this illustration and our previous analysis of marketable limit orders suggest that aggressive trades should be assumed to have positive price impact throughout the fix trading interval.

The theoretical literature on financial benchmarks begins with Duffie et al. (2014), who show that benchmarks can improve OTC-market performance by increasing pre-trade transparency. Those benefits are likely important in relatively opaque OTC settings like the market for municipal bonds. However, those benefits may be limited in forex, where pre-trade transparency is already provided by streaming online prices from retail forex dealers.

We are aware of two contemporaneous papers that model dealer conduct around the London 4 pm fix, Evans (2017) and Saakvitne (2016). In Evans' model, multiple independent dealers are free to trade either before or after the fix in a complex market structure. His model predicts that dealers do no fix trading at all before the fix. This implies that all client-service trading happens after the fix and that dealers do not take proprietary positions, both of which are at odds with evidence in the investigative reports (FCA, 2014b–f; Grabiner, 2014; CFTC, 2015).

More broadly, Evans' (2017) conclusion that rational fix dealers only trade after the fix seems to be at odds with the core finance principle that traders fully exploit private information (Samuelson, 1965). Each dealer's net fix order provides a private signal of the upcoming price trend. This signal indicates that the cost of his client-service trades will likely rise sooner or later, which implies that a rational dealer should execute those trades sooner rather than later. This signal also motivates the dealer to open a proprietary position before executing client-service trades. These information-based trades generate the pre-fix volatility that Evans' model does not predict. The partial liquidation of the proprietary position after the fix then creates the retracements that Evans' model does not predict. Note that proprietary trading is optimal regardless of the competitive setting so retracements should arise with or without information-sharing or collusion, contrary to the inference in Evans (2017).

Saakvitne's (2016) model focuses on a single dealer who only trades before the fix. This modeling constraint implicitly acknowledges that dealers routinely execute client trades before the fix. However, it also precludes trading after the fix which could explain why the model does not predict the empirically-observed proprietary trading. Saakvitne's model does predict that widening the fix window would greatly reduce price distortion, which conflicts with the empirical evidence of Ito and Yamada (date) and van der Linden (2017).

Fix prices and closing prices have much in common, and concerns about closing prices in equity markets are nothing new (Cordi et al., 2016). A tendency for U.S. equity prices to be volatile at the end of the day was documented in the mid-1980s (Harris, 1986) and a number of explanations have been suggested that involve misconduct. Carhart et al. (2002) suggest that equity fund managers may intentionally inflate quarter-end mutual fund values; Hillion and Suominen (2004) suggest that equity brokers may intentionally inflate their apparent skill. A number of the structural differences between equity and foreign exchange markets suggest that these ideas are unlikely to be important in the context of the London fix. To illustrate: the chatroom conversations did not include fund managers and forex trades are handled on a principal rather than an agency basis.

Cushing and Madhavan (2000) document negative return autocorrelation at the NASDAQ close in the late 1990s and attribute it to the common tendency for dealers to price shade, meaning to raise (lower) prices when they have insufficient (excess) inventory. Though the authors do not rigorously evaluate the price-shading hypothesis for the NASDAQ close, price shading is well-documented for equity markets (e.g., Hendershott and Menkveld, 2014). studies of currency markets, by contrast, generally find no evidence for price shading (e.g., Bjønnes and Rime, 2005), and explain the absence with reference to a simple cost-benefit analysis. Price shading does not guarantee execution and communicates sensitive information about a dealer's position; by contrast, trading in the interdealer market is anonymous, fast, and inexpensive. Indeed, existing evidence for currency markets supports a positive rather than negative response of price to lagged order flow, because small dealers tend to imitate the trading of large dealers (Menkhoff and Schmeling, 2010). Given these empirical findings, we view price shading as unlikely to have been a big contributor to retracements around the London fix. Misconduct, however – including the proprietary trading that our model identifies as the source of those retracements – seems in little doubt, given the chatroom transcripts and the banks' guilty pleas.

II. The model

This section outlines our model, highlighting its foundations in microstructure theory, evidence, and the institutional reality of the forex market. It also discusses how we evaluate the model.

A. The model

Fix dealers: Customer fix orders are managed by representative dealer d plus $1 \leq N < \infty$ other identical OTC fix dealers. The asset in question is technically the base currency but could also be a security or a commodity. The price is quoted in terms of a numeraire currency. Both the asset and the numeraire currency have zero return because all trades happen intraday and zero net supply.

The assumption that there are finite fix dealers is empirically well-supported. The top five forex dealing banks accounted for over 50% of spot dealing during the entire period of admitted collusion. The management of fix orders was yet more concentrated, because small and regional banks generally passed their client fix orders on to the dominant dealers. Fix dealing was also highly concentrated in the markets for gold and silver bullion during 2004 through early 2014, when collusion is alleged to have occurred (Case 1:14-cv-02213-UA, 2014). During this period just five banks set the gold fix (Harvey, 2014) and just three banks set the silver fix (Rice, 2014).

Fix orders: Before fix trading begins, in period 0, each fix dealer receives a random set of customer fill-at-fix orders. Representative dealer d matches off his own buy and sell orders to the extent possible and manages the remaining amount, F_d, in the interbank market. We assume for convenience and without loss of generality that the dealer's customers are net buyers at the fix, $F_d > 0$.

Dealer d's net fix order, F_d, includes a component shared by all other dealers, Φ, and a dealer-specific component, η_d : $F_d = \Phi + \eta_d$. The terms Φ and η_d are i.i.d. and mutually uncorrelated with mean zero and variances $\sigma_\Phi^2 > 0$ and $\sigma_\eta^2 > 0$, respectively. The aggregate net fix order is $F_{Tot} \equiv \sum_{N+1} F_n$, with variance $\sigma_{F_{Tot}}^2 = (N+1)^2\sigma_\Phi^2 + (N+1)\sigma_\eta^2$. Fix orders are positively correlated across dealers: the correlation between the orders of two dealers is ρ, $0 < \rho = \sigma_\Phi^2/(\sigma_\Phi^2 + \sigma_\eta^2) < 1$. A positive correlation in fix orders is consistent with Melvin and Prins' (2015) evidence that fix trading is influenced by recent returns in foreign equity markets. The covariance between a single dealer's order and the aggregate fix order is $\sigma_{d,Tot} = (1+N)\sigma_\Phi^2 + \sigma_\eta^2$.

The model takes customer fix orders as exogenous but their origin in reality is well understood. International equity funds worth \$9 trillion are benchmarked to the MSCI indexes and another \$2 trillion are benchmarked to the Citi World Government Bond Index, and these indexes are all marked to market with the WM/Reuters fix price (Cochrane, 2015). These institutions have a strong incentive to avoid tracking risk, which they can do by trading exactly at the fix price.

Customer fix orders at the major banks often accumulate to massive amounts. Publicly available chatroom transcripts (FCA 2014b–f) reveal that net fix orders for a single large bank sometimes exceeded USD 200 million and the net across a group of four banks sometimes exceeded USD 500 million. These sums are large relative to cumulative depth in the market-wide forex order book at 4 pm in London during 2008. During that specific minute cumulative depth per side averaged roughly EUR 200 mn for EUR-USD, GBP 60 mn for GBP-USD, and USD 50 mn for USD-CAD.[6]

Trading activity: After fix orders arrive in period 0, the fix dealers trade in the interdealer market for three periods, two before and one after the fix. During periods 1 and 2 dealer d trades quantities D_{1d} and D_{2d} at prices P_1 and P_2, respectively. The fix price, P_F, is set as the period-2 interdealer price, $P_F = P_2$. After the fix-calculation moment dealer d sells F_d to his customers, a trade that happens outside the interdealer market and is therefore not explicitly recorded in the model. Dealer d restores his inventory to its initial level during period 3, liquidating the amount X_d:

[6] Order-book depth figures are from RBS' private-use aggregator, which covered seven foreign exchange electronic communication networks (ECNs): EBS, Reuters, Hotspot, Lava, Currenex, FXCM, and eSpeed. Data are sampled every minute and span February 27th, 2008 to November 13th, 2008.

$$X_d \equiv D_{1d} + D_{2d} - F_d \quad . \tag{1}$$

Extensive evidence confirms that dealer inventories in major currency pairs mean-revert rapidly. The half-life of forex dealer inventory positions has been estimated at five minutes or less at major banks (Bjønnes and Rime, 2005) and half-an-hour or less at smaller banks (Osler et al., 2007). Our period 3, which should range from a few minutes to a few hours, is consistent with this time horizon even if structural changes in the market since those studies have accelerated inventory adjustment (e.g., HFT) or slowed it down (e.g., major banks now warehouse risk).

Dealer Objectives: Dealer d's quadratic utility, with risk aversion $\gamma/2$, is defined over profits π:

$$E\{\pi_d\} - \frac{\gamma}{2} Var(\pi_d) \quad . \tag{2}$$

Dealer d's profits, π_d, are:

$$\pi_d = P_F F_d - P_1 D_{1d} - P_2 D_{2d} + P_3 X_d \; . \tag{3}$$

The dealer earns the amount $P_F F_d$ when he exchanges with clients the amounts they ordered at the fix price. The next two terms represent the cost to purchase inventory in periods 1 and 2. The final term represents either a cost or a revenue, depending on whether the dealer purchases more or less than his client orders. Interest expense is irrelevant because fix trading occurs intraday. Following the literature, we abstract from the cost of bank capital and the potential costs of violating laws or regulations.

Throughout our analysis we rely on the following simplified expression for profits:

$$\pi_d = D_{1d}(P_2 - P_1) + X_d(P_3 - P_2) \quad . \tag{4}$$

The first term on the right represents the period-2 gain or loss on inventory accumulated in period 1. The second term captures the period-3 gain or loss on inventory-restoration trades. In equilibrium, the first term is positive and the second term is negative.

Price generating process: Fix dealers trade aggressively against each other and against an atomistic fringe of smaller dealers. We assume that the atomistic fringe extracts information from those trades, so fix trades have contemporaneous per-unit price impact $0 < \theta < \infty$. Returns are also driven by factors orthogonal to the fix, such as public information, that generate random order-flow shocks, ε_t. These shocks are i.i.d. with zero mean and variance $\sigma_\varepsilon^2 > 0$. The period-t price change is thus:

$$P_t - P_{t-1} = \theta\left(D_{td} + \textstyle\sum_N D_{tn} + \varepsilon_t\right), \qquad t = \{1,2,3\}. \tag{5}$$

Without loss of generality, P_0 is left unspecified. Outside of the fix trading interval the price would follow a random walk, $P_t - P_{t-1} = \theta\varepsilon_t$, with one-period return variance $\theta^2\sigma_\varepsilon^2$.

Equation (5), which is consistent with other models of optimal execution (Bertsimas and Lo, 1998; Almgren, 2012; Obizhaeva and Wang, 2013), is well-grounded in the literature. A positive, permanent price impact of informed trades is implied by models of asymmetric information (Kyle, 1985; Glosten and Milgrom, 1985) and order flow has permanent price impact for all major asset classes.[7] Asymmetric information is an integral feature of the forex interdealer market and there is no reason to expect fix trades to have less information content than trades at other times of day. Consistent with the hypothesis that fix trades carry information, Figure 1 provides evidence that the price impact of fix trades is permanent, on average.

B. Evaluating the model

We evaluate the model by assessing whether it conforms to documented features of dealer behavior and price dynamics at the fix. With respect to dealer behavior we evaluate the model's predictions for the four documented behaviors discussed previously: front-running, the timing of client-service trades, collusion, and banging-the-close.

With respect to price dynamics we evaluate the model's implications for pre-fix volatility, post-fix retracements, and convexity. We measure these three properties of price dynamics as follows:

1. *Pre-fix volatility*, denoted $\boldsymbol{\Psi}$, is measured as the variance of returns from P_0 to $P_F = P_2$:

$$\Psi \equiv E_0\{(P_F - P_0)^2\} \quad . \tag{6a}$$

2. *The extent of post-fix retracements,* denoted Λ, is measured as follows:

$$\Lambda \equiv E_0\{(P_3 - P_F)(P_F - P_0)\} \ . \tag{6b}$$

3. *Convexity*, denoted Π, is the acceleration of the pre-fix trend as the fix approaches. It is measured as the expected period-2 return relative to the expected period-1 return, quantity minus 1.

$$\Pi \equiv \frac{E\{P_2 - P_1 | F_{Tot}\}}{E\{P_1 - P_0 | F_{Tot}\}} - 1 \ . \tag{6c}$$

This measure is positive if the price accelerates to the fix and negative if it decelerates to the fix.

[7] See, e.g., Shleifer (1986) for equities, Evans and Lyons (2002) for foreign exchange, and Simon (1991) for bonds.

III. Independent trading

This section analyzes the model when dealers trade independently. For the present dealers are assumed to be risk neutral, which enables us to examine closed-form solutions while capturing most of the model's critical insights.

A. Dealer decision strategies

Representative dealer d chooses D_{1d} in period 1. In period 2 he chooses either D_{2d} or X_d, and we assume each dealer chooses X_d. To carry out this dynamic optimization problem dealer d must know the structure of his period-2 decision when making his period-1 decision. D_{1d} and X_d are proportional to F_d in equilibrium so define $\alpha \equiv D_{1d}/F_d$ and $\chi \equiv X_d/F_d$. A hat denotes dealer d's expected average value of that parameter across the other dealers: e.g., $\hat{\alpha} \equiv E_{2d}\{\sum_N D_{1n}\}/E_{2d}\{\sum_N F_n\}$.

In period 2 dealer d maximizes expected utility by choosing χ, taking α and $\hat{\alpha}$ as given:

$$\underset{\chi}{Max}\ E_{2d}\{\pi_d\} = \alpha F_d \theta \langle [(1-\alpha)F_d + (1-\hat{\alpha})E_{2d}\{\textstyle\sum_N F_n\}]\rangle + F_d(\alpha-\chi)\theta(\chi F_d + E_{2d}\{\textstyle\sum_N X_n\}). \quad (7)$$

Optimal χ depends on dealer d's expected value of the other dealers' period-3 trading, $E_{2d}\{\sum_N X_n\}$:

$$\chi = \frac{\alpha}{2} - \frac{E_{2d}\{\sum_N X_n\}}{F_d} \quad . \quad (8)$$

The rational-expectations solution for χ, derived in the Appendix, is a fraction of α we label q:

$$\chi \equiv q\alpha = \frac{\alpha}{2+\rho N}, \quad 0 < q = \frac{1}{2+\rho N} < \frac{1}{2} \ . \quad (9)$$

In period 1 dealer d maximizes expected utility by choosing α to solve:

$$\underset{\alpha}{Max}\ E_{1d}\{\pi_d\} = \alpha F_d \theta[(1-\alpha)F_d + (1-\hat{\alpha})\rho N F_d] + F_d \alpha(1-q)\theta[q\alpha F_d + E_{1d}\{\textstyle\sum_N X_n\}]. \quad (10)$$

The first-order condition shows that α depends on the other dealers' expected behavior, $\hat{\alpha}$ and $\hat{q}$:

$$\alpha = \frac{(1+\rho N) - \hat{\alpha}\rho N[1-\hat{q}(1-\hat{q})]}{2[1-q(1-q)]} \ . \quad (11)$$

Dealer symmetry implies that $\alpha = \hat{\alpha}$ and $q = \hat{q}$ in market equilibrium, which closes the model.

B. Market equilibrium under independent trading

Lemma 1 summarizes a risk-neutral dealer's optimal strategy in an independent-trading equilibrium (henceforth denoted by superscript IN).

Lemma 1: When risk-neutral dealers trade independently, the expected profits and optimum trades of representative dealer d are:

a. $E_0\{\pi_d^{IN}\} = \theta\sigma_{d,Tot}F_d\left[\alpha^{IN}[1-(1-q^{IN})\alpha^{IN}] - \alpha^{IN^2}q^{IN^2}\right], \quad E\{\pi_d^{IN}\} > 0$. (12a)

b. $D_{1d}^{IN} = \alpha^{IN}F_d$, $\alpha^{IN} = \frac{(2+\rho N)(1+\rho N)}{(2+\rho N)^2-(1+\rho N)}$, $\frac{2}{3} < \alpha^{IN} < 1$. (12b)

c. $D_{2d}^{IN} = [1-(1-q^{IN})\alpha^{IN}]F_d$, $q^{IN} = \frac{1}{2+\rho N}$, $0 < q^{IN} < \frac{1}{2}$. (12c)

d. $D_{3d}^{IN} = -X_d^{IN} = -q^{IN}\alpha^{IN}F_d$, $0 < q^{IN}\alpha^{IN} < \frac{1}{3}$. (12d)

In period 1 a profit-maximizing fix dealer opens proprietary position $\alpha^{IN}F_d$. In period 2 he carries out the full amount of his client-service trading, F_d, and liquidates $(1-q^{IN})\alpha^{IN}F_d$ of his proprietary position. After the fix dealer d has excess inventory of $q^{IN}\alpha^{IN}F_d$, which he liquidates in period 3.

Dealer d's profits come from the period-2 price move of $\theta[1-(1-q^{IN})\alpha^{IN}]F_{Tot} > 0$, which appreciates his proprietary position. The price moves in response to the fix dealers' aggressive trades, which are the net of client-service trades, F_{Tot}, and the partial liquidation of proprietary positions, $(1-q^{IN})\alpha^{IN}F_{Tot}$. The period-2 profits are partially offset in period 3 when the price declines by $\theta q^{IN}\alpha^{IN}F_{Tot}$, depreciating the value of inventory remaining at the end of period 2. Expected profits are rising in price impact, θ, and in the covariation between dealer d's order and F_{Tot}, $\sigma_{d,Tot}$.

Expected profits are positive due to a special feature of fill-at-fix orders, a feature they share with market-on-close orders: the trade's price is determined after its quantity. The strategy of Lemma 1 could not be profitable with a regular (a.k.a. "arrival-price") OTC trade, in which the client's traded price and quantity are set simultaneously, because it is logistically impossible to front-run an unknown client trading interest. Profits on arrival-price trades are also inversely rather than positively related to price impact, because price impact makes it costly to restore inventory to its desired level after the trade.

The dealer's optimal strategy includes free-riding, a strategy we introduce to the literature on benchmark prices. To understand free-riding it is helpful to examine the equilibrium without free-riding, which would occur if fix orders were uncorrelated or $\sigma_\Phi^2 = 0$. In this case dealer d's proprietary position is minimized, $\alpha_{\sigma_\Phi^2=0}^{IN} = \frac{2}{3}$, and the share of that position liquidated in period 3 is maximized, $q_{\sigma_\Phi^2=0}^{IN} = \frac{1}{2}$.

Free riding is each dealer's effort to exploit the likely trading of other dealers. He can forecast those trades because his own order provides information the other dealers' net fix orders. Dealer d's estimate of their aggregate orders is $E_d\{\sum_N F_n\} = \rho N F_d$. This allows him to forecast that these orders' execution will intensify the period-2 price trend by $\theta\rho NF_d[1\text{–}(1\text{-}q)\alpha]$, a forecast he exploits with two adjustments to his trading strategy. First, he takes a bigger proprietary position: $\alpha^{IN} = \frac{2}{3} + \frac{\rho N(1+\rho N/3)}{(2+\rho N)^2-(1+\rho N)} > \frac{2}{3}$. Second, he liquidates more of that position in period 2: $1\text{–}\,q^{IN} \equiv \frac{1}{2} + \frac{\rho N}{2(2+\rho N)} > \frac{1}{2}$.

Proposition 1 summarizes key features of risk-neutral dealer behavior under independent trading:

Proposition 1: When risk-neutral dealers trade independently, representative dealer d's equilibrium trading strategy exhibits the following features:

a. *Proprietary trading*: Dealer d opens a proprietary position before trading for his clients, a position that is at least 2/3 of F_d ($\alpha^{IN} > \frac{2}{3}$).

b. *Distributed liquidation of proprietary positions: D*ealer d liquidates at least one-half of his proprietary position before the fix and the remainder after the fix.

c. *Client-service trading before the fix but after proprietary trading*: Dealer d accumulates the inventory required to fulfill his client orders, F_d, in period 2, after he opens a proprietary position but before the fix is calculated.

d. *Front-running*: The amount $(2/3)F_d$ of dealer d's proprietary position is intended to front-run his own client fix orders. Without information about the other dealers he would liquidate half of this position immediately before and immediately after the fix.

e. *Free-riding*: The rest of dealer d's proprietary position, $\frac{\rho N(1+\rho N/3)}{(2+\rho N)^2-(1+\rho N)}F_d$, is intended to exploit the expected client-service trades of other dealers. Dealer d also exploits those expected trades by increasing the share of his proprietary position liquidated in period 2 by $\frac{\rho N}{2(2+\rho N)}$.

C. Fix price dynamics when dealers trade independently

The model under independent trading predicts high pre-fix volatility and post-fix trend reversals. Pre-fix volatility, Ψ^{IN}, is:

$$\Psi^{IN} \equiv E_0\{(P_F - P_0)^2\} = 2\theta^2\sigma_\varepsilon^2 + \theta^2\sigma_{F_{Tot}}^2(N+1)(1+q^{IN}\alpha^{IN})^2. \tag{13}$$

The first term on the right of Equation (13), $2\theta^2\sigma_\varepsilon^2$, captures volatility that would be observed under regular OTC trading outside the fix interval. The second term of Equation (13) can be disaggregated into two terms that capture the fix dealers' effects on volatility: $\theta^2\sigma_{F_{Tot}}^2(N+1)$ is the effect of their client-service trades; $\theta^2\sigma_{F_{Tot}}^2(N+1)q^{IN}\alpha^{IN}(2+q^{IN}\alpha^{IN})$, is the effect of their proprietary trades.

Post-fix retracements: Return autocorrelation around the fix is negative under independent trading:

$$\Lambda^{IN} \equiv E_0\{(P_3 - P_F)(P_F - P_0)\} = -\frac{\theta^2\sigma_{F_{Tot}}^2(N+1)(1+q^{IN}\alpha^{IN})q^{IN}\alpha^{IN}}{\Psi^{IN}} < 0 \quad . \tag{14}$$

Post-fix retracements are due entirely to the liquidation of the dealers' proprietary positions (Expression (14)): if dealers neither front-run nor free-ride then $\alpha^{IN} = q^{IN} = \Lambda^{IN} = 0$.

Convexity: When risk-neutral fix dealers trade independently the pre-fix price trend is concave:

$$\Pi^{IN} = -\frac{\rho N}{1+\rho N} < 0 \ . \tag{15}$$

If risk-neutral dealers did not free-ride the expected price path would be linear ($\Pi^{IN} = 0$) because expected price moves would be the same for periods 1 and 2. = $E_{0d}\{P_2 - P_1\}$. In period 1, each dealer's proprietary position would be intended solely to front-run his own client orders, so D_{1d}^{IN} = $\alpha_{\rho=0}^{IN} F_d$ = $(2/3)F_d$ and $E_{0d}\{P_1 - P_0\}$ = $\theta(2/3)F_{Tot}$. In period 2, each dealer's purchase of F_d would be offset by $(1/3)F_d$ as he liquidated exactly half of his proprietary position. However, the rational free-riding of risk-neutral dealers creates a strictly concave price path. They strengthen the period-1 price move by opening a larger proprietary position and they moderate the period-2 price move by liquidating more of that position before the fix. Proposition 2 summarizes the model's implications so far for fix-price dynamics:

<u>Proposition 2</u>: When risk-neutral dealers trade independently, equilibrium price dynamics display the following features:

a. *High pre-fix volatility*: Volatility is higher before the fix than after the fix and at other times of day. The price moves before the fix in response to normal random order flow, the dealers' client-service trading, and the dealers' proprietary trading.

b. *Post-fix retracements*: In expectation the pre-fix trend is partially reversed after the fix, as fix dealers finish liquidating proprietary positions.

c. *Strict concavity of the pre-fix price path*: In expectation the pre-fix trend decelerates as the fix approaches because dealers free-ride on each others' expected client-service trades.

D. Discussion

Front-running is illegal in well-regulated financial markets because it is costly to clients. In currency markets front-running is discouraged but not explicitly prohibited (Bank of England, 2011; B.I.S., 2017). The absence of any prohibition reflects a simple cost-benefit analysis. There is no institution with authority to enforce trading rules worldwide, so banks can always find a legal domicile from which to trade as preferred. But authorities would rather keep the business at home because it is lucrative and non-polluting. In addition, currencies are neither securities nor financial instruments, so they are not covered by Europe's MIFID or similar legislation elsewhere. The fix dealers' proprietary trades were sometimes substantial by any yardstick. Figure 2, for example, shows the Citibank dealer placing orders for at least EUR 33 mn more than his underlying client fix orders (FCA, 2014b). CFTC transcripts report one forex dealer telling another, "haha i [sic] sold a lot up there and over sold by 100" (CFTC, 2015, pp. 2-3), meaning 100 million of the base currency.

The front-running and free-riding predicted by our model essentially represent trade-based manipulation (Hart, 1977; Allen and Gale, 1992; Aggarwal and Wu, 2006; Vitale, 2000). However, this research generally focuses on just one informed trader who exploits the market power that arises from superior information. Our model assumes, in contrast, multiple fix dealers with imperfectly correlated private information and our findings involve those dealers' strategic interactions. The profitability of trade-based manipulation on arrival-price trades is driven by lags or instabilities in the expectation formation process (e.g., Hart, 1977; Aggarwal and Wu, 2006), but these are irrelevant for the profitability of trade-based manipulation at the fix.

The model's prediction that volatility will be high before the fix under independent trading has an important implication: high pre-fix volatility and retracements cannot support a strong legal case for misconduct, including collusion. These features of fix-price dynamics will arise even if dealers trade independently and avoid misconduct, because client-service trading happens before the fix. Even if independent dealers take proprietary positions their client-service trades will contribute a minimum 66% of pre-fix volatility, according to the model, which arises in the extreme conditions of with zero random trading ($\sigma_\varepsilon^2 = 0$) and the slowest rational liquidation of proprietary positions (q^{IN}= 1/2).

IV. Information sharing and collusion

This section examines market equilibrium when risk-neutral dealers share information about client orders or collude outright in executing a joint trading strategy.

A. Information sharing

It is considered unethical for dealers to share information about client orders because it puts the client at risk of manipulation. Forex dealers know this because bank compliance officers remind them of it regularly. This message came through loud and clear in the Non-investment Products Code signed by all the major banks in 2011 (Bank of England, 2011, see p. 18) as well as the recent Forex Global Code (Bank for International Settlements, 2017).

Nonetheless, transcripts of private chat-room conversations revealed by investigative reports show that forex dealers were accustomed to sharing information about client fix orders (FCA, 2014b–f). We assume that each fix dealer shares his own net fix order with the other fix dealers before fix trading begins. (The possibility of deceit is discussed below.)

Lemma 2 describes dealer d's optimal fix trading under information sharing (superscript IS).

<u>Lemma 2</u>: When risk-neutral dealers share information, the expected profits and optimum trades of representative dealer d are:

a. $E\{\pi_d^{IS}\} = \theta\sigma_{F_{Tot}}^2\bar{F}[\alpha^{IS}[1-(1-q^{IS})\alpha^{IS}] - {\alpha^{IS}}^2 {q^{IS}}^2] > 0$. (16a)

b. $D_{1d}^{IS} = \alpha^{IS}\bar{F}$, $\alpha^{IS} = \frac{(1+N)(2+N)}{(2+N)^2-(1+N)}$, $\frac{6}{7} \leq \alpha^{IS} < 1$. (16b)

c. $D_{2d}^{IS} = F_d - (1-q^{IS})\alpha^{IS}\bar{F}$, $q^{IS} = \left(\frac{1}{2+N}\right)$, $0 \leq q^{IS} < \frac{1}{2}$. (16c)

d. $D_{3d}^{IS} = -X_d = -q^{IS}\alpha^{IS}\bar{F}$, $0 < q^{IS}\alpha^{IS} \leq \frac{3}{7}$. (16d)

Equilibrium trading under information sharing has the same outline as equilibrium trading under independent trading: dealer d opens a proprietary position in period 1; he carries out his client-service trades in period 2, when he also begins liquidating his proprietary position; he finishes liquidating that position in period 3. The proprietary position once again includes a component intended to front-run client fix orders and another intended to free-ride on the other dealers; a dealer also free-rides by accelerating the liquidation of his proprietary position.

Nonetheless, the fix dealer' strategy is critically different. First, each proprietary position is now proportional to the market's average net fix order, $\bar{F}$, rather than to a dealer's own net fix order, so every fix dealer takes the identical proprietary position. Second, market-wide front-running remains unchanged at $(2/3)F_{Tot}$ but free-riding intensifies because dealers no longer need to estimate each others' client-service trading, they know it with certainty. The larger proprietary positions, $\alpha^{IS} > \alpha^{IN}$, and the accelerated liquidated of those positions, $1-q^{IS} > 1-q^{IN}$, would increase dealer d's expected profits if he were the only one to free-ride. When all dealers free-ride, however, expected profits decline, despite the larger proprietary position, because the extra speriod-2 liquidation sales moderate the period-2 return and thus the proprietary position's per-unit appreciation.

The conclusion that information sharing is costly to dealers may be robust to the possibility that dealers are not fully truthful. Gal-Or (1985) analyzes such a model and concludes that in Nash equilibrium firms that optimally lie still do worse by sharing information. Clarke (1983) likewise finds that firms profit most if they collude on trading strategies as well as share information.

Proposition 3 summarizes the key features of equilibrium behavior when dealers share information:

Proposition 3: When risk-neutral fix dealers share information:

a. *Conditional expected profits* are lower than under independent trading.
b. *Proprietary trading* remains an optimal strategy. However, the positions are now perfectly correlated because they are proportional to $\bar{F}$ rather than a dealer's own net fix order. Proprietary positions are larger, on average, than under independent trading, $\alpha^{IS} > \alpha^{IN}$.
c. *Client-service trading* is still optimally carried out before the fix but after proprietary trading.
d. *Front-running* is still optimally 2/3 of each dealer proprietary position.
e. *Free-riding becomes more pronounced than under independent trading*: This explains the larger proprietary positions and why those positions are liquidated more quickly, $q^{IS} < q^{IN}$.

The adjustments in trading strategies under information sharing bring corresponding changes in fix-price dynamics, as summarized in Proposition 4:

Proposition 4: In equilibrium when risk-neutral dealers share information about client fix orders,

a. *Pre-fix volatility is less pronounced* than under independent trading:

$$0 < \Psi^{IS} = \theta^2 2\sigma_\varepsilon^2 + \theta^2 \sigma_{F_{Tot}}^2 (N+1)(1+\ q^{IS}\alpha^{IS})^2 < \Psi^{IN}. \tag{17a}$$

b. *Post-fix trend retracements* are less pronounced than under independent trading:

$$\Lambda^{IN} < \Lambda^{IS} = -\frac{\theta^2 \sigma_{F_{Tot}}^2 (N+1)(1+q^{IS}\alpha^{IS}) q^{IS}\alpha^{IS}}{\Psi^{IS}} < 0\ . \tag{17b}$$

c. *The pre-fix price path is less convex* (more concave) than under independent trading:

$$\Pi^{IS}\ = -\frac{N}{1+N} < \Pi^{IN} = -\frac{\rho N}{1+\rho N} < 0\ \ . \tag{17c}$$

These shifts all reflect the increase in free-riding under information sharing. The two dimensions of free-riding exert opposing forces on volatility and retracements: larger proprietary positions intensify both while faster liquidation reduces them. As with dealer profits, faster liquidation dominates in equilibrium so these features of price dynamics become less pronounced. The two dimensions of free-riding have reinforcing effects on convexity: larger proprietary positions strengthen the period-1 trend and faster liquidation weakens the period-2 price trend.

B. Discussion

It may seem surprising that information sharing reduces dealer profits at the fix: indeed, Evans asserts that dealers would benefit from sharing information if they had the opportunity to do so (Evans, 2017; p. 46). Nonetheless, this represents yet another way in which our model is consistent with microeconomic research on cartels. Clarke (1983), whose model of oligopoly is closest to ours, shows that in a full Bayes-Cournot equilibrium firms have no mutual incentive to share information unless they also collude in setting traded quantities. The parallels are striking given that the mechanisms supporting these results have little in common: fix profits derive from price changes across time while oligopoly profits derive from a high price level at a given time.

C. Collusion

We assume that all risk-neutral fix dealers join a single cartel and collaborate in the execution of a single trading strategy that maximizes expected aggregate profits, consistent with transcripts of the dealers' private conversations (FCA, 2014b–f; CFTC, 2015). (The outcome with K separate cartels is isomorphic to independent trading with $N = K$ -1). We also assume that cartel members do not cheat in fulfilling their mutual agreements (the possibility of cheating is discussed below). Lemma 3 summarizes the cartel's optimal trading strategy under collusion (superscript C), with its total trading in period t denoted $D_{t\,Tot}$.

Lemma 3: When risk-neutral dealers execute a collusive trading strategy, the cartel's aggregate expected profits and optimum trades are:

a. $E\{\pi^C_{Tot}\} = \frac{1}{3}\theta\sigma_{d,Tot}F_{Tot} > 0$. (18a)

b. $D^C_{1Tot} = \alpha^C F_{Tot} = \frac{2}{3}F_{Tot}$, $\alpha^C = \frac{2}{3}$. (18b)

c. $D^C_{2Tot} = [1-(1-q^C)\alpha^C]F_{Tot}$, $q^C = \frac{1}{2}$. $[1-(1-q^c)\alpha^C] = \frac{2}{3}$ (18c)

d. $D^C_{3Tot} = X^C = -q^C\alpha^C F_{Tot} = \frac{1}{3}F_{Tot}$, $q^C\alpha^C = \frac{1}{3}$. (18d)

Equilibrium trading under collusion has the same outline as equilibrium trading under independent trading and information sharing: in period 1 the cartel opens a proprietary position; in period 2 the cartel carries out all client-service trades and begins to liquidate the proprietary position; in period 3 the cartel finishes liquidating that position. Beyond this common outline, optimal trading under collusion has one crucial difference from its predecessor strategies: to maximize profits dealers shut down free-riding. The shared proprietary position is therefore just 2/3 of the aggregate net fix order and the position is liquidated in equal amounts. Proposition 5 summarizes the features of the optimal strategy in collusive equilibrium and Proposition 6 summarizes the properties of fix-price dynamics:

Proposition 5: In equilibrium, when risk-neutral dealers collude,

a. *Aggregate conditional expected profits* are maximized across the three competitive settings, $E_0\{\pi_{Tot}^{C}\} > E_0\{\pi_{Tot}^{IN}\} > E_0\{\pi_{Tot}^{IS}\}$.

b. *Proprietary trading* remains an optimal strategy. The dealers' aggregate proprietary position is smaller than the average position under either independent trading or information sharing: $\alpha^C < \alpha^{IN} < \alpha^{IS}$.

c. *Client-service trading* is still optimally carried out before the fix but after proprietary trading.

d. *Front-running* is optimally 2/3 of the dealers' aggregate proprietary position.

e. *Free-riding* is shut down. This explains why the aggregate proprietary position is minimized and why that position is liquidated most slowly, $q^{IS} < q^{IN} < q^{C}$.

Proposition 6: In equilibrium when risk-neutral dealers collude:

a. *Pre-fix volatility* is highest across the three competitive settings:

$$\Psi^C = 2\theta^2\sigma_\varepsilon^2 + \theta^2\sigma_{F_{Tot}}^2(N+1)[1+q^C\alpha^C]^2 > \Psi^{IN} > \Psi^{IS} \quad . \tag{19a}$$

b. *Post-fix retracements* are most pronounced across the three competitive settings:

$$\Lambda^C = -\frac{\theta^2\sigma_{F_{Tot}}^2(N+1)(1+q^C\alpha^C)q^C\alpha^C}{\Psi^C} < \Lambda^{IN} < \Lambda^{IS} < 0 \quad . \tag{19b}$$

c. *The pre-fix path is linear* in expectation and thus has the highest convexity across the three competitive settings:

$$\Pi^C = 0 > \Pi^{IN} = -\frac{\rho N}{1+\rho N} > \Pi^{IS} = -\frac{N}{1+N} \quad . \tag{19c}$$

D. Discussion

The model under collusion continues to predict front-running of client orders and client-service trading before the fix. It also shows that dealers have a strong incentive to collude: higher profits. The model also continues to predict high pre-fix volatility and partial retracements of the pre-fix trend immediately after the fix. Under collusion pre-fix volatility and trend reversals will be more pronounced, and the pre-fix price path will be more convex, than under information sharing or independent trading. Indeed, collusion is missing just one of the characteristic features of the fix: banging-the-close.

Collusion at the fix shares some notable features with shrouding (Gabaix and Laibson, 2006), a pricing strategy in which producers hide the overall cost of a product. Manufacturers of home printers, for example, advertise a low price for the physical machine and hide the high price of ink, which is the costliest part of home printing. Shrouding may be relevant to fix dealing insofar as the true cost of liquidity at the fix is not visible to the dealers' clients. Prior to reforms, the bid-ask spread on fix trades was zero even though such orders are often quite large and large forex orders are typically charged wider spreads (Cochrane, 2015). In effect, the price of the headline liquidity product associated with a fix trade, meaning a transaction at a specific price, was zero. The clients' total cost of liquidity was not zero, however, because the dealers' strategic trading was costly to their clients. Clients were unaware of this behavior because dealers colluded in private electronic chat rooms and the market is opaque.

The possibility of cheating by cartel members cannot reasonably be ruled out given the dealing banks' admission that dealers violated bank ethical standards and anti-trust laws. Indeed, given the strong incentives for dealers to cheat on each other identified by the model, fix trading is isomorphic to a prisoner's dilemma and cheating could perhaps be expected. A cheating dealer could have understated his fix orders to minimize the other dealers' front-running and then traded for his own account separately from the cartel. In equilibrium, however, the influence of cheating might be limited as rational lying dealers anticipated the lying of other dealers.

As a repeated game fix trading can be analyzed in terms of dynamic collusion. If forex demand and supply functions are known with certainty, equilibrium cartel behavior would be determined by the fact that cheating can be identified unambiguously. In reality, however, forex dealers face many sources of uncertainty and signals of cheating would be noisy. Green and Porter (1984) and Abreu et al. (1986) show that Bertrand competitors facing such uncertainty can rationally adopt both carrots and sticks. They cooperate if and only if the price remains within a certain range, but if the price breaches that range they perceive a high likelihood of cheating. In this case the other cartel members retaliate for a finite number of rounds and then revert to collusion. In fix trading the trigger for retaliation could have been an observed price path that was sufficiently inconsistent with the expected path under collusion.

The price trend might begin earlier than expected, for example. A dealer suspicious of a co-conspirator could be un-cooperative with that other dealer in a variety of ways outside the fix: quoting slower or wider prices in direct trading; accommodating smaller amounts; or engaging in social exclusion.[8]

Given randomness in the price process, the cartel theories of Green and Porter (1984) and Abreu et al. (1986) predict an irregular cycle of cheating, retaliation, and renewed cooperation which, in our contest, could generate irregularities in fix-price dynamics. However, empirical research on cartels does not entirely support these cartel theories. Cartels typically survive for at least a few years – many last beyond ten years – and price wars are less frequent and less intense than predicted by theory (Levenstein and Suslow, 2006). The empirical research also highlights conditions under which collusion tends to thrive, at least two of which were met by forex dealing at the fix. First, in successful cartels colluding agents avoid disagreement over how to adjust collusive rents, one approach to which is to make compensation responsive to market conditions (Levenstein and Suslow, 2011). In the fix cartel, profits necessarily varied by market conditions such as volatility. Second, members of successful cartels typically apply low discount rates for the future (Levenstein and Suslow, 2016). This could be relevant to the fix cartel because dealers were secure within their banks and interest rates were generally low.

E. Evaluation of the model

Our model under risk neutrality is highly successful at predicting the key documented features of dealer behavior and price dynamics at the London 4 pm fix. It predicts three of the dealer behaviors identified in regulatory reports: proprietary trading, client-service trading before the fix, and collusion. The next section shows that the fourth and last documented behavior, banging-the-close, can arise when dealers are risk averse. The model under risk neutrality also predicts both of the fix-price dynamics documented empirically – high pre-fix volatility and post-fix retracements. It also predicts that convexity is highest under collusion, consistent with the rise in convexity after 2007 documented in Section VI.

[8] We are grateful to Alexis Stenfors for these insights.

V. Risk-averse dealers: Convexity and banging-the-close

This section examines the full model exposited in Section II, in which fix dealers are risk-averse with risk-aversion coefficient $\gamma/2$ (see Equation (2)). This version of the model captures all four of the characteristic dealer behaviors and all three of the characteristic features of fix-price dynamics. As before, we consider independent trading, information sharing, and collusion in sequence.

A. Independent trading under risk aversion

Consider risk-averse dealer d in period 2 as he decides how much to trade in period 3. He does not know the fix orders of the other dealers because they are neither colluding nor sharing information. This decision relies on the variance of profits conditional on period-1 information, $Var_1(\pi_d)$ (see Equation (2)). In addition to the model's three risk primitives – σ_ε^2, σ_Φ^2, and σ_η^2 – $Var_1(\pi_d)$. depends on dealer d's error in forecasting the other dealers' aggregate net fix orders, $\vartheta_d \equiv \sum_N F_n - E_{2d}\{\sum_N F_n\}$. The variance of ϑ_d, $\sigma_\vartheta^2 > 0$, proves important for this analysis, but its dependence on other variables does not prove important so we leave it unspecified. Using the superscript R to indicate risk aversion, $Var_1(\pi_d)$ is:

$$Var_1(\pi_d) = \sigma_\varepsilon^2[2 - q^{R,IN}(1 - q^{R,IN})] + \sigma_\vartheta^2[(1 - \widehat{\alpha^{R,IN}})(1 - q^{R,IN})]^2. \tag{20}$$

Equilibrium fix-dealer trading depends on the three risk primitives indirectly, through $Var_1(\pi_d)$ and three other composite risk term. The first, σ_ϑ^2, was just introduced. The second is σ_μ^2, the variance of dealer d's error in forecasting other dealers' period-3 trades, $\mu_d \equiv \sum_N X_n - E_{2d}\{\sum_N X_n\}$. The third is $Cov(\vartheta_d, \mu_d) > 0$.[9]

Equilibrium trading shares cannot be expressed in closed form because proprietary trading under risk aversion, $\alpha^{R,IN}$, and the share of the proprietary position liquidated in period 3, $q^{R,IN}$, depend non-linearly on each other and on the composite risk terms, as shown in Lemma 4.

Lemma 4: In equilibrium when risk-averse dealers trade independently, dealer d with net fix orders F_d trades the following amounts:

a. $D_{1d}^{IN} = \alpha^{R,IN} F_d$, $\quad \alpha^{R,IN} = \frac{(1+\rho N)}{(2+\rho N)[1-q^{R,IN}(1-q^{R,IN})]+\gamma\theta Var_1(\pi_d)}$, $\quad \frac{1}{2} < \alpha^{R,IN} < 1.$ (21a)

b. $D_{3d}^{IN} = -q^{R,IN}\alpha^{R,IN} F_d$, $\quad q^{R,IN} = \frac{1+\gamma\theta(\sigma_\varepsilon^2+\sigma_\mu^2)+\gamma\theta(1-\alpha^{R,IN})Cov(\mu_d,\vartheta_d)}{2+\gamma\theta(\sigma_\varepsilon^2+\sigma_\mu^2)+\rho N}$. (21b)

[9] In equilibrium the last two errors and their statistical properties are closely tied, but the form of those links prove immaterial to the qualitative analysis presented here.

Equations (21a) and (21b) are not highly informative and comparative statics are also inconclusive. Figures 3A through 3D present simulations that show how equilibrium proprietary trading and convexity vary with risk aversion, risk primitives, and the number of dealers. Proprietary positions are declining in the risk primitives and in risk aversion, which implies that the period-1 price trend is more moderate under risk-aversion than under risk-neutrality. More broadly, as uncertainty rises fix dealers tend to trade more in period 2 and less in periods 1 and 3.

Figures 3A through 3D also show that convexity is rising both in the risk primitives and risk aversion. The pre-fix price path is strictly convex when risk aversion is high or when risk aversion is modest and risk primitives are high. This convexity is consistent with the well-known tendency of dealers to trade most aggressively during the fix calculation window or, equivalently, to bang the clos as noted in the original *Bloomberg* article (Vaughan et al., 2013). Formal and informal evidence suggests that forex fix dealers used this strategy commonly at the London 4pm fix and ECB fix. Evans (2016) provides evidence that trading volume surged during the fix calculation window and the FCA presentations describe multiple specific episodes of banging-the-close. Outside of forex, the CFTC has identified banging-the-close by specific traders in palladium and platinum in 2007 and 2008 (Doering and Rampton, 2010).

B. Information sharing and collusion under risk aversion

Information sharing: When risk-averse dealers share information their certainty about the orders of other dealers streamlines the analysis considerably, because $\sigma_\vartheta^2 = Cov(\vartheta, \mu) = 0$ and $Var_1^{IS}(\pi_d) = \theta^2\sigma_\varepsilon^2(D_{3d}^2 + D_{1d}^2)$. Equilibrium trading shares can be expressed in closed form as shown in Lemma 5.

Lemma 5: In equilibrium when risk-averse dealers share information about client orders, dealer d with net fix orders F_d trades the following amounts:

b. $D_{1d} = \alpha^{R,IS}\bar{F}$, $\qquad \alpha^{R,IS} = \frac{(2+N+\theta\gamma\sigma_\varepsilon^2)}{(2+N+\theta\gamma\sigma_\varepsilon^2)^2-(1+N)} \qquad \frac{1}{3} < \alpha^{R,IS} < \alpha^{IS} < 1.$ (22a)

c. $D_{3d} = -q^{R,IS}\,\alpha^{R,IS}\bar{F}$, $\qquad q^{R,IS} = \frac{1}{2+N+\theta\gamma\sigma_\varepsilon^2}$, $\qquad 0 < q^{R,IS} < q^{IS} < \frac{1}{2}$. (22b)

Risk-averse dealers who share information take smaller proprietary positions in period 1 and liquidate a smaller share of that position in period 3 than their risk-neutral counterparts. Convexity is rising in dealer risk aversion and in random non-fix trading, σ_ε^2. Convexity is falling in N because the free-riding incentive still operates, so rising N brings larger proprietary positions and accelerated liquidation.

When dealers collude, equilibrium trading strategies can again once again be expressed in closed form, as shown in Lemma 6.

Lemma 6: With a cartel's proprietary positions is smaller with risk-averse than with risk-neutral dealers. With risk-averse dealers the cartel bangs-the-close and the average pre-fix price path is strictly convex:

a. $D_{1Tot} = \alpha^{R,C} F_{Tot},\qquad \alpha^{R,C} = \frac{(2+\theta\gamma\sigma_\varepsilon^2)}{(2+\theta\gamma\sigma_\varepsilon^2)^2-1},\qquad \alpha^{R,C} < \frac{2}{3} = \alpha^C < 1$. (23a)

b. $D_{3Tot} = -q^{R,C}\alpha^{R,C}F_{Tot},\qquad q^{R,C} = \frac{1}{2+\theta\gamma\sigma_\varepsilon^2}$, $\qquad 0 < q^{R,C} < q^C < \frac{1}{3}$ (23b)

C. Evaluation of the model under risk aversion

if dealers are risk-averse and competition is limited (low N) the model is consistent with all four documented dealer behaviors and all three documented features of fix-price dynamics. Under collusion the model necessarily predicts all four behaviors and all three features of fix-price dynamics. As discussed in Section II, the other existing models of dealer behavior at the fix (Evans, 2017; Saakvitne, 2016) are inconsistent with some or all of these dealers behaviors and fix-price dynamics.

VI. A simple test of the model

We next take the model to the data by testing its prediction that convexity should be higher under collusion than under other independent trading or information sharing. Five major forex dealing banks have pleaded guilty to collusion and market manipulation, admitting that it occurred between 2008 and 2013 (Department of Justice, 2015). According to the model, this collusion would have brought higher convexity, other things equal. We develop a rigorous measure of convexity and test statistically whether it was higher after December 2007 than before.

Our high-quality data comprise tick-by-tick OTC prices from a Thompson-Reuters price aggregator that takes the best executable bid and ask quotes from EBS, Reuters Matching, Reuters Dealing, and other platforms.[10] The data begin in February 1996 for JPY, GBP, CHF, CAD, NZD, and DKK and in January 1999 for EUR. Data for all currencies end in May 2013 but for CHF we drop all observations after October 2011, when the Swiss National Bank began to support a floor on EUR-CHF. We focus exclusively

[10] Reuters considers the full list of platforms to be proprietary information.

on end-of-month trading days following Melvin and Prins (2015) and use log mid-quotes at the end of each minute.

To capture the shape of the average pre-fix path we first index each path to 100.0 at exactly 3:45, reverse the direction of any price path that declines overall between 3:45 and 4:00 pm, and take averages of those paths. We then measure the convexity of a given average path. As depicted in Figure 4A, our measure involves two areas: the area between the actual path to the fix and the no-change path, $Area$; and the area of the triangle formed by the linear price path and the no-change path, ΔABC. Convexity is defined as $(\Delta\mathrm{ABC} - Area)/\Delta\mathrm{ABC}$. Like the measure of convexity used to evaluate the model, this measure is positive If the price path accelerates to the fix, zero if the path is linear, and negative otherwise.

Figure 4B shows convexity for the exchange rates of Figure 1. The first observation for each series is convexity using data from the beginning of the sample through December 2002. Each subsequent observation shows convexity using a progressively longer sample. Early in the sample convexity was negative for some currencies, consistent with the model when dealers with low risk or moderate aversion trade independently. By the end of the sample period convexity was positive for all currencies. Critically, convexity rose over the sample period for all currencies as predicted by the model.

To test the statistical significance of this finding we first calculate convexity separately for the periods before and after December 2007. Our null hypothesis is that each currency's convexity was drawn from the same distribution after 2007 as before, which implies that each currency's convexity had a 50% chance of being higher from 2008 to 2013 than from 1996 through 2007. Each currency thus represents a single Bernoulli trial with $p = 0.50$ and the number of currencies with rising convexity has a binomial distribution with $N = 7$ and $p = 0.50$ so long as convexity is independent across currencies.

The assumption that convexity is independent across these exchange rates might seem questionable given the tight links among currency returns forged by triangular arbitrage. However, convexity is many steps removed from returns: it is a complex property of the underlying return process calculated from an average of price series sampled on month-end dates over many years. We examine whether independence is a reasonable assumption by calculating the convexity of each end-month pre-fix path for each currency using the full sample of data. This gives 208 convexity values for each of GBP, JPY, CAD, NOK, and DKK; 191 values for CHF; and 173 values for EUR. Of the 21 bilateral correlations across these convexity series, only eight are positive and the overall average is just -0.005. Independence appears to be a reasonable assumption.[11]

Each currency's convexity before and after December 2007 is presented in Figure 4C. As one would expect given Figure 4B, convexity is higher during the period of admitted collusion for each currency. The binomial test rejects the null hypothesis of no change in convexity with marginal significance 0.008. We conclude that convexity was higher during the period of alleged collusion.

VII. Practical lessons from the model

Before closing we use the model to analyze three questions of practical relevance. First: can the striking price dynamics associated with the London 4 pm fix be consistent with an efficient market? Second: Why did these properties of fix-price dynamics survive the reforms of 2015? Third: could price dynamics and dealer behavior be moderated under alternative fix-calculation methodologies?

A. Market efficiency

The model highlights a potential gap between two dimensions of market efficiency that are usually conflated: speculative efficiency and informational efficiency. These two need not occur simultaneously at the fix because the fact that client fill-at-fix orders are priced after their quantity has been agreed. To clarify this we compare regular arrival-price trades with fill-at-fix orders.

Consider a dealer who makes a regular arrival-price trade with a client, in which quantity and price are set simultaneously. In general, the dealer cannot directly profit from the transaction, beyond the half-spread. If he restores his inventory to its original level or purchases more for his own account in the interbank market, any interbank trades will generate slippage that is unprofitable to himself. Further, this dealer is a strategic substitute with other dealers: the trades mentioned above will move the market towards informational efficiency, leaving fewer profits for other dealers adopting the same strategy.

[11] We also conducted this test assuming that the probability of a change under the null exceeds 50%, in case convexity was pushed upward for reasons unrelated to collusion. The null is still rejected at the 5% level if the underlying probability of a rise in convexity was as high as 65%.

Now consider a dealer who receives a fill-at fix order from a client, in which the client and dealer agree initially on a quantity but not a price. This dealer can profit from the transaction in multiple ways that do not involve the half-spread. First, when he purchases the amount needed to service the client he moves the price the price in a direction that is profitable to himself, though adverse for the client, because the amounts purchased early in the pre-fix interval will ultimately be sold to the client at a higher price. In addition, the dealer can profitably exploit that trend by opening a proprietary position. Nonetheless, the dealer's rational behavior pushes the price beyond its informationally-efficient value and that level will not be arbitraged away because his trading magnifies rather than diminishes the other fix dealers' expected profits from adopting the same strategy. Thus each individual fix is speculatively efficient, because every dealer's trading is incentive-compatible, but informationally inefficient, because the fix price consistently differs from the asset's fundamental value. Indeed, the model provides a closed-form measure of the divergence, $E_{0d}\{P_F - P_3\} = \theta q \alpha F_{Tot} \neq 0$, the dependence of which on $q\alpha F_d$ shows explicitly that it arises from the amount of the dealers' proprietary trading that is not liquidated prior to the fix.

Prior to 2015 the strategic complementarities at the fix were not limited to fix dealers. Clients and smaller dealing banks (Bulow et al., 1985) were also linked to the dealers in interlocking chains of mutually reinforcing incentives. Chain 1: The clients chose to avoid risk by placing fill at fix orders; the dealer's execution of fix orders generated pre-fix price trends and thus additional volatility; the additional volatility encouraged clients to place more orders. Chain 2: The conditionally-predictable trends motivated the dealers to open proprietary positions; those positions generated yet more volatility and encouraged funds to place yet more fix orders. Chain 3: Small and mid-sized dealers, observing the pronounced pre-fix volatility, chose to avoid executing fix orders by passing their fix orders on to the bigger banks; this would have increased the concentration of fix orders at the major banks which strengthened those banks' information advantage, encouraging them to take larger proprietary positions; larger proprietary positions would have brought further volatility, thereby encouraging more clients to place fix orders.[12]

[12] The smaller banks' exit from fix trading was confirmed in conversation by a former trader, Alexis Stenfors.

Evans (2017) presents evidence that a trader could profitably exploit the post-fix retracements by trading against the informational inefficiency. This would imply that the market was both speculatively and informationally inefficient at the fix prior to reforms. According to our analysis, however, that strategy would have left money on the table. The profit-maximizing strategy was, instead, to exploit the entire fix trading context rather than just the informational inefficiency at the 4:00 pm. The broader strategy involved proprietary trading before the fix, and of course such proprietary trading was common (Grabiner, 2014). When dealers exploit the entire fix context the market is informationally inefficient, as noted by Evans (2017), but speculatively efficient.

B. Price dynamics since reforms

Pre-fix volatility remains high and retracements are still common despite the reforms of 2015 (Ito and Yamada, 2017b; van der Linden, 2017). The model provides a ready explanation for this, to understand which it is helpful to distinguish two sets of reforms: those of the regulators and those of the banks. The regulators extended the fix calculation window from one to five minutes, a change that would be captured in the model as an increase in the variance of non-fix trading, σ_ε^2. This could have reduced the dealers' proprietary positions or accelerated the liquidation of those positions. However, it brought no structural change in the dealers' incentives and, thus, in their overall optimum strategy.

The banks themselves instituted two reforms that did change dealer behavior. First, they prohibited the fix dealers from conversing in private with competing dealers. This certainly impeded collusion and information sharing, though ironically it could have enhanced dealer profits by eliminating free-riding. Second, the banks required all fix trades to be processed via automated algorithms that distribute trades over the pre-fix interval. This essentially prohibits strategic trading by fix dealers.

If fix dealers had been the only agents to exploit their private information about client fix orders, as assumed in existing theoretical models including our own, the banks' reforms would probably have brought substantial changes in fix-price dynamics. However, non-dealers may have taken over from the dealers in strategic trading, and thereby sustained the characteristic features of fix-price dynamics. The early pre-fix price trend always provided a signal to non-dealers of the market's aggregate net fix order, which could have enabled rational non-dealers to adopt the speculative strategy identified by the model: open a proprietary position immediately and liquidate it progressively before and after the fix. Fix dealers were chagrinned to observe such behavior among other market participants as early as 2006 (Grabiner, 2014). The automated execution of fix orders in 2015 increased the precision of that pre-fix signal, after which more non-dealers could have found it profitable to adopt the model's speculative strategy. SmartFix, a software package released after the fix reforms and marketed to active traders, facilitates fix-based speculative strategies among non-dealers (Albinus, 2016). Such trading by non-dealer would generate and thus sustain excess volatility and retracements.

C. Alternative fix structures

The model also provides useful insights regarding other proposed reforms to the London 4 pm fix, most notably a clearing auction for fix orders. This could certainly provide a mechanism for matching off the maximum possible number of fix buy and sell orders. However, the clearing auction would be of no use in identifying an equilibrium price because fill-at-fix orders are perfectly price inelastic: clients have instructed their banks to trade a specific amount regardless of the market price. The price for the matched orders would be indeterminate and would therefore be taken from some other market source, which once again opens the price to manipulation. The "clearing" auction would also not identify a price and counterparties for the remaining unmatched orders.

Non-fix trading is price elastic, so one proposal for fix reform involves eliciting non-fix orders to match the net fix order imbalance. This idea essentially describes the status quo and has already been tried on the NASDAQ with disappointing results. In the late 1990s the NASD began publishing market-on-close imbalances shortly before 4:00 pm (Cushing and Madhavan, 2000). Our analysis suggests that this could intensify instead of dampen fix-price dynamics because rational non-dealers, when informed of the dealers' net order imbalance, will front-run. The NASDAQ dropped this approach and instituted a closing call in 2004.

The foregoing analysis has the following implication: fill-at-fix orders have negative externalities. According to our analysis the execution of such orders brings higher volatility, higher risk for anyone trading at the fix, and high client execution costs even if dealers do not engage in misconduct. Each of these consequences brings incentives for dealers to front-run or collude.

These observations highlight an uncomfortable trade-off. On the one hand, fill-at-fix orders are a natural and ex-ante reasonable solution to challenges faced by buy-side clients. On the other hand, long-established economic logic points to a specific remedy for negative externalities: calibrated disincentives for the behavior that generates them. If some sort of calibrated disincentive were considered with respect to client reliance on fix orders it would be important to keep in mind that decades of experience confirm that every attempt at financial regulation introduces its own set of negative externalities.

VII. Summary

This paper examines dealer behavior at the London 4 pm fix in forex, an exchange rate benchmark price relevant to index funds valued at $11 trillion (Cochrane, 2015). We develop a model of dealer conduct and misconduct at the fix. The model's assumptions are based on core microstructure theories and evidence. It is also based on the actual structure of the forex market, which allows us to correctly identify dealer incentives and constraints. We examine the dealer strategies and fix-price dynamics under independent trading, information sharing, and collusion.

The outline of the optimal fix-dealer trading strategy is the same under all circumstances. Before the each fix dealers opens a proprietary position (or equivalently, front-runs his fix orders) and then executes his client-service trades. The dealer begins liquidating his proprietary position before the fix and finishes doing so after the fix.

The paper introduces a new form of free-riding to the literature, in which fix dealers attempt to exploit the anticipated trading of their competitors. Free-riding dealers open larger proprietary positions and liquidate a larger share of those positions before the fix.

Collusion maximizing profits because it shuts down free riding; information sharing is associated with the most free-riding and thus minimizes profits. This ranking of profitability is consistent with traditional models of collusion among oligopolists. The mechanisms behind them are entirely distinct, however. Fix profits are determined in part by the price path while profits in traditional models are determined by the price level.

We evaluate the model in part by comparing its predictions to seven documented features of the fix that are not common to regular trading. Regulatory reports document four unusual dealer behaviors:

taking proprietary positions, executing client-service trades before the fix, and colluding (FCA, 2014b–f; CFTC, 2015). Earlier studies document three unusual properties of fix-price dynamics: high pre-fix volatility, post-fix retracements, and an acceleration of the pre-fix trend (Michelberger and White, 2016; Evans, 2017; Ito and Yamada, 2017b). The model predicts all but two of these seven features of the fix unconditionally. The model predicts the remaining two features – banging the close and an acceleration of the pre-fix trend – when competition is limited, as it arguably was prior to June of 2013.

Five major banks have acknowledged that their dealers colluded at the fix beginning around 2008, and the model implies that convexity is highest under collusion. We test the model by examining whether convexity rose after 2007 in seven highly-liquid currencies vis-à-vis USD. The results reject the hypothesis that convexity was unchanged after 2007 in favor of the alternative hypothesis that convexity was higher under collusion.

Future research could usefully endogenize customer fix orders within the context of this model. This would clarify some of the interlocking chains of strategic complementarities that allowed these dealer behaviors to persist for years.

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Appendix A: Details of the Model Solution

A.1. Rational expectations equilibrium under risk neutrality and independent trading

In period 2, dealer d must forecast the amount the other dealers will sell in period 3, $E_{2d}\{\sum_N X_n\}$. This forecast necessarily depends on his period-2 information set, $\Omega_{2d} \equiv \{F_d, D_{1d}, P_1 - P_0\}$. To identify the functional form of this expectation, we assume that it is linear and apply the method of undermined coefficients:

$$E_{2d}\{\textstyle\sum_N X_n\} = A(P_1 - P_0) + BD_{1d} + CF_d. \tag{A.1}$$

The coefficients A, B, and C are identified from rationality constraints. The first is that dealers should expect their own period-3 inventory, as a share of their net fix order, to be neither more nor less than the unconditional expected value of that share:

$$\frac{E_{1d}\{X_d\}}{F_d} = E_0\left\{\frac{\sum_{N+1} X_n}{\sum_{N+1} F_n}\right\}. \tag{A.2}$$

This implies the following equality which can only be satisfied if A = 0:

$$\frac{E_{1d}\{X_d\}}{F_d} = \frac{\alpha}{2}\left[1 - B - \frac{A}{B}(1+\rho N)\right] = \frac{\alpha}{2}\left[1 - B - \frac{A}{B}(1+N)\right] = E_0\left\{\frac{\sum_{N+1} X_n}{\sum_{N+1} F_n}\right\} \quad . \tag{A.3}$$

A second rationality constraint is that dealers should not make predictable errors in forecasting the other dealers' aggregate position at the beginning of period 3, ξ.

or: $E_{1d}\{\xi_d\} \equiv E_{1d}\{\sum_N X_n - E_{2d}\{\sum_N X_n\}\} = 0$. With A = 0, this implies:

$$E_{1d}\{\xi_d\} = E_{1d}\left\{\left(\frac{1-B}{2}\right)\textstyle\sum_N D_{1n} - BD_{1d} - \frac{C}{2}\left(\sum_N F_n + 2F_d\right)\right\} = 0. \tag{A.4}$$

This can be solved for B and C by considering (a) the model's symmetry, which implies that $\alpha_n = \alpha_m$ for all n, and (b) the structure of fix orders, which implies $E_{1d}\{F_n\} = \rho F_d$. Equation (A.4) becomes $E_{1d}\{\xi_d\} = 0 = \alpha_1[B(2+\rho N) - \rho N] - C(2+\rho N)$ or

$$C = \alpha\left(\frac{\rho N}{(2+\rho N)} - B\right). \tag{A.5}$$

Applying this to Equation (A.1) reveals that $E_{2d}\{\sum_N X_n\}$ depends only on D_{1d}:

$$E_{2d}\{\textstyle\sum_N X_n\} = \frac{\rho N}{2+\rho N} D_{1d} \quad . \tag{A.6}$$

Thus $B = \rho N/(2+\rho N)$ and C =0. In combination with Equation (5), this implies:

$$X_d = \frac{1}{2+\rho N} D_{1d} \equiv qD_{1d}. \tag{A.7}$$

A.2 Rational expectations equilibrium with risk aversion under independent trading

Risk-averse dealer d begins optimizing by evaluating expected profits and the variance of profits for the period-2 trading decision. Unexpected profits for the period-2 decision are:

$$\pi_d - E_{2d}\{\pi_d\} = D_{1d}\theta(1 - \widehat{\alpha^R})(\vartheta + \varepsilon_2 + \varepsilon_3) + (D_{1d} - X_d)\theta(\mu - \varepsilon_3). \tag{A.8}$$

Dealer d's sources of risk include his error in forecasting the other dealers' net fix order, $\vartheta \equiv \sum_N F_n - E_{2d}\{\sum_N F_n\}$, with variance σ_ϑ^2, and his error in forecasting the other dealers' excess inventory at the beginning of period 3, $\mu \equiv \sum_N X_n - E_{2d}\{\sum_N X_n\}$, with variance σ_μ^2. The variance of profits conditional on period-2 information, $Var_2(\pi_d)$, also depends on the covariance of these forecast errors, $\sigma_{\vartheta_d,\mu_d}$:

$$Var_2(\pi_d) = \theta^2\left\{\widehat{\alpha^R}^2 F_d^2\left[2\sigma_\varepsilon^2 + (1-\widehat{\alpha^R})^2\sigma_\vartheta^2\right] + (\alpha^R F_d - X_d)^2(\sigma_\varepsilon^2 + \sigma_\mu^2)\right\} \tag{A.9}$$

$$+\,\theta^2\{2\alpha^R F_d(\alpha^R F_d - X_d)[(1-\widehat{\alpha^R})\sigma_{\vartheta_d,\mu_d} - \sigma_\varepsilon^2]\}.$$

The prediction-error properties are partially endogenous because they depend on the dealers' proprietary trading. In equilibrium these errors, as well as their variances and covariance, necessarily depend on the three underlying sources of randomness: φ, η, and ε.

To identify utility-maximizing period-2 trading, dealer d applies Equations (4) and (5) to his overall optimization problem, Equation (2). The first-order condition for X_d implies:

$$X_d = D_{1d}\frac{1+\gamma\theta(\sigma_\varepsilon^2+\sigma_\eta^2)+\gamma\theta(1-\alpha^R)\sigma_{\vartheta_d,\mu_d}}{2+\gamma\theta(\sigma_\varepsilon^2+\sigma_\eta^2)} - E_{2d}\{\textstyle\sum_N X_n\}\frac{1}{2+\gamma\theta(\sigma_\varepsilon^2+\sigma_\eta^2)}. \qquad \text{(A.10)}$$

$$\chi = \frac{\boldsymbol{\alpha}}{2} - \frac{E_{2d}\{\sum_N X_n\}}{F_d} \quad .$$

Dealer d's proprietary trading is again linear in his period-1 trading but the proportionality coefficient, q^R, now depends non-linearly on risk. To identify $E_{2d}\{\sum_N X_n\}$we again assume that it is linear in the dealer's information:$E_{2d}\{\sum_N X_n\} = A(P_1 - P_0) + BD_{1d}$ (this excludes F_d based on the analysis of A.1).[13] We once again infer that $A = 0$ from the rational expectation constraint that a dealer expects his proprietary trading, as a share of his fix orders, to equal the unconditional average share of proprietary trading. B can once again be identified from the rational expectation constraint that the dealer's period-2 expectation error should have expected value of zero conditional on period-1 information: $B = \rho N[1+\gamma\theta(\sigma_\varepsilon^2+\sigma_\eta^2)+1+\gamma\theta(1-\alpha^R)\sigma_{\vartheta_d,\mu_d}]/(2+\gamma\theta(\sigma_\varepsilon^2+\sigma_\eta^2)+\rho N)$.

Applying this to Equation (A.10) gives the following solution for period-3 inventory:

$$X_d = q^R\alpha^R F_d \ , \quad q^R = \frac{1+\gamma\theta(\sigma_\varepsilon^2+\sigma_\eta^2)+1+\gamma\theta(1-\alpha)\sigma_{\vartheta_d,\mu_d}}{2+\gamma\theta(\sigma_\varepsilon^2+\sigma_\eta^2)+\rho N} \qquad \text{(A.11)}$$

Dealer d next identifies the variance of profits from the perspective of period 1, $Var_1(\pi_d)$:

$$Var_1(\pi_d) = \theta^2\sigma_\varepsilon^2[2-q^R(1-q^R)] + \sigma_\vartheta^2[(1-\widehat{\alpha^R})(1-q^R)q^R\widehat{\alpha^R}]^2 \quad . \qquad \text{(A.12)}$$

The period-1 trading strategy in Lemma 5 solves the dealer's period-1 optimization problem:

$$\underset{\alpha}{Max}\ \alpha F_d\theta[F_d(1-\alpha)+\textstyle\sum_N F_n(1-\hat{\alpha})] + \alpha F_d(1-q)\theta[q\alpha F_d + \sum_N X_n\}] - \frac{\gamma}{2}(\alpha F_d)^2 Var_1(\pi_d)\ , \qquad \text{(A.13)}$$

where the superscript "R" is suppressed for brevity.

[13] The irrelevance of F_d is confirmed in unreported analysis.

Figure 1: Exchange rate dynamics around the London 4 pm fix

Mean price path from 60 minutes before to 60 minutes after the London 4 pm fix using tick-by-tick quotes from Reuters Dealing, and interbank trading platform, for EUR-USD, GBP-USD, USD-JPY, USD-CHF, CAD-USD, NZD-USD, and DKK-USD. The series begin in February 1, 1996, except EUR-USD, which begins January 1, 1999. All series end on December 31, 2013 except CHF-USD, which ends in October 2011. All series are indexed to 100 at 3:45 pm. Declining prices have trends reversed for the average.

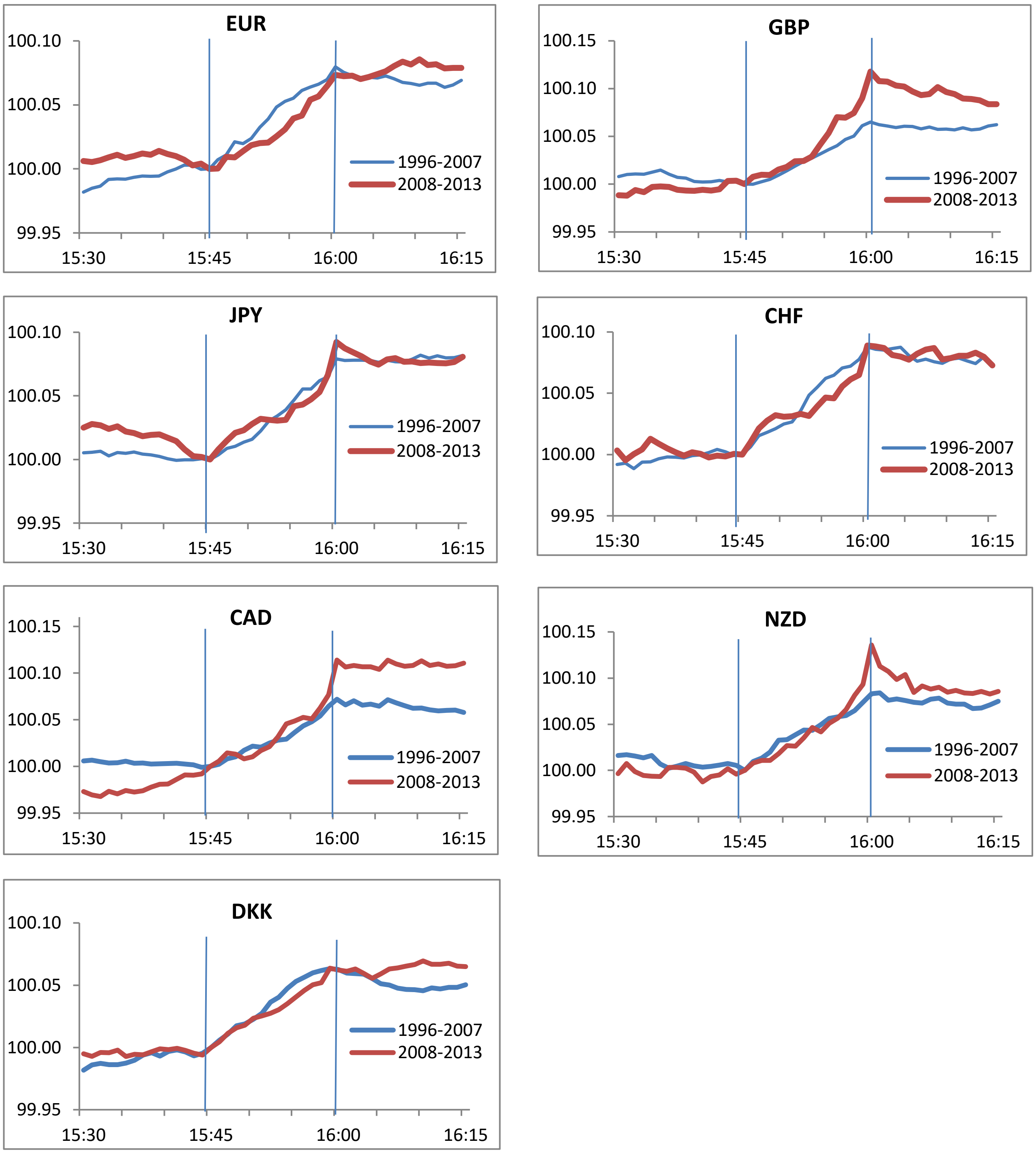


Figure 2: Banging-the-close under risk aversion

Reproduction of the image at minute 10:00 from the fix trading history in FCA (2014b) showing fix-dealer trading before an ECB fix (1:15 CET) during the period of admitted collusion. Best bid (ask) shown as thin blue (red) line; best Size and time of marketable limit orders (MLOs) are indicated by vertical green bars.

Figure 3: Banging-the-close under risk aversion

Charts show simulated levels of proprietary trading and period-2 trading relative to period-1 trading for N risk-averse dealers trading independently ($N \geq 1$) or colluding (N=0). For all simulations $\rho = 0.5$ and $\sigma_\varepsilon^2 = 1$. Charts end at $N = 9$ to ensure differences at N=0 are readily apparent. With high (moderate) risk aversion $\gamma\theta = 0.5$ ($\gamma\theta = 0.25$). With high (low) fix-order risk $\sigma_\eta^2 = \sigma_\varepsilon^2 = 1$ ($\sigma_\eta^2 = \sigma_\varepsilon^2 = 0.1$).

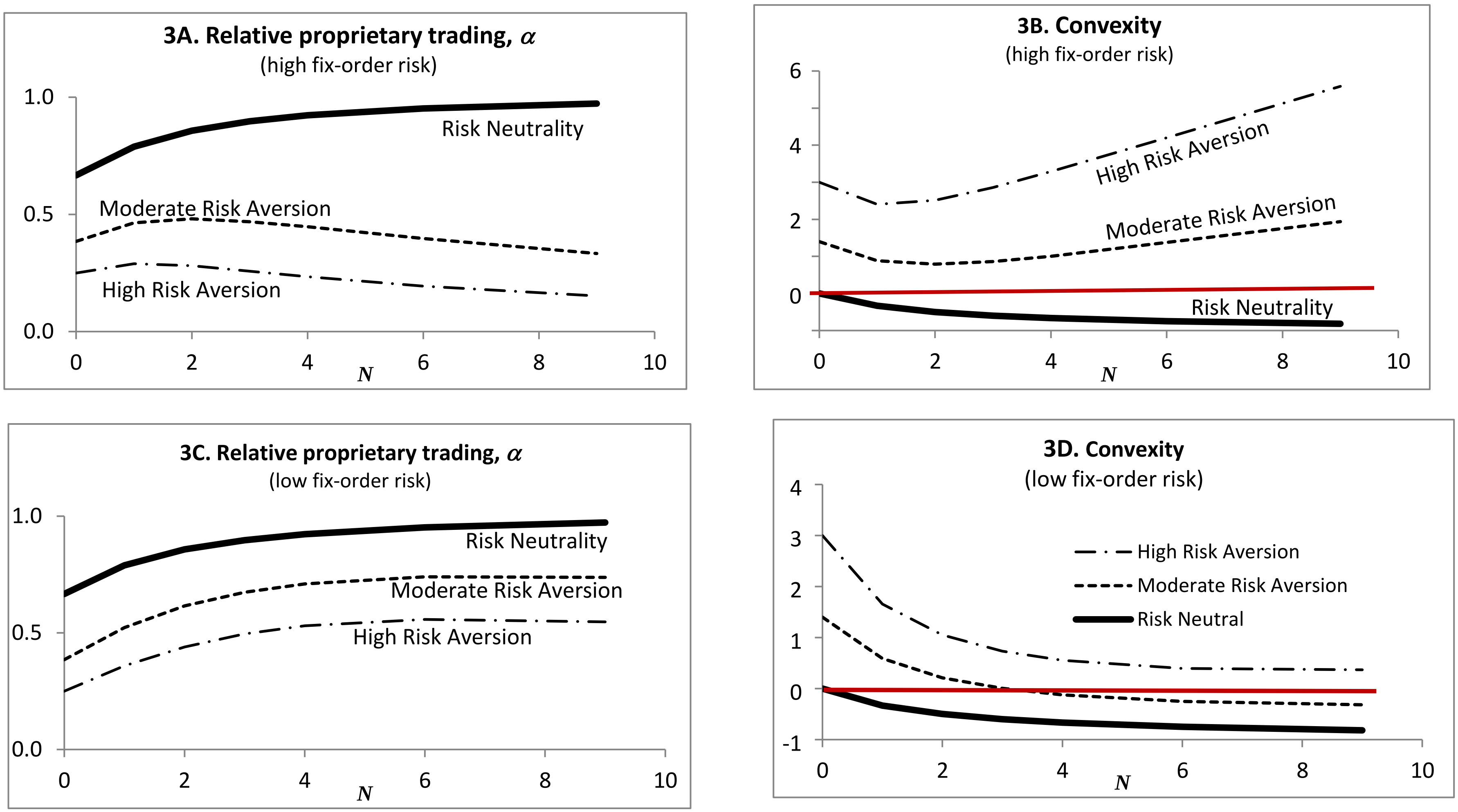
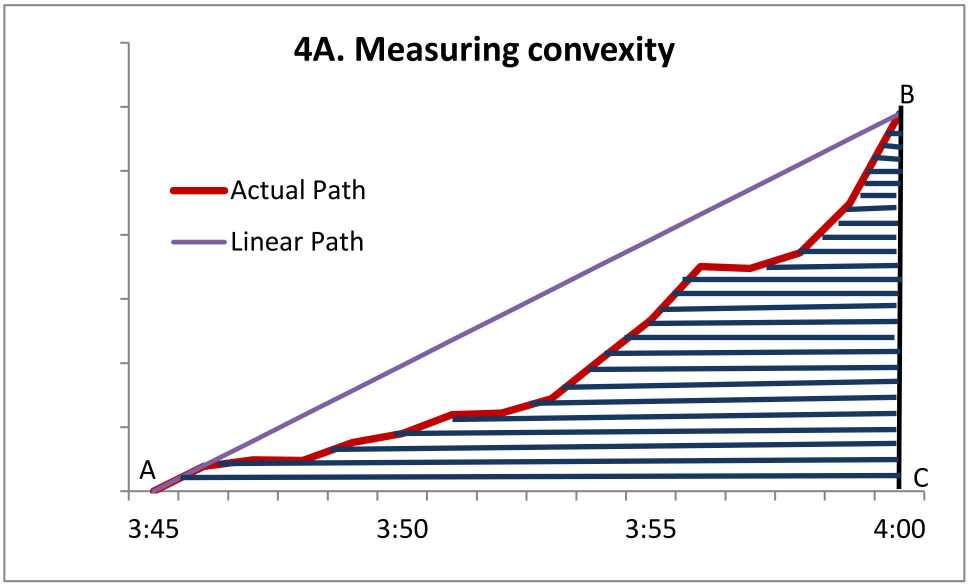
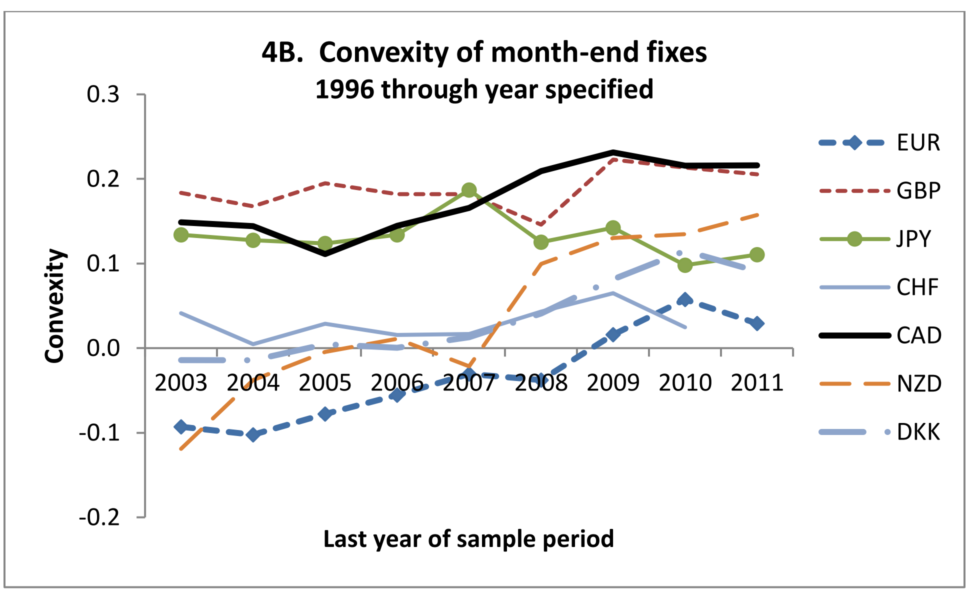


Figure 4. Testing for rising in convexity after 2007

4A. Measuring convexity. We measure convexity as the ratio of (i) the difference between the area of triangle ABC and the shaded area; (ii) the area of triangle ABC. If the price path accelerates to the fix, on average, this measure is positive and vice versa.



4B. Convexity time series: Chart shows convexity of the average month-end price path over 3:45-4:00. Data for each observation span the beginning of the sample through the end of December in the year specified. One-minute returns calculated from tick-by-tick data from Reuters Dealing. For most currencies these begin January 1996, and end May 2013. For EUR the data begin January 1999. For CHF the data end October 2011.



4C. Convexity before and after December 2007.