Easy Money: the Inefficient Supply of Inside Liquidity*

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Abstract

The money supply composition has shifted towards liquid securities created by financial intermediaries. However, the recent financial crisis has highlighted the fragility of this source of liquidity. Therefore, I create a model where currency, safe liabilities and risky liabilities all provide liquidity services. During normal times, intermediaries are able to fully satiate the demand for liquidity. This corresponds to a large drop in liquidity supply during a crisis because of the defaults from risky liabilities. Nevertheless, a welfare maximizing planner would like to reduce or eliminate these changes in the supply of liquid asset. Liquidity and capital requirements can restore efficiency, but they are sensitive to calibration and may be ineffective when analyzed individually.

Keywords: Currency, Inside Money, Liquidity Requirements, Capital Requirements

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1 Introduction

A commonly held view is that money aggregates should include liquid securities issued by intermediaries, such as money market deposit accounts. Otherwise, empirical relationships about these aggregates broke down after the 1980s, as documented in Lucas and Nicolini (2015). Money aggregates should also include other types of liquid securities issued by intermediaries, such as repo and commercial paper, because they provide liquidity for financial transactions (Lucas (1990), Geromichalos, Licari, and Suárez-Lledó (2007), Bigio (2015), Herrenbrueck and Geromichalos (2017), Piazzesi and Schneider (2018), and Lagos and Zhang (2018)). However, the 2008 financial crisis highlighted that the liquid securities issued by intermediaries differ in an important way from other components of money aggregate: they entail substantial liquidity risk. Indeed, as the financial system conditions deteriorated, these securities quickly lost both their value and their ability to provide liquidity services.[1] Many have argued that the sudden stop in the provision of liquidity was a key driver of the Great Recession. The crisis led to a revision of the financial system regulation with the objective of reducing the financial fragility caused, among other things, by too few high-quality, liquid assets. The result was the Dodd-Frank act domestically and the Basel III agreement internationally.

Motivated by these observations, I study the equilibrium and socially optimal composition of liquidity supply. Households demand liquid securities to finance their consumption needs. These liquid securities are supplied by a central bank in the form of fiat currency or by intermediaries-issued liabilities. Some intermediaries issue safe or riskless liabilities, which always provide liquidity services.[2] Others issue risky liabilities, whose liquidity value is lost in the event of a default. Therefore, default events are associated with sudden collapses in the availability of liquidity. The endogenous supply of safe and risky liquidity by intermediaries is a key feature that distinguishes this work from the previous literature. I find that endogenous fluctuations in the aggregate supply of liquidity are ex-ante inefficient. Therefore, a planner would implement policies like liquidity or capital requirements with the objective of reducing defaults in the economy.

Given this framework, I first study the composition of aggregate liquidity resulting from a competitive market. Every period there are two states of the world. A normal state, where capital productivity is high. And a bad state, where capital productivity is low and risky securities default. In an equilibrium in which currency has positive value and only safe

[1] Gorton and Metrick (2012) and Gorton, Laarits, and Metrick (2018) document this phenomenon in the repo market

[2] See Gorton (2017) for the historical evidence about the liquidity provision of safe assets

securities are issued, overall real balances adjust to keep the aggregate amount of liquidity constant. However, in an equilibrium where risky securities are also issued, real balances adjust to keep only the expected marginal value of liquidity services constant. That is, the amount of currency and of safe and risky liabilities changes in order to keep expected benefit of holding an additional unit of any liquid asset constant. But households' consumption is tied to the amount of available liquid securities, so how liquidity is allocated among the possible states of the economy feeds directly into welfare.

The households' desire to hold liquid assets is reflected in a liquidity premium that intermediaries can capture by issuing safe or risky liabilities. However, issuing safe securities is costly since equity is necessary to be always able to deliver on the promised return. Additional balance sheet costs, like the FDIC insurance, also contribute to increase the cost of funding with safe liabilities. On the other hand, issuing a risky security is subject to both a default premium, from the lost value after a default, and to a smaller liquidity premium, since any liquidity service is lost after default. In equilibrium these effects balance out and intermediaries issue both safe and risky securities. Furthermore, the amount of liquidity is so abundant in normal times that household's demand is fully satiated. This is the first prediction of the model, as "easy money" is available to satisfy all consumption needs.

A consequence of this equilibrium is that the economy is exposed to liquidity risk, since intermediaries' default causes the aggregate amount of liquidity to change over time. In fact, the second prediction of my model is that these changes in the levels of liquidity are always inefficient, since the collapse in consumption after risky securities default more than compensates the increase in consumption in normal times. The externality arises from the issuance of risky securities. Intermediaries do not internalize that their presence in the market depresses the amount of safe securities and currency needed to keep the expected marginal amount of liquidity constant. A welfare maximizing planner would generally prefer to fully equalize the amount of available liquidity across states or at least decrease its variability compared to the competitive equilibrium.

This inefficiency creates a role for government intervention through regulation. The third contribution of this paper is to study the impact of liquidity and capital requirements as revised in the Basel III agreement. While global compliance is voluntary, both the Federal Reserve and the European Union have implemented the accord as part of the financial system regulatory reforms.

In this model, liquidity requirements mandate intermediaries to back up a fraction of their liabilities with government issued currency. I find that they can increase welfare and take the economy to the planner's solution when appropriately designed. Nevertheless, a necessary condition to achieve efficiency is to impose stricter requirements on the issuers of risky securities. Intuitively, the planner would like to discourage the issuance of risky liabilities and therefore make it more expensive to issue them.

Similarly, capital requirements direct intermediaries to issue an amount of equity equal to at least a given fraction of their assets. I find that they can increase welfare when the requirements are sufficiently large. However, they are not necessarily effective in steering the economy towards the desired allocation of liquidity. That is, multiple equilibria exist after the implementation of the policy. The economy can either transition to the welfare-improving equilibrium or stay in the same inefficient equilibrium where "easy money" is readily available.

Related Literature

My work is related to the new monetarist literature started by Lagos and Wright (2005), in which households demand liquid assets for transaction services. While this literature focuses on micro-foundations for valued fiat currency, I take a reduced from approach similar to Lucas and Stokey (1987) to focus on the role of intermediaries. Within this class of models, Lagos and Rocheteau (2008) study how money and capital can be a competing medium of exchange. Their analysis is further refined in Gu, Mattesini, Monnet, and Wright (2013) and Gu, Mattesini, and Wright (2016), whose work focuses on the role of banking and credit in expanding the set of feasible allocations. In particular, Gu et al. (2016) compare money with credit to show how real balances adjust to keep the amount of liquidity constant.[3] My framework provides a similar result, with the additional feature that intermediaries can default on their liabilities.[4] In this context, the key equilibrium condition is that the expected marginal value of liquidity is constant, implying that liquidity can fluctuate across states and create a welfare loss. Geromichalos and Herrenbrueck (2016) consider an economy where an illiquid asset can be exchanged for a liquid one in a frictional search model. Similarly, my model includes securities that can provide liquidity and therefore demand a liquidity premium that increases with inflation. Finally, Andolfatto, Berentsen, and Waller (2016) study monetary policy where money is backed by an illiquid capital, which is exactly the type of asset-backed security that intermediaries intermediaries issue in my model.

This paper also relates to a long literature in the creation and demand for liquidity, starting

[3]Lacker and Schreft (1996) look at a similar problem, but where money and credit have different user cost.

[4]These liabilities are claims backed by a risky asset, as in Lagos (2011).

from the work in Gorton and Pennacchi (1990). Holmström and Tirole (1998) address whether governments should create or regulate liquidity to stimulate efficient investment. Eisfeldt (2004), Bigio (2015) and Kurlat (2017) identify adverse selection as the key elements for asset illiquidity. Bianchi and Bigio (2017) then look at how intermediaries manage their liquidity risk and how monetary policy affects the issuance of credit. I combine all of this work to inform a stylized model where securities can lose their liquidity properties. Finally, Benigno and Robatto (2018) consider the efficient supply of liquidity when safe government bonds[5] are available together with safe and risky securities from intermediaries.[6] They then study tax-based policies to correct potential inefficiencies. This paper follows their base framework, while using fiat currency as the asset of choice for the public supply of liquidity. This lets me address different regulatory policies that have not been studied from a consumption-based point of view.

The safe assets literature (such as Caballero and Farhi (2017); Diamond (2016); Farhi and Maggiori (2017); Li (2017); Magill et al. (2016); Stein (2012); and Woodford (2001)) has modeled how these assets provide liquidity services. Given this setting, I extend the liquidity provision property to risky assets as well, as long as the economy is in the good state. Gorton and Metrick (2012) show how risky assets quickly lose their liquidity value in the repo market during the 2008 financial crisis.

My results also point out that there might be an excessive amount of credit, as in Lorenzoni (2008) and Moreira and Savov (2017). Furthermore, the inefficiency can be understood as a lack of safe assets, whose effects have been studied in Caballero (2006) and Caballero and Farhi (2017). These papers all stress the importance of fiscal capacity to implement active policies, while I focus my attention on the outcomes from regulation that does not require direct government intervention.

2 Environment

Time is discrete over an infinite horizon. As in the new monetarist models pioneered in Lagos and Wright (2005), each time period is divided into two sub-periods, morning and evening. There is a single consumption good, which is produced in the morning of every period and can be freely stored until the evening, after which it fully depreciates. Production transforms a fixed and non depreciating supply of capital $\bar{K}$ into the consumption good through a linear

[5] Krishnamurthy and Vissing-Jorgensen (2012) document how the treasury bond market is driven by the demand for safe and liquid assets.

[6] Magill, Quinzii, and Rochet (2016) consider the case where only private debt provides liquidity service and analyze the consequences for monetary policy.

technology $Y_t = A_t \bar{K}$, where A_t is an aggregate shock on capital productivity and the only source of uncertainty in the model. The shocks are independent and identically distributed according to

$$A_t = \begin{cases} A_h & \text{with probability } 1-\pi \\ A_\ell & \text{with probability } \pi \end{cases},$$

with $A_h > A_\ell$. Define $\bar{A} \equiv (1-\pi)A_h + \pi A_\ell$ as the average productivity of capital. In what follows, I will refer to a realization of A_h as the good or high state and a realization of A_ℓ as the bad or low state.

The economy is populated by an infinitely lived representative household, a central bank, and a continuum of two-period lived, overlapping generations of intermediaries. The central bank controls the supply of fiat currency, M_t, through lump-sum transfers to households. Intermediaries manage capital, while supplying debt securities and equity in the economy. The household cannot manage capital,[7] thus it invests in intermediaries to transfer resources intertemporally. Furthermore, the household is subject to a liquidity constraint, where it must finance its morning consumption with a combination of currency and securities. These two securities are not perfect substitutes when it comes to their liquidity value. First, only a fraction θ of a security face value can be used to finance morning consumption. Second, a security may be defaulted upon in the bad state, in which case it loses all of its liquidity value and cannot be used in morning transactions. However, a default does not imply a total loss for the security holder, since the value of the assets backing the security can still be recovered in the evening.[8]

I assume that intermediaries honor their obligations as long as they are below the value of the assets in the balance sheet. Consequently, a default event in this model occurs whenever the asset side of an intermediary's balance sheet is insufficient to cover the issued securities face value. For clarity of exposition, I will call the intermediary issuing of a safe security "commercial bank" and the issuer of a risky security "shadow bank". The corresponding securities will be denoted as b^c and b^s respectively. To create a safe security, the commercial bank will have to issue equity n^c, so that the limited liability constraint is never binding.

Table 1 summarizes the timing of the model. At the beginning of the morning the aggregate shock realizes, resolving all uncertainty for the time period. Therefore, production occurs and all prices are determined, so that the household can make its morning consumption

[7]That is, the household has an infinitely high management cost for capital. A similar setup can be found in Gertler and Kiyotaki (2010) and in Gertler and Karadi (2011).

[8]Intermediaries securities can be interpreted as collateralized loans, as in Gorton and Ordonez (2014). Thus, in the event of a default, the borrower can still obtain the value of the collateral.

Morning	Evening
• Aggregate shock is publicly observed • Production • Prices & default determined • Household's morning consumption	• New generation of intermediaries enters • Lump sum monetary transfer • Household's evening consumption and portfolio choice • Old generation of intermediaries dies

Table 1: Model Timing

choice. Then in the afternoon, a new generation of bankers is born and the central bank makes the lump-sum monetary transfer. Subsequently, the household makes its portfolio choice, so that the new generation of bankers can acquire capital from the old generation of bankers.

2.1 Household's Problem

Household's preferences are characterized by a quasi-linear per period utility over morning and evening consumption[9]

$$U_t = \log c_t^{am} + c_t^{pm}.$$

Morning consumption c_t^{am} is subject to a liquidity constraint: the household can use only currency or securities to finance morning consumption. The two assets are imperfect substitutes in terms of their liquidity value. While currency can be freely used to finance any morning transaction, securities use is subject to two constraints. First, I assume that only a fraction $0 < \theta < 1$ of the face value of a security can be immediately redeemed. That is, securities provide less liquidity services than fiat currency.[10] Barnett (1982) argues that different securities provide different amount of liquidity services, as measured by their user cost, and this should be taken into account when measuring aggregate liquidity. Second, securities come into two different varieties. Some securities are risk free; thus they will never be defaulted upon. Others are risky and will be subject to default in the bad state. If that is the case, the defaulted security loses its liquidity value and cannot be used in morning transactions. While this might look as a stark assumption, since it ignores any secondary market or debt collection services that may be available for defaulted securities, it is a useful

[9]The log utility is a useful device to recover closed form solutions, but the results of this paper can be extended to a more general quasi-linear utility $U = u(c^{am}) + c^{pm}$, where $u(\cdot)$ is an increasing and concave function that satisfies standard Inada conditions.

[10]This is a also a common assumption in the Kiyotaki and Moore (1997) literature and in new monetarist models with multiple sources of liquidity, like Lagos and Rocheteau (2008).

simplification to highlight the main forces of the model. Furthermore, it is the case that liquidity of an asset deteriorates quickly as its rating declines, as documented in Benmelech and Bergman (2018).

The morning liquidity constraint can therefore be written as

$$c_t^{am} \leq \varphi_t M_{t-1} + \theta \left(1 + r_{t-1}^c\right) b_{t-1}^c + \theta \left(1 - \mathbb{I}_t\right) \left(1 + r_{t-1}^s\right) b_{t-1}^s, \tag{1}$$

where $\mathbb{I}_t$ is an indicator function such that $\mathbb{I}_t = 1$ when the shadow bank defaults on its securities. Taking the morning consumption as the numeraire good, the real value of a unit of money is denoted by φ_t.

The asset timing follows the cash-in-advance and new monetarist tradition. The asset allocations that determine the morning liquidity constraint need to be made in the previous period, before the aggregate capital productivity realizes. The household lends b_{t-1}^c to the commercial bank and its first order condition pin down the equilibrium interest rate $1+r_{t-1}^c$. Another way to interpret these amounts is to normalize the price of the liability to one at issuance. Then $1 + r_{t-1}^c$ represents the face value of an equivalent zero coupon bond issued by the commercial bank and b_{t-1}^c is the quantity issued. The same holds true for a shadow bank issued security b_{t-1}^s.

Once the period moves forward to the evening, the household makes its evening consumption and asset allocation decisions subject to a budget constraint

$$c_t^{pm} + \varphi_t M_t + b_t^s + b_t^c + n_t^c \leq W_t + T_t, \tag{2}$$

where W_t is the household's wealth at the beginning of the evening, which can be expressed as

$$W_t = \varphi_t M_{t-1} + \left(1 + r_{t-1}^c\right) b_{t-1}^c + \left(1 - \chi_t\right) \left(1 + r_{t-1}^s\right) b_{t-1}^s + (1 + r_t^n) n_{t-1}^c - c_t^{am}, \tag{3}$$

That is, household wealth is defined by the value of the assets carried over from the previous period minus the amount used for morning consumption. The transfer T_t includes the real value of the monetary transfer from the central bank and any taxation levied on the intermediaries and then rebated to the household. The monetary component of the lump-sum transfer implements the desired monetary policy, either as a helicopter drop if the central bank is expanding the available currency in circulation or as a tax if currency is reduced.[11]

[11] Clearly, it is necessary to assume that the central bank is credible when announcing its policy and can then enforce it with the household. This assumption is not crucial for any of the paper results. The model can be adjusted to account for imperfect enforcement, but it would be an additional mechanism that masks some of the economic forces.

The realized return on commercial bank equity is denoted as r_t^n . This is a random variable and determined as a residual from the intermediaries' profits, as I will detail in the following section. Finally, $1-\chi_t$ represents the recovery rate of a security that might default. If no default happens, then the full face value of the risky security is paid and $\chi_t = 0$. If instead the intermediary is in a state of default, then $\chi_t > 0$ and the recovery rate is determined endogenously as the value of the assets in the defaulted intermediary's balance sheet.

The household chooses a plan for state contingent consumption and asset holdings to maximize its expected utility over the infinite time horizon, or

$$\max E\left[\sum_{t=0}^{\infty}\beta^t U_t\right]$$

Subject to the morning liquidity constraint (1) and the evening budget constraint (2), where $0<\beta<1$ represents the discount factor. Define η_t as the Lagrange multiplier for the liquidity constraint (1) and λ_t as the multiplier for the budget constraint (2) at the time period t. Taking the first-order conditions gives

$$\begin{aligned}
\text{w.r.t. } c_t^{am}: \quad \tfrac{\beta^t}{c_t^{am}} &= \lambda_t+\eta_t \\
\text{w.r.t. } c_t^{pm}: \quad \beta^t &= \lambda_t \\
\text{w.r.t. } b_t^{s}: \quad \lambda_t &\geq E\left\{\left[\theta\left(1-\mathbb{I}_{t+1}\right)\eta_{t+1}+\left(1-\chi_{t+1}\right)\lambda_{t+1}\right]\left(1+r_t^s\right)\right\} \\
\text{w.r.t. } b_t^{c}: \quad \lambda_t &\geq E\left\{\left[\theta\eta_{t+1}+\lambda_{t+1}\right]\left(1+r_t^c\right)\right\} \\
\text{w.r.t. } n_t^{c}: \quad \lambda_t &\geq E\left\{\lambda_{t+1}\left(1+r_{t+1}^n\right)\right\} \\
\text{w.r.t. } M_t: \quad \varphi_t\lambda_t &\geq E\left\{\varphi_{t+1}\left(\eta_{t+1}+\lambda_{t+1}\right)\right\}
\end{aligned}$$

where all inequalities hold with equality if the choice variable is strictly positive. Divide both sides by the value of λ_t to get the Euler equations

$$n_t^c: \quad 1 \geq E\left\{\beta\left(1+r_{t+1}^n\right)\right\} \tag{4}$$

$$M_t: \quad 1 \geq E\left\{\frac{\varphi_{t+1}}{\varphi_t}\left(\frac{\eta_{t+1}}{\beta^t}+\beta\right)\right\} \tag{5}$$

$$b_t^c: \quad 1 \geq E\left\{\left[\theta\frac{\eta_{t+1}}{\beta^t}+\beta\right]\left(1+r_t^c\right)\right\} \tag{6}$$

$$b_t^s: \quad 1 \geq E\left\{\left[\theta\left(1-\mathbb{I}_{t+1}\right)\frac{\eta_{t+1}}{\beta^t}+\left(1-\chi_{t+1}\right)\beta\right]\left(1+r_t^s\right)\right\} \tag{7}$$

These equations illustrate the structure of the returns in the economy. Equity return is pinned down by a simple stochastic discount factor, as in traditional asset price models. Then currency and securities have a liquidity premium component that is governed by the Lagrange multiplier η_t on the morning liquidity constraint. Of course, the liquidity premium is larger for currency, which is the most liquid asset in the economy. Securities follow, with risky securities always having a smaller liquidity premium than safe ones, as implied by the presence of the indicator function. Furthermore, risky securities returns are also subject to a default premium, measured by the default loss rate χ_{t+1}.

From the first order conditions of the problem, one can also derive the following relationship

$$c_t^{am} = \frac{\beta^t}{\lambda_t + \eta_t} = \frac{\beta^t}{\beta^t + \eta_t}, \tag{8}$$

which means that morning consumption c_t^{am} is decreasing in the Lagrange multiplier $\eta_t \geq 0$. Therefore, the maximum level of morning consumption is $c_t^{am} = 1$ and it can be achieved if and only if $\eta_t = 0$ (i.e. the morning liquidity constraint is not binding). If so, marginal utilities of consumption are equalized between morning and evening.

The intuition behind this result is simple, since it would be a standard outcome in a utility maximization problem with two goods subject to a budget constraint. Therefore, the household would always like to increase its holdings of assets that can relax the morning constraint, even though it results in higher demanded returns for those assets. Whether or not the supply of liquid assets is positive in equilibrium is an outcome of the interaction between monetary policy and intermediaries.

2.2 Intermediaries' Problem

Financial intermediaries' main role is to manage capital and to provide liquidity in the economy. That is, they take an asset as capital that households cannot manage directly and transform it into a different asset that relaxes household's morning liquidity constraint. Thus, in the context of this paper, liquidity coincides with facilitating transactions, rather than the classic definition of liquidity as the ability to quickly transform an asset into currency with little to no losses on its face value. Examples of assets with such properties include demand deposits, certificates of deposit, and commercial paper.

How risky is a security is determined by the operational choices of an intermediary, given that they can commit to a contract, but they have limited liability. If an intermediary operates as a commercial banker, then the balance sheet needs to be structured so that issued securities that never default. I assume this is achieved by issuing equity, an asset that only has a residual claim to the intermediary's profits. If instead the intermediary chooses to operate as as shadow banks, whose liabilities default in the bad state, then he has no need to raise equity to back security returns.

Intermediaries live only for two periods, as in an overlapping generations model. At a given time period t there is a measure one of competitive intermediaries that issue securities and raise equity (if needed) to invest in capital. At time $t+1$, the intermediary observes the return on capital, repays its creditors and liquidates its equity with dividends, if any.

More formally, a newborn intermediary has a choice between two contracts. If the intermediary chooses to operate as a commercial bank, it will have to raise debt and some equity in order to make its securities default free. Furthermore, a commercial banker is subject to some balance sheet costs τ proportional to the size of its assets, to be paid in units of consumption good. The balance sheet costs reflect the regulatory cost of the banking activity. In the United States, the the major source of regulatory costs for institutions that offer demand deposits and other safe instruments is the FDIC insurance.[12] If instead the intermediary chooses to operate as a shadow banker, he is not going to be subject to as much regulation and can issue risky securities that default in the bad state of the world. Thus, a shadow banker not subject to the balance sheet costs and does not need to raise equity. In both cases debt is implicitly collateralized by capital, and the modeling choice can be thought as a simplified version of the collateral equilibrium framework described in Geanakoplos and Zame (2002), Geanakoplos (2003), and Geanakoplos and Zame (2014).

In this environment, an intermediary that chooses to operate as a commercial bank is subject to the balance sheet constraint

$$(1+\tau)\, q_t^k K_t^c = b_t^c + n_t^c, \tag{9}$$

where τ represents the balance sheet costs. The price of capital q_t^k is an "ex-dividend" price, since the capital production for the current period is enjoyed by the older generation of intermediaries. Securities b_t^c offer a real return $1+r_t^c$ promised at the time of issuance, thus before the new aggregate shock A_{t+1} realizes. The return on capital $1+r_{t+1}^k$ and the one on equity $1+r_{t+1}^n$ are instead stochastic and depend on the state realization. Consequently, the expected profits of a newborn commercial banker are given by

$$E\left[\Pi_{t+1}^c\right] = E\left[1+r_{t+1}^k\right] q_t^k K_t^c - (1+r_t^c)\, b_t^c - E\left[1+r_{t+1}^n\right] n_t^c, \tag{10}$$

[12]See Afonso, Armenter, and Lester (2018) and Banegas and Tase (2017).

where the expected return on capital and the expected return on equity are to be determined in equilibrium. The return on securities $1 + r_t^c$ is not in an expectation term, since the promised return on securities must always be delivered in full in order to create a safe security. Then, it must be the case that the limited liability constraint is never binding for the commercial banker. That is, the return on capital in the event of a bad shock is sufficient to repay the promised return on securities, or

$$\left(1 + r_\ell^k\right) q_t^k K_t^c \geq \left(1 + r_t^c\right) b_t^c, \tag{11}$$

where $1 + r_\ell^k = {}^{\left(A_\ell + q_{t+1}^k\right)}/_{q_t^k}$ is the return on capital after drawing the low aggregate productivity state in period $t+1$. The problem of the commercial banker is to choose capital K^c, securities b^c, and equity n^c in order to maximize equation (10) subject to the balance sheet constraint (9) and the liability constraint (11), taking as given prices and returns.

The balance sheet constraint for a shadow banker at time t is given by

$$q_t^k K_t^s = b_t^s, \tag{12}$$

and its expected profits in the following period are

$$E\left[\mathit{\Pi}_{t+1}^s\right] = E\left[1 + r_{t+1}^k\right] q_t^k K_t^s - E\left[\left(1 - \chi_{t+1}\right)\left(1 + r_t^s\right)\right] b_t^s, \tag{13}$$

where $1 + r_t^s$ is the promised return on the shadow bank securities and $0 < 1 - \chi_{t+1} < 1$ is the recovery rate on the promised return. By design of the contract, the shadow bank securities never default in the good state of the world. Thus $\chi_h = 0$ if the economy is in the good state. However, when a negative one is drawn, the shadow banker will partially default of its liabilities and the recovery rate $1 - \chi_\ell$ is pinned down by the limited liability constraint

$$\left(1 + r_l^k\right) q_t^k K_t^s = \left(1 - \chi_\ell\right)\left(1 + r_t^s\right) b_t^s. \tag{14}$$

The objective of a shadow banker is to choose securities b^s and capital K^s in order to maximize its profits (13), given the balance sheet (12), taking as given the recovery rate pinned down by (14), prices and returns.

To wrap up the problem of a newborn intermediary, the decision between commercial and shadow banking operation is taken according to the contract that returns the highest expected profits, or

$$E\left[\Pi_{t+1}\right] = \max\left\{E\left[\mathit{\Pi}_{t+1}^c\right], E\left[\mathit{\Pi}_{t+1}^s\right]\right\}.$$

2.3 Market Clearing and Equilibrium Definition

To close the model, the central bank sets a constant monetary policy described by

$$M_{t+1} = (1+\mu)\,M_t, \tag{15}$$

where μ is positive when the central bank is expanding the quantity of nominal currency in circulation and negative when reducing it. As this is fiat currency, thus unbacked by any asset, the central bank can potentially implement any policy it wants, as long as the implied returns of money do not violate the household transversality conditions. Since the objective of this paper is to study ex-ante prudential regulation, I will simply take central bank policy as a given and solve for the resulting equilibrium.

Now that the description of all the economic actors is complete, I can define an equilibrium in this economy

Definition 1. An equilibrium is a set of

- Sequences of state contingent prices $\varphi_t,\ q_t^k$and returns $r_t^s,\ r_t^c,\ r_t^n$
- Default rates χ_{t+1}
- Household's choices of $M_t,\ b_t^s,\ b_t^c,\ n_t^c,\ c_t^{am}, c_t^{pm}$
- Intermediaries' choices of $K_t^c,\ b_t^c,\ n_t^c,\ K_t^s,\ b_t^s$

Such that:

- Households maximize their utility, given prices, returns and default rates
- Intermediaries maximizes profits, given prices, returns and default rates
- The free entry condition holds
- The government budget is satisfied

$$T_t = \mu M_{t-1} + \tau q_t^k K_t^c$$

- Markets clear, including

$$\begin{aligned}\bar{K} &= K_t^c + K_t^s \\ Y_t &= c_t^{am} + c_t^{pm} = A_t \bar{K}\end{aligned}$$

While this definition allows for a variety of equilibrium paths, I will focus on stationary equilibria, where prices (except for the price of money φ_t), returns and quantities (except for fiat currency) are constant over time. Since the quantity and price of currency are not constant, in a stationary equilibrium only the real value of money $m_t = \varphi_t M_t$ is constant over time.

3 Main Economic Forces

In this section I am going to consider simplified versions of the model to illustrate the basic economic forces at work and derive some basic results that will be helpful to study the complete model. First, I am going to look at the case where only a commercial bank operates. Second, at the case where only a shadow bank operates. As I show in the following section, both cases are actual corner solutions of the general model, so the following discussion is also useful to fully characterize the results.

The purpose is to highlight the degree of complementarity and substitutability between currency and securities, and what are the consequences on consumption outcomes. This also sheds some light on the debate on the role of privately issued forms of money. That is, what are the advantages and the limitations of a free banking system. While this has been the object of economic research since the inception of the discipline,[13] the modern debate around the topic can be summarized into two positions. On one hand there is Hayek (1976), who argued that we should have competition among different types of money within a country until the best one prevailed. On the other, Friedman (1960) argues that liquidity provision should be tightly controlled by the government.

Before moving to the specific cases of the model, it is useful to derive some general results that apply regardless of the simplifications that I am going to make in the following sections. First consider any monetary policy that satisfies constant growth of nominal currency from (15), where I am going to refer to $\mu \geq \beta - 1$ as the growth rate of money. The lower bound for it is pinned down by the Friedman rule, where the nominal interest rates hit zero. In order to achieve a stationary equilibrium, where the real value of money is constant, it must be the case that the price of currency φ_t is moving in equal and opposite direction, or

$$\frac{\varphi_{t+1}}{\varphi_t} = \frac{1}{1+\mu}. \tag{16}$$

Note that this growth rate is exactly the inverse of the inflation rate, thus in this model

[13] See Smith (1776) or Bagehot (1873).

issuance of currency generates an equal amount of inflation.

Having established a path for money prices, I can revisit the money holding Euler equation (7). Assuming that the household wants to hold a strictly positive quantity of currency, the equation simplifies to

$$1+\mu = E\left[\frac{\eta_{t+1}}{\beta^t} + \beta\right].$$

For the rest of the paper, I am going to redefine the morning constraint multiplier η_{t+1} to its current value multiplier form, such that $\eta_{t+1} = \beta^t \kappa_{t+1}$. $\kappa_t \geq 0$ then represents the value of relaxing the morning constraint, that is, the marginal value of liquidity for the household. Thus equation (7) further simplifies to

$$\bar{\kappa} = E\left[\kappa\right] = 1+\mu-\beta, \tag{17}$$

where I will refer to $\bar{\kappa}$ as the expected or average liquidity premium in the economy. This equation is capturing a crucial aspect of this model: in any equilibrium, the expected value of the marginal utility of aggregate liquidity must be constant.[14] The real balances of all the liquid assets will then adjust in all states of the world to satisfy this condition.

As I am going to show in the next sections, the average liquidity premium is one the essential quantities that determines the welfare outcomes in the economy. In fact, morning consumption from equation (8) can be written as a function of the wedge created by the realized liquidity premium ${}^{\kappa_t}/_{\beta}$:

$$c_t^{am} = \frac{1}{1+{}^{\kappa_t}/_{\beta}}. \tag{18}$$

Given that the average value of liquidity is constant,

3.1 A Model with Only Commercial Banks

Consider the model where the only choice for an intermediary is to offer a safe security, thus operate as a commercial banker. To further simplify the analysis, also assume that the commercial banker does not sustain any balance sheet costs ($\tau = 0$). Using equation (5) from the household problem, in a stationary equilibrium the commercial bank security needs to offer a return equal to

$$1+r^c = \frac{1}{\theta\bar{\kappa}+\beta}. \tag{19}$$

Given that equity has an expected return $E\left[1+r^n\right] = 1/\beta$, the term $\theta\bar{\kappa}$ is a measure for the liquidity premium of securities. In this context, it is even more transparent how the param-

[14] This result is in line with Gu et al. (2016)

eter θ can be interpreted as the relative liquidity value between currency and commercial bank securities, since the implied return of currency is $^{1}/_{(\bar{\kappa}+\beta)}$.

Having solved for the returns of assets, I can now look at the profit maximization problem of a commercial banker. Because the expected profits of a commercial bankers are linear in the asset allocation and that the return on securities is lower than the expected return of equity, the banker would issue only securities if he had no further constraints. However, the commercial banking contract requires him to provide safe securities that never defaults, as implied by constraint (11). Thus, the constraint must be binding, and I can solve for securities expressed as a fraction of the total assets

$$\frac{b^c}{q^k \bar{K}} = \frac{1+r_\ell^k}{1+r^c}, \tag{20}$$

where I replace $K^c = \bar{K}$ to account for the market clearing conditions. Another way to interpret this equation is as the debt fraction of a banker's liabilities, with the remaining fraction being equity. Along the equilibrium path this fraction needs to be less than one, which implies that in any equilibrium where a commercial bank is present there is no way to offer a safe security without raising some equity.

Because of free entry, expected profits need to be zero in equilibrium, thus

$$E\left[1+r^k\right] q^k \bar{K} = (1+r^c)\, b^c + E\left[1+r^n\right] n^c.$$

Divide this equation by the total value of capital $q^k K$ to write the expected return on capital as a weighted average between the safe return on securities and the expected return on equity

$$E\left[1+r^k\right] = (1+r^c)\,\frac{b^c}{q^k \bar{K}} + E\left[1+r^n\right]\left(1-\frac{b^c}{q^k \bar{K}}\right), \tag{21}$$

where the portfolio weights are given by the composition of the balance sheet liabilities. This is a relevant feature of the model, as the return on capital is only pinned down by the demand coming from the financial sector of the economy. In an economy without banks in which the households can hold capital but cannot do liquidity transformation, the expected return on capital would be equal to the expected return of equity in the model. Thus, in the presence of liquidity premium, the return on capital must be below the return implied by the household's discount factor. That implies that there is some charter value in the banking activity that would be destroyed without the financial system. General equilibrium forces connect this charter value directly to the demand for liquidity from the representative household.

Therefore, when only a commercial bank operates, the expected return on capital is

$$E\left[1+r^k\right]=\frac{1}{\beta}-\frac{1-\beta}{\beta}\frac{\bar{A}-\theta(\bar{A}-A_\ell)\bar{\kappa}}{\bar{A}\beta+A_\ell\theta\bar{\kappa}}. \tag{22}$$

First note how the expected liquidity premium $\bar{\kappa}$ appears in this formula. With no liquidity premium, the formula collapses to the standard return $1/\beta$. Otherwise, capital returns are a decreasing function of the average liquidity premium. Indeed, this is what the general equilibrium forces would suggest, as the increased but unsatisfied demand for securities depresses their returns and in turn reduces the return for capital. Second, the low state productivity A_l appears explicitly in the formula, and not only as part of the average productivity of capital $\bar{A}$. This is because the limited liability constraint pins down the structure of the balance sheets, thus the weights in equation (21).

The return on capital also gives a solution for the price of capital, that can be used to solve for the real amount of securities and equity issued by the commercial bank. More importantly, these securities never default, so the liquidity conditions for the household must be the same regardless of the state. In other words, there is no variance in the liquidity premium term, so $\bar{\kappa}=\kappa_l=\kappa_h=1+\mu-\beta$. Therefore, equation (18) implies that morning consumption is equalized between the two states and expected one period utility in the stationary equilibrium is

$$E\left[U\right]=\log\left(\frac{\beta}{\beta+\bar{\kappa}}\right)+\bar{A}\bar{K}-\frac{\beta}{\beta+\bar{\kappa}}, \tag{23}$$

where the linear term is expected evening consumption. This illustrates how the liquidity premium in the economy is the determinant factor for welfare. Furthermore, it also highlights how monetary policy has welfare implications, since the expected utility is decreasing in the inflation rate μ.

The non neutrality of money in this environment arises mechanically from the presence of the liquidity constraint that governs morning consumption. However, it still highlights the role of an inflation tax in the economy. Namely, higher levels of inflation in this model push consumption into the future (i.e. the evening), as the return of holding currency becomes smaller and the representative household does not want to hold as much of it.

A second consequence is that the highest expected utility is achieved when the growth rate of currency μ is the smallest, at $\underline{\mu}=\beta-1$. This is not a surprising result, since it is standard in monetary models. As the nominal interest rate hits zero, the return of currency is equalized to the expected return of a Lucas tree, thus the rate of return dominance disappears. Then also the wedge from the morning liquidity constraint must disappear, otherwise the household would demand more money to satisfy its liquidity needs. If the morning constraint is no longer binding, then the morning and evening marginal utilities are equalized, which is the general condition for optimality.

After discussing the banker and the household problem, I can close the model by looking at the real value of money m. All the previous discussion assumed that the household is willing to hold a strictly positive amount of real currency. However, it is possible that the supply of liquidity from a commercial bank is so large that money is worthless.[15] Using the optimal level of consumption in equation (18) and the morning liquidity constraint (1), it is possible to solve for the real value of money and derive the following result:

Proposition 1. *Given a monetary policy μ and parameters of the model, there exist a threshold for the security liquidity $\bar{\theta}^c$ such that*

- *If $\theta \geq \bar{\theta}^c$, no monetary equilibrium exists*
- *If $\theta < \bar{\theta}^c$ there is a monetary equilibrium and the value of money is decreasing in θ*

For the proof and the closed form solution for the real value of money m see appendix A.1. Intuitively, if securities can provide abundant liquidity services, then there is no need for fiat currency. Under the interpretation suggested in Hayek (1976), this is as if commercial banks' securities emerged as the dominant currency after competition. If instead securities provide little liquidity services, then the household demands more aggregate liquidity than what the commercial bank can supply. Thus, fiat currency and securities must coexist.

3.2 A Model with Only Shadow Banks

Now consider an environment where only shadow bankers operate. That is, the only assets that can relax the household morning liquidity constraint are fiat currency and securities that default when the bad state of the world is drawn. Unlike the previous case, consumption cannot be equalized between the two states, as one asset loses its liquidity value in the event of a negative shock. Then holding currency becomes an insurance instruments against negative shocks. However, the expected return of currency is still limited by equation (17), so the insurance value of currency is constrained by the expected return that currency needs to have in a monetary equilibrium.

As in the previous section, the return of money is dominated by the return on shadow bank

[15] A non monetary equilibrium always coexists with the monetary equilibrium I am describing, as it is the case for models with fiat currency.

securities. However, the sources of dominance are different. From equation (4), the promised return on shadow bank securities is

$$1 + r^s = \frac{1}{\theta \left(1 - \pi\right) \kappa_h + \left(1 - \pi \chi_\ell\right) \beta}. \tag{24}$$

In the high state, shadow bank securities do not default, thus they enjoy a liquidity premium as measured by $(1 - \pi) \kappa_h$ and no risk premium component. On the other hand, in the low state these securities lose their liquidity value and gain a default premium component $-\pi \chi_\ell \beta$ that measures the amount of return lost in the event of a bankruptcy. So, the shadow banker is only able to capture a fraction of the liquidity premium and needs to pay an additional amount to compensate for the risk of default. Since the value lost in the bankruptcy χ_ℓ is determined in equilibrium, the overall return of the risky security may exceed the discount rate.

The return on securities then directly pins down the return on capital. As implied in equation (12), securities are the only source of financing for the shadow bank, and the free entry condition still implies zero expected profits. Then combining (12), (13), (24), and the zero profit condition solves for the expected return of capital

$$E\left[1 + r^k\right] = \frac{1 - \pi \chi_\ell}{\theta \left(1 - \pi\right) \kappa_h + \left(1 - \pi \chi_\ell\right) \beta}, \tag{25}$$

where the numerator is the result of the expansion of the expected value of the recovery rate $1 - \chi$. Note how if the liquidity needs of a household are completely satisfied in the high state ($\kappa_h = 0$), then the expected return to pay out for a banker is exactly equal to $1/\beta$, the return of an asset when there are no liquidity concerns and the expected return a consumer demands on equity.

While equation (24) pins down the promised return to the household, the shadow bank is only paying it in full in the high state. In the low state only a fraction $1 - \chi_\ell$ of the promised return is paid out as pinned down by combining (12) with (14) to get

$$1 + r^k_\ell = \frac{1 - \chi_\ell}{\theta \left(1 - \pi\right) \kappa_h + \left(1 - \pi \chi_\ell\right) \beta}. \tag{26}$$

This equation, together with (25), summarizes the interaction of the different general equilibrium forces in the financial assets market. Taking the price of capital as given, the liquidity premium in the high state κ_h and the default loss rate χ_ℓ adjust to jointly guarantee that no more securities are issued and that the limited liability constraint is binding. If equation

(25) fails, then the supply of securities, thus the overall supply of liquidity, must change to bring profits to zero. If (26) fails, then default loss rate χ_ℓ adjusts so that no value is destroyed in the bankruptcy process.

Unlike the model with commercial bankers only, in any equilibrium it must be that the household is more liquidity constrained in the bad state, or $\kappa_\ell > \kappa_h$. This is a consequence of having securities that default, and therefore the aggregate amount of liquidity changes between the states. Another difference is that money fully derives its value from the morning consumption a household can afford in the low state. Thus, the real value of money is tied to the liquidity premium in the low state κ_ℓ. Of course, it may still be the case that the corresponding issuance of securities is too high to result in an average liquidity premium of $\bar{\kappa} = 1+\mu-\beta$. I defer the details on how to solve for the high state liquidity premium κ_h and the low state loss rate after default χ_ℓ to appendix A.2. Also, since the liquidity premium κ is a factor of the Lagrange multiplier η, it must also be the case that $\kappa \geq 0$ in all states. Studying these conditions leads to the following proposition:

Proposition 2. *Given a monetary policy μ and parameters of the model, there exist a threshold for the security liquidity $\bar{\theta}^s$ such that*

- *If $\theta > \bar{\theta}^s$, no monetary equilibrium exists*
- if $\theta = \bar{\theta}^s$ *there is a monetary equilibrium with $\kappa_h = 0$ and $\kappa_\ell = {}^{(1+\mu-\beta)}/_{\pi}$*
- *If $\theta < \bar{\theta}^s$ there is a monetary equilibrium with $\kappa_\ell > \kappa_h > 0$*

This result is similar to the one derived under commercial banking, since when securities have very high liquidity value θ they can fully replace currency. However, the real value of money is no longer necessarily decreasing in security liquidity. When θ is small enough, the value of money is increasing in security liquidity. At those initial levels for θ, the liquidity value of securities is so small that the insurance motive for holding money dominates, driving up demand and therefore its price. Low security liquidity also makes them more expensive to issue, as the shadow banker can only capture a small fraction of the liquidity premium. This reduces the supply of securities and drives the household towards currency.

A second difference concerns morning consumption and welfare. In a monetary equilibrium, welfare depends on the amount of liquidity a shadow bank can issue, as parameterized by θ. Moreover, welfare can either increase or decrease as the amount of shadow bank securities, as measured by θ, increases. Starting from the case where $\theta = \bar{\theta}^s$, the liquidity needs of the household are fully satisfied in the high state, since $\kappa_h = 0$. Thus, morning consumption is also maximized in the same state. The downside of higher morning consumption in the good state is variance in consumption, as the liquidity premium in the low state is the highest possible in any equilibrium. Therefore, morning consumption is minimized in the low state. Now look at the case where $\theta < \bar{\theta}^s$. While morning consumption in the low state is still smaller than the one in the high state, the difference between the two is smaller. While reducing the variance in marginal utilities is always welfare improving, level effects may prevail. I will discuss equilibrium ranking, thus whether the supply of liquidity is efficient or not, in the following section after studying the outcomes in the more general version of the model.

4 General Case Results

In the previous section I have established the role of each type of security in providing liquidity services. Safe securities and fiat currency are close to perfect substitutes, therefore there is a role for both only if the general equilibrium forces constrain the commercial banker to a limited issuance. On the other hand, risky securities provide the household with additional consumption only in one state of the world, thus complementing fiat currency but never being a substitute for it.

In light of these facts, I will now consider the case where all securities can be issued. That is, where an intermediary has the choice of operating either as a commercial banker or as a shadow banker. The economy that emerges has a positive supply of all of the assets types, with consumption outcomes similar to the ones observed in Proposition 2. However, a different structure may emerge, with only one type of banker as described previously. In fact, under some parameterizations, there may even be multiple equilibria. This is going to lead me to the following section, where I will discuss the welfare implications of the model.

4.1 No Balance Sheet Costs ($\tau = 0$)

First consider the case where there are no additional balance sheet costs on the commercial bank operations. The main reason to look at this special case is to evaluate the equilibrium that emerges purely from the different funding structure of each intermediary. The shadow banker issues an asset that usually demands a lower return than equity. However, safe securities ask for an even lower return, so running a commercial bank may be the more profitable option for a given return on capital.

These forces combine with the linearity of the intermediary's problem to generate a set of indifference conditions that need to hold in order to achieve an equilibrium where both safe and risky securities circulate and complement currency. First, the commercial and the shadow banker must make the same profits, or one way of operating in the financial markets would dominate the other. Second, free entry implies that the expected return of capital is equal to the expected payout for risky securities and to the average cost of issuing safe securities and equity of Equation (21). Third, the total amount of securities (i.e. both safe and risky) issued must be small enough to require a positive value for fiat currency, in which case the average liquidity premium in the economy is pinned down by Equation (17).

These three conditions also provide the framework to solve for an equilibrium. The general solution method follows a guess and verify approach, where I postulate the structure of the liquidity premia and which intermediaries operate in the stationary equilibrium. I illustrate this procedure in the following example, where I show how it can also be used to rule out candidate equilibria. Suppose there exists an equilibrium where safe and risky securities are issued and the liquidity premium in the high state is zero ($\kappa_h = 0$). Monetary policy is away from the Friedman rule ($\mu > \beta - 1$). Equation (25) needs to hold, as the shadow banker must make zero profits. Since $\kappa_h = 0$, Equation (25) implies that the expected cost of issuing risky securities for the shadow banker is equal to the discount rate. Thus, the shadow bank zero profit condition requires the expected return on capital to be equal to the discount rate as well. From (4), the return on equity is also equal to the discount rate. Nevertheless, there is still a positive liquidity premium that the commercial banker is able to capture from the low state, thus the return on commercial bank securities is lower than the return on equity. Consequently, the portfolio weighted cost of funding for a commercial bank is always lower than the expected return on capital, which means that the commercial bank's expected profits are strictly positive. As a result, the shadow banker should operate as a commercial one, which is a violation of the equilibrium conditions.

The previous example rules out any equilibrium where the liquidity demand from the household is fully satisfied in one state by any combination of liquid securities. Thus, in equilibrium the household morning constraint (1) is always binding and the associated liquidity premium must be strictly positive. More generally, it is possible to construct the following equilibria involving a commercial bank:

Proposition 3. *If an equilibrium with positive issuance of safe securities exists, then it takes one of the following forms*

- *If* $\theta < \bar{\theta}^c$ *and* $\pi \geq \bar{\pi}$, *then only safe securities are issued with* $\kappa_\ell = \kappa_h = 1 + \mu - \beta$

- *If $\underline{\theta}^{cs} < \theta < \bar{\theta}^{cs}$ and $\pi < \bar{\pi}$, then both safe and risky securities operate with $\kappa_\ell > \kappa_h > 0$*

The details of the derivation can be found in the appendix A.3. Importantly, the threshold for the probability of a low state π is the same for the two possible equilibria, so the two equilibria are mutually exclusive. However, the liquidity threshold is different, since, when the both bankers operate, the increased availability of privately issued liquid instruments reduces changes the role for currency.

The first equilibrium is the same as the one described in section 3.1. However, now that the intermediary has a choice about on whether to issue safe or risky liabilities, I need to verify that a shadow banker does not find it optimal to enter and issue risky bonds. That is, issuing risky bonds must return negative expected profits. This will happen if the shadow banker can capture enough of the existing liquidity premium, so that the expected return paid on risky securities goes below the average cost of funding for a commercial bank. As risky securities only have access to the liquidity premium in the high state, the less is the high state likely, the less premium they can capture. A highly unlikely good state also increases the risk premium, further increasing the cost of issuing a risky security. Therefore, an equilibrium with a commercial bank exists only if the probability of a low state is large enough.

If instead the probability of a low state is small, then the issuer of risky securities wants to enter the market. The result is an equalization of the cost of funding, as long as one type of banker is not incentivized to expand its balance sheet beyond feasibility. This is the mechanism that drives the existence of the lower bound for the liquidity of securities $\underline{\theta}^{cs}$. As the liquidity value of securities θ decreases, the shadow banking sector controls a larger share of the capital in the economy. This is driven by an increase in the difference between the expected return on capital and the realized return in the low state. This forces the commercial bank to issue more equity to insure the return of its safe securities. The increased operational cost reduces the size of commercial banking to the point that shadow bankers would want to control more than the available capital. On the contrary, the role of the upper bound $\bar{\theta}^{cs}$ is the same as the one seen in the previous sections, where if securities bring too much liquidity value, then there is no place for money.

In terms of consumption, both types of securities circulate in the second equilibrium, thus morning consumption is differentiated between the two states. As in the equilibrium described in Proposition 2, the household is able to consume more in the high state mornings, but not as much as to completely fulfill its liquidity needs. However, the mechanism is different from the one in Proposition 2. There fiat currency was fully responsible for the consumption in the low state, so the value of currency was more sensitive to the changes in the values of parameters like the liquidity value of securities θ. In this case, the value of currency is less elastic to such changes, since part of the change is absorbed by the commercial bank's securities. In other words, the real value of money is more stable with respect to changes in the environment when other similarly safe sources of liquidity are available.

Finally, let me discuss the possibility of an equilibrium where only the shadow banker operates. This amounts to verifying whether a commercial bank would have any incentive to enter in the equilibrium describe in Proposition 2. The commercial bank does not have an incentive to enter in extreme regions of the parameter space. One way to have negative profits for a commercial bank entrant is to use an unrealistically low discount factor. If at the same time the probability of a low state is small, then a potential commercial bank would fund almost the entirety of its assets with securities, achieving an average cost of funding below the one of a shadow banker. For more reasonable values of the discount factor, an equilibrium with only risky securities requires a close to zero probability of a bad state and high levels of inflation, with growth rate of money μ in excess of 50% per period.

When such an equilibrium exists, a commercial bank only equilibrium might also exist under the same parameterization. In other words, this model allows for multiple equilibria within the class of stationary monetary equilibria. This is driven by the linearity of the problem, that pushes intermediaries to corner solutions (i.e. either zero or infinite supply of liquid assets) outside the zero profit conditions that characterize an equilibrium.

4.2 Positive Balance Sheet Costs ($\tau > 0$)

The previous sections serve as a baseline to understand the interaction between different types of liquid debt securities. However, the issuance of different types of securities is often connected to management and regulatory costs that go beyond the simple difference in returns. For instance, a financial intermediary may implement stronger monitoring practices when investing in capital backed by high grade debt (as in Benigno and Robatto (2018)). In terms of regulatory costs, the biggest one for bank holding companies is the Federal Deposit Insurance Corporation (FDIC) insurance fee (see Afonso et al. (2018) and Banegas and Tase (2017)).

To model these differences in the cost of funding, I assume that the commercial banker needs to pay an additional cost measured as a fraction $\tau > 0$ of the capital he acquires. This is effectively a capital tax, that is paid in units of the consumption good and then rebated as a lump sum to the household.[16] All things equal, the additional cost increases the return on capital required for a commercial bank to break even, thus it creates the space for new types of equilibria that were impossible in the previous case. Furthermore, it opens the possibility for multiple equilibria over the same parameter space. As different equilibria have different welfare implications, government policies also have the role of addressing selection among the multiple equilibria.

First, I am going to describe the main equilibrium that emerges under this market structure. Suppose that in equilibrium both securities are issued and that shadow bankers expand their balance sheet up to the point where the liquidity premium in the high state drops to zero. Since the commercial banker is subject to an additional cost, its profits do not become strictly positive, as it was the case in section 4.1. Then, because the average liquidity premium is constant in any equilibrium, a decrease in the liquidity premium in the high state requires an increase of the liquidity premium in the low state. The conditions for the equilibrium to exist are detailed in the following proposition, which I prove in appendix A.4.

Proposition 4. *Given a monetary policy $\mu > \beta - 1$ and parameters of the model, if $\tau < \bar{\tau}^{sc}$ and $\theta \geq \underline{\theta}^{sc}_{\tau}$, or $\tau \geq \bar{\tau}^{sc}$ and $\underline{\theta}^{sc}_{\tau} \leq \theta \leq \bar{\theta}^{sc}_{\tau}$, then there exists an equilibrium where both bankers operate and the liquidity premia are given by*

$$\kappa_h = 0 \text{ and } \kappa_\ell = \frac{1 + \mu - \beta}{\pi}.$$

The intuition for the boundary is the same as Proposition 3. If the liquidity of securities is low, then the shadow banker has an incentive to expand its balance sheet beyond what is feasible in the economy. The same holds true if the balance sheet costs τ are large. If that is the case, then the financial sector as a whole may also issue too many securities, rendering money useless. Thus, an upper bound for security liquidity exists in this parameter region.

A second possible equilibrium involves shadow banks only. As detailed in section 3.2, this equilibrium is characterized by the circulation of risky securities only, thus currency is necessary to achieve positive morning consumption in the low state. Furthermore, the equilibrium liquidity will generally not lead to the zero liquidity premium in the high state of Proposition 4. The existence of this equilibrium is limited by the commercial banker's incentives to entry. As expected, the positive balance sheet cost strongly reduces the expected profits

[16] The results would not be different if the capital tax was simply destroyed, but the assumption of a lump-sum rebate makes facilitates the welfare analysis.

of the commercial banker, making the equilibrium feasible in a more realistic part of the parameter space.

Finally, the equilibria with commercial banks only or with both bankers but positive liquidity premium in both states detailed in Proposition 3 are also possible with positive balance sheet costs. They are still mutually exclusive, but they can each exist in regions where the equilibrium of Proposition 4 exists.[17] This is where the multiplicity of equilibria for this model comes into play. The linearity of the problem makes it so that the financial sector can divide the ownership of capital, and the consequent issuance of liquid securities, in different ways that are all compatible with the equilibrium definition. Which one is picked can be the outcome of a sunspot, or the result of government policies that constrain asset issuance. This is the goal of the final section of this paper, after I show some numerical examples of the above equilibria and discuss the welfare effects of the liquidity premium distribution.

4.3 Calibration

Before discussing welfare in detail, let me calibrate the model to illustrate the equilibrium that emerges among the ones detailed in the previous section. In order to effectively calibrate the model, I will introduce a small extension, where I allow for the liquidity of bank securities to differ. That is, safe commercial bank securities have a liquidity θ^c and risky shadow bank securities have a liquidity θ^s in the household's morning cash-in-advance constraint (1).

The model is then calibrated at a quarterly frequency. I choose the balance sheet cost τ to match the cost of the FDIC insurance in the United States. Banegas and Tase (2017) estimate this cost at 7 basis points over the entire asset composition of the average balance sheet of an insured intermediary. Then I pick the productivity in the low state A_ℓ such that consumption must be positive in both sub-periods in any equilibrium. To have a significant difference between a boom and a crisis, I impose the high state productivity A_h to be approximately 10% larger than the low state one. A numerical exploration of the model suggests that the outcomes are not strongly dependent on the size of the shock.[18] Furthermore, my choice of productivity parameters is consistent with Queralto (2019), who estimates a drop of 9% in the total factor productivity after banking crises in a panel of advanced and emerging economies. The probability of a low state is chosen so that the average time between two crises is 6 years and one quarter. Given that the realization of a low state is a Bernoulli random variable with independent draws, the expected time between crises (in quarters) is given by $1/\pi$. Jordà, Schularick, and Taylor (2011) look at a panel data

[17] See appendix B.1 for a numerical illustration.

[18] The model is robust for values of A_h up to 30% larger than A_ℓ.

Parameter	Value	Source
A_ℓ	1.0007	Model condition
A_h	1.1	Queralto (2019)
$\bar{K}$	1	Normalization
τ	0.0007	Banegas and Tase (2017)
β	0.97	Real Interest Rates
μ	0.005	Federal Reserve 2% Inflation Target
π	0.04	6 Years and 1 Quarter Between Crises

Table 2: Calibration: Chosen Parameters

of financial crises to unveil an average duration between crises of 28 years, which reduces to 15 years once the no financial crises period from 1940 to 1973 is removed from the sample. Equilibrium conditions imply that the probability of a low state cannot be calibrated to these values. Nevertheless, under the calibration choice the time between recessions is longer than the approximately 4 years and 3 quarters observed in the US economy after the Great Depression.[19] Finally, I pick a standard discount factor[20] and set the growth rate of currency to the Federal Reserve inflation target of 2%. The set of parameters is summarized in Table 2.

With this choice of parameters, I then use the banking data from the Federal Reserve FR Y-9C Consolidated Report of Condition and Income form to recover appropriate values for the liquidity of safe securities θ^c and for the liquidity of risky securities θ^s. Specifically, I extract the consolidated balance sheet from Schedule HC in the third quarter of 2017 and construct the composition of liabilities in the financial sector as a whole to target the ratio between deposits and equity, and the ratio between risky liabilities and equity[21].

In the extended model, the first ratio is equivalent to the ratio between securities and equity issued by the commercial banking sector, which is given by

$$\frac{b^c}{n^c} = \frac{\theta^c (1 + \mu - \beta) + \beta}{\theta^c (1 + \mu - \beta) - \beta\tau}\tau. \tag{27}$$

On the contrary, there is no direct match for the second ratio in the model. However, since

[19]See the NBER US Business Cycle Expansions and Contractions

[20]Compared to common calibration exercises at a quarterly level, the discount factor here implies a much higher real rate, but the choice of discount factor is constrained by the characteristics of the model.

[21]Further details on the data are postponed to Appendix C

Parameter	Value	Target
θ^c	0.0233	$\frac{b^c}{n^c} = 5$
θ^s	0.0405	$\frac{b^s}{n^c} = 2.8$

Table 3: Calibration: Targeted Parameters

the data I have takes a snapshot of to the financial industry as a whole, then it is appropriate to consider the equity issued by commercial banks and the securities issued by the shadow bank as the total equity and total risky securities in the economy respectively. Note that targeting the second ratio is also equivalent to targeting the total amount of non-equity liabilities over the total amount of equity in the model economy. The target values and the resulting parameters are summarized in Table 3.

The calibration results may seem surprising, since to match the composition of bank liabilities I need to have risky securities to be more liquid than safe ones. However, recall how the securities in the model also represent a much wider class of bank liabilities than the more liquid banks. Therefore, the parameter θ combines these two elements into one constant. Consequently, the calibration suggests that there is a large fraction of deposits that are not demand deposits. In fact, in the third quarter of 2017 the timed and money market deposits were 2.5 times larger than demand deposits.[22] On the other hand, long terms risky liabilities are about twice the size of short term liabilities,[23] which illustrates the obtained result.

Furthermore, the general equilibrium forces in the model also push towards a calibration where risky securities are individually more liquid than safe ones. The mechanism is the household's aggregate demand for liquidity. Equation (17) defines what the average marginal value of liquidity must be in any equilibrium. Then, the liquidity demand satiation in a good state implies that the marginal value of liquidity is zero. Thus, the marginal value of liquidity in the bad state is pinned. This is the liquidity value that currency and safe securities provide. Similarly, the difference between the two marginal values is the liquidity value provided by risky securities. Given that the liquidity provided is proportional to θ, the higher its value, the lower is going to be the demand for the corresponding security. Conversely, since in the calibration the amount of safe securities is much larger than the amount of risky ones, then the liquidity θ^s of a single risky security should be larger than the liquidity of a safe security θ^c.

[22]This measure includes interest and non-interest bearing deposits, NOW accounts and other transaction accounts.

[23]Short term risky liabilities include Federal Funds, reverse repo and trading liabilities. Long term risky liabilities are made of other borrowings, subordinated notes and other liabilities.

Variable	Description	Equilibrium Value
K^c	Commercial Capital	68.17%
K^s	Shadow Capital	31.83%
$1-\chi_\ell$	Recovery Rate	82.75%
$m/\bar{A}\bar{K}$	Real Currency/GDP	3.9%
c_h^{am}	High State am Consumption	1
c_ℓ^{am}	Low State am Consumption	0.5257

Table 4: Calibration: Equilibrium Outcome

Under the calibrated parameterization, the competitive equilibrium is characterized as expected by the presence of both types of financial intermediaries. Specifically, shadow bankers flood the market with liquidity in good times, such that the demand for liquidity is completely satiated and morning consumption reaches its maximum value $c_h^{am} = 1$. However, these securities are unable to provide liquidity services in a crisis, thus morning consumption collapses after a negative aggregate shock.

The equilibrium outcome is summarized in Table 4. Approximately 68% of capital is held by the commercial banking sector, with the remaining part in the shadow banking sector. While the shadow banking sector is roughly half the size of the commercial banking sector, its default causes the morning consumption in the low state to be about half of the good state one. The impact on morning consumption is so large because the shadow bankers issue slightly less than half of the aggregate amount of privately issued liquid securities. Of course, the total of morning and evening consumption in the low state only falls by 10%, which is equal to the drop in production. This is reflected in a substantial recovery rate on defaulted securities, as the household is able to recover approximately 83% of the promised payment. Finally, the real value for money implies a currency to average GDP ratio of about 3.9%. This value is a slightly below what can be observed in the data,[24] especially in the light of the monetary expansion that followed the financial crisis.

5 Inefficient Supply of Liquidity

In the previous sections I have detailed the equilibrium structure of the financial sector and what is the liquidity provision in each equilibrium. The numerical illustration showed how, keeping all the parameters constant, reallocating liquidity between states can be welfare

[24] See the currency component of M1 from the Board of Governors of the Federal Reserve System

improving. Therefore, I will now study the optimal provision of liquidity.

To do so, consider the household's optimal morning consumption as written in Equation (18). Since morning consumption depends on the state contingent liquidity premium κ_t and the average liquidity premium in a monetary equilibrium is pinned down by (17), I can write stationary equilibrium welfare as a function of the high state liquidity premium κ_h. Furthermore, market clearing and the definition of transfers imply that evening consumption is $c_t^{pm} = A_t\bar{K} - c_t^{am}$. Thus, the per period expected utility in a stationary equilibrium given a monetary policy μ and high state liquidity premium κ_h is

$$W = (1-\pi)\left[\log\left(\frac{\beta}{\beta+\kappa_h}\right) + \left(A_h\bar{K} - \frac{\beta}{\beta+\kappa_h}\right)\right] + \pi\left[\log\left(\frac{\beta}{\beta+\frac{1+\mu-\beta-(1-\pi)\kappa_h}{\pi}}\right) + \left(A_\ell\bar{K} - \frac{\beta}{\beta+\frac{1+\mu-\beta-(1-\pi)\kappa_h}{\pi}}\right)\right], \quad (28)$$

where $\kappa_h \in [0, 1+\mu-\beta]$. The bounds on the liquidity premium arise from the characterization of the liquidity premia from the household's problem and the structure of the financial system, that also requires $\kappa_\ell \geq \kappa_h$.

First, let me discuss which monetary policy achieves the highest levels of welfare. In other words, what would be the monetary policy chosen by a welfare maximizing central bank. As shown in section 2.1, the highest level of welfare in a given state is achieved only if the liquidity premium is zero. Thus, if there exists a monetary policy such that the liquidity premium is zero in both states, that would immediately be a candidate for the first best monetary policy. That policy is the Friedman rule, or setting the money growth rate to $\mu = \beta - 1$. Any other policy, with $\mu > \beta - 1$, requires a positive average liquidity premium, thus morning consumption must be less than optimal in at least one of the productivity states.

While the Friedman rule is the best monetary policy in terms of welfare, it has a number of drawbacks that might make it infeasible, both in this model and as a real world tool. First, since $\mu < 0$, it requires that the central bank has the power to tax the household. Second, the Friedman rule would endogenously create a system where private agents do not engage in the transformation of liquidity. This is because no banker as defined in the model would find it profitable to enter under the Friedman rule. To have a competitive equilibrium, households must be able to hold capital directly or through a specialized manager who acts as a pass-through entity. Under these assumptions, the central bank becomes the only supplier of liquidity.

The first best nature of the Friedman rule also implies that welfare is subject to an inflation or liquidity cost that is necessary to sustain a monetary equilibrium. This cost can be divided equally, thus keeping consumption constant across states, or concentrated in the low state to increase morning consumption in the high state. The next section investigates which allocation is preferred by the household.

5.1 Second Best and Inefficient Liquidity

Since the Friedman rule cannot be implemented in a competitive equilibrium, the central bank is forced to choose a policy such that $\mu > \beta - 1$. For a given choice of the money growth rate, I define a second best supply of liquid assets which translates into the state contingent liquidity premia. While the second best outcome may not be implementable in a competitive equilibrium, it is informative of the outcomes that government policies should aim for.

First, Equation (18), combined with Equation (17), implies that the expected marginal utility with respect to morning consumption is constant in any equilibrium and equal to

$$E\left[U'\right] = \frac{1 + \mu - \beta}{\beta} = \frac{\bar{\kappa}}{\beta}.$$

Therefore, there are two channels that operate in selecting the welfare maximizing liquidity premium. On one hand, having different liquidity premia in the two states increases the variance in the marginal utility realizations, which negatively impact welfare for a risk averse household. On the other hand, having a positive liquidity premium moves consumption from the morning to the evening. Therefore, reducing the liquidity premium brings consumption back to the morning (where the household values it the most) and increases the state contingent utility level. The welfare maximization problem, shown in detail in appendix A.5, reflects these two forces and leads to the following result:

Proposition 5. *If* $\sqrt{\frac{(1+\mu)(2\beta-\mu-1)}{\beta^2}} > 1 - 2\pi$ *, then* $\kappa_h = 1 + \mu - \beta$ *is the unique welfare maximizer. If not, then welfare is maximized for some interior liquidity premium* $\kappa_h \in (0, 1 + \mu - \beta]$.

The propositions states that if the growth rate of money is small enough, then the best outcome is achieved by equalizing the liquidity premium, and therefore consumption, across all states. That is exactly what one would expect from a risk averse consumer that would always like to even out consumption over uncertain outcomes. However, as monetary policy selects higher levels of inflation and the required average liquidity premium increases, consumption does not decrease as much when you concentrate the cost of liquidity in one state. Thus, it becomes beneficial to cluster the reduction in consumption in the unlikely state and consume as much as possible in the good state.

The proposition also implies that setting the liquidity premium to zero in one state is never welfare maximizing. Therefore:

Corollary 1. *The competitive equilibrium in Proposition 4 is inefficient, in the sense that there exists a different liquidity allocation that improves on household's welfare.*

The consequences of this result can be counter-intuitive. In fact, an economy that relies only on risky securities may achieve higher levels of welfare than the one that mixes safe and risky securities. This apparent puzzle is solved by noting that in an equilibrium with only risky securities aggregate liquidity is less volatile, which is preferable when the growth rate of money μ is small. As shown in the numerical example, the welfare gains from liquidity reallocation can be sizable. Therefore, regulation that imposes further restrictions on the assets or liabilities on an intermediary's balance sheet can be a powerful tool to address inefficiencies.

The result can also be interpreted as an argument in favor of the Friedman (1960) position. If we let the markets supply a variety of liquid securities, as suggested by Hayek (1976), the competitive equilibrium involves an oversupply of liquid assets that dries out in a crisis and leads to deeper recession. However, it may not be the case that tightly controlling liquidity or imposing narrow banking is the optimal policy, especially when inflation is high or households are impatient. If that is the case, the competitive equilibrium would still be the one in Proposition 4, but the optimal policy involves only some restrictions on the issuance of risky securities. That is, with high inflation Proposition 5 states that it is optimal to have some variation in the levels of aggregate liquidity between states. Therefore, there would still be scope for a financial sector that is involved in liquidity transformation.

6 Government Policies

Having determined that the competitive equilibrium in Proposition 4 is inefficient, I turn my attention to institutional realistic interventions that have been proposed or implemented to strengthen the financial system. I choose to focus on that competitive equilibrium since it is the one that provides a better description of reality. There is abundant liquidity during normal times, but during a crisis privately created liquidity dries up, but does not completely disappear. Because the welfare analysis from the previous section is ex-ante, I will study if and how macro-prudential policies can achieve the second best welfare of Proposition 5. In particular, I will focus on liquidity requirements and equity requirements. Liquidity requirements reduce the volatility of assets and therefore reduce losses in the event of a default.[25] Similarly, equity requirements protect security holders by issuing a junior asset that is the first to absorb the losses.

6.1 Liquidity Requirements

In this section I am going to focus on liquidity requirements as a policy to address the inefficiencies in the financial markets. As currently implemented, the main objective of liquidity requirements is to avoid self-fulfilling prophecies that would lead to a bank run.[26] In fact, an intermediary can be solvent, with assets valued more than liabilities, but not able to cover unexpected cash flows.

As such, the Basel III accords introduce a liquidity coverage ratio (LCR) requirement, where intermediaries need to hold an amount of liquid assets greater or equal to their net cash flow over a 30-day stress period.[27] To implement this regulation in the model, I will consider cash as the only asset that counts toward the liquidity requirement and require intermediaries to hold a fraction of their liabilities in the liquid asset. Thus, every intermediary will have to hold an amount of fiat currency greater or equal than a fraction $0 < \delta < 1$ of the issued securities, or

$$m^c \geq \delta^c b^c \text{ and } m^s \geq \delta^s b^s.$$

This notation allows for potentially different regulation to be imposed on the two banking sectors. It is clear that the constraint is always going to be binding for both banking sectors, since in equilibrium the return on currency is always lower than the expected return on capital. Thus, liquidity requirements can be interpreted as an additional cost that is imposed on intermediaries with the objective of pushing the intermediaries to issue more or only safe liabilities.

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[25] Of course, liquidity requirements may also be useful to prevent other causes of a financial collapse, such as bank runs.

[26] As intended in the literature stemming from the Diamond and Dybvig (1983) model.

[27] As in my model, different assets have different likelihood of losing their liquidity value and are consequently classified differently. Level 1 assets are the safest and most liquid, thus they fully count towards the liquidity requirements. Examples include cash, central bank reserves, and high quality government securities. Level 2 assets carry some risk of losing their liquidity value, thus only a fraction of their value counts towards the liquidity coverage ratio, with haircuts up to 50%. Starting from the most liquid instruments, examples include securities issued or guaranteed by specific multilateral development banks or sovereign entities, securities issued by U.S. government-sponsored enterprises, publicly traded common stocks, and investment-grade corporate debt securities issued by non-financial sector corporations.

After the new regulation is imposed, I let the economy adjusts to a new stationary competitive equilibrium. Then, the following proposition holds:

Proposition 6. *Suppose that the parameters are such that the second best welfare prescribes* $\kappa_h = \kappa_\ell = 1 + \mu - \beta$. *If*

$$\delta^c < f(\delta^s)$$

and $\delta^c \geq \underline{\delta^c}$, *the welfare maximizing liquidity allocation can be achieved as a competitive equilibrium where only the commercial banks operate.*

Where $f(\cdot)$ is an increasing function. Shadow bankers do not find it profitable to enter the market if $\delta^c < f(\delta^s)$. The second condition, $\delta^c \geq \underline{\delta^c}$, is necessary to ensure that the household is holding a positive amount of currency. However, it is always verified (i.e. $\underline{\delta^c} < 0$) for realistic levels of inflation μ. For reasonable values of δ^c[28] and low values of the inflation rate, this proposition implies that liquidity regulation must be stricter on shadow banks, or $\delta^s > \delta^c$.

There are two consequences for policy. First, differentiating the regulation between commercial and shadow banks is necessary in order to achieve the efficient welfare allocation. Secondly, achieving the efficient allocation requires a policy that imposes a stricter liquidity constraint on the issuers of risky securities. It can be shown that, if the opposite is true, the competitive equilibrium with regulation allocates liquidity as in the inefficient equilibrium of Proposition 4. The intuition is the following: an equal liquidity requirement would increase the marginal cost of funding by the same amount for both intermediaries. Thus, the shadow banker would still find it profitable to enter the market when it is not socially optimal. The additional requirement on shadow banks is then necessary to make their expected profits negative.

However, this approach presents strong limitations in practical applications. Since different liquidity requirements need to be imposed on the different financial firms, there may be issues with the incentive compatibility of such plan. Any financial institution in the real economy offers a mix of safe and risky securities and the regulator may not be able to distinguish the two without monitoring. Thus, any intermediary's manager has an incentive to increase profits by overstating the fraction of safe securities issued. A supervising authority would need to be able carry detailed audits of every security issued by a given intermediary to

[28] According to the Quarterly Trends for Consolidated U.S. Banking Organizations, in the 4th quarter of 2017 the banking industry was holding 1.36% of their assets in cash and 9.14% in reverse repo and Fed Funds. Government bonds represent another 1.39% of the total assets. Thus having $\delta^c < 0.2$ would achieve a realistic value of overall liquid securities in the balance sheet.

identify any deviation from the imposed regulations. The cost of such activity may outweigh the benefit of a more efficient allocation of liquidity.

6.2 Capital Requirements

After establishing that liquidity requirements can restore efficiency but may be difficult to implement, I will turn my attention to capital requirements. These have been long used to ensure the stability of the financial sector, and they have been subject to numerous revisions. The underlying principle is to make sure financial institutions have enough skin in the game to avoid excessive risk taking and enough resources to withstand a negative shock.

Under the Basel III agreement, financial institutions must hold a minimum amount of capital relative to their risk weighted assets.[29] This type of regulation can be almost directly implemented in the model, by mandating intermediaries to issue equity for at least a fraction $0 < \gamma < 1$ of their assets, or

$$n^c \geq \gamma^c q^k K^c \text{ and } n^s \geq \gamma^s q^k K^s.$$

As in the previous section I allow for differential regulation between the two sectors. Additionally, this constraint will always be binding for the shadow banker in equilibrium, but not for the commercial one. Issuers of safe liabilities already issue some equity because of Equation (11), so market forces may be sufficient to make the equity requirement not binding for the commercial bank.

After the new regulation is imposed, I let the economy adjusts to a new stationary competitive equilibrium. Assuming that the welfare optimum is to equalize consumption between the two states, the following proposition holds:

Proposition 7. *Suppose that the parameters are such that the second best welfare prescribes $\kappa_h = \kappa_\ell = 1 + \mu - \beta$. If*

$$\gamma^c > \underline{\gamma}^c$$

and $\gamma^s > \underline{\gamma}^s$, the welfare maximizing liquidity allocation can be achieved as a competitive equilibrium where only the commercial banks operate. However, an inefficient competitive equilibrium may also be possible under the same policy choice.

[29] Financial institutions must have a ratio of common equity tier 1 over risk weighted assets greater than 4.5%. That means, an intermediary must have an amount of common stocks and earnings greater than 4.5% of the value of its assets, weighted by the risk. A broader requirement also mandates a Tier 1 capital (which includes equity-like securities such as non-redeemable non-cumulative preferred stocks) ratio over the risk weighted assets over 6%.

Under this policy, the capital requirement is binding for the commercial bank. Therefore, the issuance of equity is higher than in an equilibrium without the policy. The lower-bound for the capital requirement on commercial banks is a necessary condition to have a monetary equilibrium. Much like Proposition 1, a monetary equilibrium does not exist if safe securities provide high liquidity services. The capital requirement offsets this mechanism by limiting the quantity of safe securities in circulation and forcing the commercial bank to issue more equity to acquire assets. The downside of the policy is that the minimum capital ratio $\underline{\gamma}^c$ may be pushed to unrealistically high levels. This occurs when θ is large, where safe securities provide a lot of liquidity services. Therefore, issued quantities must be tightly limited to have a positive demand for currency. Finally, the lower bound on the capital requirement for shadow bankers ensures that they do not find entering the market profitable.

Unrealistically high capital requirements are the first issue with implementing this policy. A second and more relevant one is that capital requirements may not eliminate the inefficient equilibrium. It can be shown that, if the capital requirement on shadow banks is not too large, the inefficient equilibrium of Proposition 4 still exists. Therefore, if the economy starts from that equilibrium, capital requirements are not sufficient to induce a more efficient allocation of liquidity. While capital requirements solve many of the incentive compatibility issues that affect liquidity requirements, their implementation may only lead to a redistribution of resources within the financial sector that leaves the allocation of liquidity unchanged. Therefore, capital requirements are an ineffective policy tool under the lens of the model.

7 Conclusion

I have shown a model where households' liquidity demands are satisfied by a combination of publicly issued fiat currency and intermediaries issued safe and risky liabilities. As risky liabilities circulate, consumption increases but it then collapses if the issuers of risky securities default. This outcome is ex-ante inefficient, since the existence of fiat currency forces the economy to keep the average amount of liquid securities constant, thus introducing more fluctuations than what a planner would desire.

Consequently, government regulation can be used to address the inefficiency. In particular, I concentrate on liquidity and capital requirements. As for the former, they can restore efficiency when appropriately designed, but they require separate regulation for each type of security, which is likely to generate moral hazard if the regulator is unable to verify intermediaries' balance sheets. As for the latter, they are insufficient on their own, as the economy would stay in the inefficient equilibrium, even if a more efficient one exists.

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A Omitted Proofs

A.1 Proof of Proposition 1

Solve for b^c using Equations (19), (20), and (22). Then combine the result with Equation (1) and $\kappa_\ell = \kappa_h = 1 + \mu - \beta$ to get

$$\frac{\beta}{1+\mu} = \varphi_t M_{t-1} + \theta \frac{b^c}{\theta(1+\mu-\beta)+\beta}.$$

Multiply and divide the money term M_{t-1} by φ_{t-1}, noting that ${}^{\varphi_{t-1}}/_{\varphi_t} = 1 + \mu$ and $m = \varphi_{t-1} M_{t-1}$, to get

$$\frac{\beta}{1+\mu} = \frac{m}{1+\mu} + \theta \frac{b^c}{\theta(1+\mu-\beta)+\beta}.$$

Replace the value of b^c and simplify to get

$$m = \frac{\beta - \beta^2(1-\theta) - \beta\theta(1+\mu)\left(1+\bar{A}\bar{K}\right) - A_\ell \bar{K}\theta(1+\mu)(1-\beta)}{1-\beta-\theta(1+\mu-\beta)},$$

which, given the general assumptions about the parameters, is positive if and only if

$$\theta < \frac{(1-\beta)\beta}{A_\ell \bar{K}(1+\mu)(1-\beta) + \beta\left[1+\mu-\beta+\bar{A}\bar{K}(1+\mu)\right]} = \bar{\theta}^c.$$

Which proves the first part of the proposition. Then differentiate m with respect to θ to get

$$\frac{\partial m}{\partial \theta} = -\frac{(1-\beta)(1+\mu)\left[\bar{A}\beta + A_\ell(1-\beta)\right]}{\left[\beta(1-\theta)+\theta(1+\mu)-1\right]^2}\bar{K}.$$

Numerator and denominator are both positive, thus the real value of money is decreasing in security liquidity θ.

A.2 Solution Steps for the Equilibrium in Proposition 2

Note that

$$E\left[1+r^k\right] = \frac{\bar{A}+q^k}{q^k} \text{ and } 1 + r_\ell^k = \frac{A_\ell + q^k}{q^k}.$$

Then Equations (25) and (26) both solve for the price of capital q^k as a function of the liquidity premium in the good state κ_h and the loss rate of default χ_ℓ:

$$\begin{aligned} q^k &= \frac{\bar{A}\left[\beta\left(1-\pi\chi_\ell\right)+\theta\kappa_h\left(1-\pi\right)\right]}{1-\pi\chi_\ell-\beta(1-\pi\chi_\ell)-\theta\kappa_h\left(1-\pi\right)} \\ q^k &= \frac{A_\ell\left[\beta\left(1-\pi\chi_\ell\right)+\theta\kappa_h\left(1-\pi\right)\right]}{1-\chi_\ell-\beta(1-\pi\chi_\ell)-\theta\kappa_h\left(1-\pi\right)} \end{aligned}$$

Equating the two solves for the liquidity premium κ_h as a function of the default loss rate χ_ℓ

$$\kappa_h = \frac{\bar{A}\left[1-\beta\left(1-\pi\chi_\ell\right)-\chi_\ell\right]-A_\ell\left(1-\beta\right)\left(1-\pi\chi_\ell\right)}{\theta\left(1-\pi\pi\right)\left(A-A_\ell\right)}.$$

Thus $\kappa_\ell$is obtained by solving $\bar{\kappa}=1+\mu-\beta=\left(1-\pi\right)\kappa_h+\pi\kappa_\ell$. Since in an equilibrium it must be that $\kappa_h\geq 0$ and $\kappa_\ell>\kappa_h$, then it must be that $0<\chi_\ell<1$ and

$$\frac{\left(\bar{A}-A_\ell\right)\left[1-\beta+\beta\theta\left(1-\pi\right)-\theta\left(1+\mu\right)\left(1-\pi\pi\right)\right]}{\bar{A}-\pi A_\ell-\left(\bar{A}-A_\ell\right)\beta\pi}<\chi_\ell\leq\frac{\left(\bar{A}-A_\ell\right)\left(1-\beta\right)}{\bar{A}-\pi A_\ell-\left(\bar{A}-A_\ell\right)\beta\pi}.$$

The only unknown left to solve for is the default loss rate χ_ℓ. To do so, I can solve for the amount of securities issued by the shadow banks in two ways. First, using the shadow bank balance sheet constraint (12), it must be that $b^s=q^k\bar{K}$. Second, using the household morning liquidity constraint (1) realization in the low state

$$\frac{\beta}{\beta+\kappa_\ell}=\frac{m}{1+\mu}$$

with the same constraint in the high state and the return on securities (24), shadow banks securities must satisfy

$$b^s=\beta\frac{\kappa_\ell-\kappa_h}{\left(\beta+\kappa_\ell\right)\left(\beta+\kappa_h\right)}\frac{\theta\left(1-\pi\right)\kappa_h+\left(1-\pi\chi_\ell\right)\beta}{\theta}. \tag{29}$$

Equating the two expressions for securities b^s returns a cubic equation in the object of interest, the default loss rate χ_ℓ. While this equation is unwieldy to even report in this paper, it can be decomposed into a linear and a quadratic factor. The solution to the linear term is never acceptable in equilibrium, which leaves the two solutions from the quadratic equation. One of these solutions either is negative or implies negative value for liquidity premium in the low state κ_ℓ, which numerically verifies that solution is unique, when it exists.

To determine the security liquidity threshold $\bar{\theta}$, suppose you construct an equilibrium with

$\kappa_h = 0$ and $\kappa_\ell = {}^{(1+\mu-\beta)}/_{\pi}$. This hugely simplifies the previous analysis, since many elements of the solutions can be solved for directly. In particular, the return on equity is now ${}^1/_\beta$, thus the price of capital is

$$q^k = \frac{\beta}{1-\beta}\bar{A}$$

securities are defined as in Equation (29), which are then used to solve for the default loss rate χ_l from the bank balance sheet constraint (12) as a function of the parameters. However, the limited liability constraint (26) can also be used to obtain another closed form solution for χ_l as a function of the parameters. Since the two expressions for χ_l are not the same algebraically, there must be a parameter value that makes them equal in an equilibrium. The parameter of choice is of course arbitrary, but focusing on the liquidity of the securities returns

$$\bar{\theta}^s = \frac{(1-\beta)(1-\pi)(1+\mu-\beta)}{\bar{K}\left[1+\mu-\beta(1-\pi)\right]\left[\bar{A}-\pi A_l-(A-A_l)\beta\pi\right]}$$

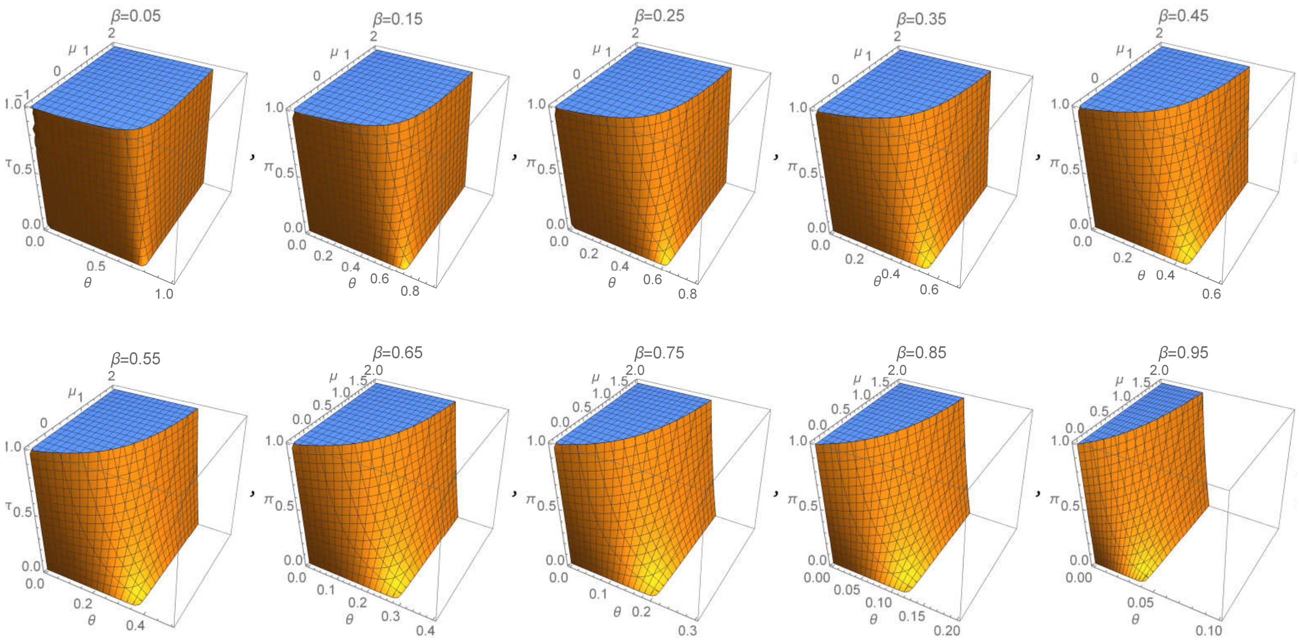
Finally, I take a numerical approach in order to verify that an equilibrium exists only if $\theta \leq \bar{\theta}^s$. While a closed form solution exists, it is as impractical as the equation generating it to study it how it evolves over the entire parameter space. Thus, I test the hypothesis with the assistance of Mathematica to span a reasonable set of the parameter space. The threshold, and therefore the region, is much more sensitive to the discount factor β, the probability of a low state π, and the money growth rate μ, rather than capital $\bar{K}$ and productivity levels A_h and A_l. Consequently, I focus my analysis on the first group of parameters. Here I graphically report the results when setting $\bar{K} = 1$, $A_\ell = 1$, and $A_h = 1.2$. The shaded region is where the equilibrium is satisfied, that is where $\kappa_h \geq 0$, $\kappa_\ell > \kappa_h$, and $0 < \chi_\ell < 1$. The visible boundary is the threshold value $\bar{\theta}^s$.

The scale of the graphs over θ changes to keep the boundary visible, while the scale for μ increases to account for the shifting Friedman Rule. Indeed, the shaded region is to the left of the boundary, or where the value of the security liquidity is below the threshold.

A.3 Deriving the Equilibrium in Proposition 3

As a first step to prove Proposition 3, consider first the commercial bank only equilibrium from Proposition 1 with the details provided in appendix A.1. The only step missing is verifying that there is no incentive to operate as a shadow banker. First compute what the recovery rate after default $1-\chi_\ell$ is using Equation (26) to get

$$1-\chi_\ell = \frac{(1-\pi)\left[A_\ell(1-\beta)+\bar{A}\beta\right]\left[\beta(1-\theta)+\theta(1+\mu)\right]}{\bar{A}\beta(1-\beta\pi)+A_\ell\theta(1+\mu-\beta)-A_\ell(1-\beta)\beta\pi}.$$



Then compute the expected profits as in (25)

$$E\left[\Pi^s\right] = E\left[1 + r^k\right] - \frac{1 - \pi\chi_\ell}{\theta\left(1-\pi\right)\kappa_h + \left(1 - \pi\chi_\ell\right)\beta},$$

with the expected return on capital defined by Equation (22). To see where a deviation exists, set $E\left[\Pi^s\right] > 0$ and solve the inequality for the probability of the low state π to get

$$\pi < \frac{\left(\bar{A} - A_\ell\right)\left[1 - \beta\left(1-\theta\right) - \theta\left(1+\mu\right)\right]}{\bar{A}\beta + A_\ell\left(1-\beta\right)} = \bar{\pi}.$$

The shadow bank profits are positive if the probability of a low state π is below the threshold $\bar{\pi}$, thus an equilibrium with commercial banks exists only if $\pi \geq \bar{\pi}$.

Now move to the second equilibrium in the proposition, where both type of banks operate.

Equations (25) and (26) pin down the expected return on capital and the return on capital in the low state respectively as a function of the default loss rate χ_ℓ and the liquidity premium in the good state κ_h. Then Equation (20) pins the fraction of the commercial bank assets financed with safe securities. The complement fraction then identifies the equity issuance as a fraction of the commercial bank assets. Use Equation (21) to solve for the liquidity premium in the high state κ_h as a function of the default loss rate χ_ℓ

$$\kappa_h = \frac{(1+\mu-\beta)(1-\chi_\ell)}{1-\pi}.$$

Since $\bar{\kappa} = 1+\mu-\beta$, the liquidity premium in the low state is

$$\kappa_\ell = \frac{(1+\mu-\beta)\,\chi_\ell}{\pi}.$$

Plug the liquidity premia in Equation (29) to get and expression for the securities issued by the shadow bank b^s. Also, the liquidity premia can be used back in Equations (25) and (26) to obtain two expressions for the price of capital q^k as in appendix A.2. Equating the two expressions solve for the default loss rate

$$\chi_l = \frac{\left(\bar{A}-A_\ell\right)\left[1-\beta(1-\theta)+\theta(1+\mu)\right]}{\bar{A}\left[1-\theta(1+\mu-\beta)-\beta\pi\right]+A_\ell\left[\theta(1+\mu-\beta)-(1-\beta)\pi\right]}.$$

Given the solution for χ_l, check for the necessary but not sufficient condition for equilibrium $\kappa_\ell > \kappa_h > 0$ and $0 < \chi_\ell < 1$. The inequalities are verified if either

$$\theta < \frac{\bar{A}(1-2\beta)-2A_\ell(1-\beta)}{\left(\bar{A}-A_\ell\right)(1+\mu-\beta)}$$

or

$$\begin{aligned}
\pi \quad &< \quad \frac{\left(\bar{A}-A_\ell\right)\left[1-\beta(1-\theta)-\theta(1+\mu)\right]}{\bar{A}\beta+A_\ell(1-\beta)} = \bar{\pi} \\
\frac{\bar{A}(1-2\beta)-2A_\ell(1-\beta)}{\left(\bar{A}-A_\ell\right)(1+\mu-\beta)} \quad &< \theta < \quad \frac{1-\beta}{1+\mu-\beta},
\end{aligned}$$

where these last two inequalities must hold jointly. The first inequality is relevant only if the capital productivity in the high state A_h is at least twice the productivity in the low state, and for low values of the discount factor β, thus I focus on the second set of conditions. These define the upper bound for π from the proposition and conditions on θ that end up being irrelevant for the equilibrium.

To find the relevant conditions on the security liquidity θ come from solving for the last unknowns in the model. Equation 12 solves for the amount of capital held by the shadow banker K^s, given the solution for the shadow bank securities b^s and the price of capital q^k. Market clearing then returns the capital held by the commercial banker K^c. The equilibrium condition $0 < K^c < \bar{K}$ implicitly determines the equilibrium lower bound for the security liquidity $\underline{\theta}^{cs}$.

After determining the asset side of a commercial bank's balance sheet, Equation (20) pins down the amount of safe securities b^c issued and thus Equation (9) solves for the amount of equity n^c issued. Finally, the household's morning liquidity constraint (1) at the low state solves for the real value of money

$$m = \left(\frac{\beta}{\beta + \kappa_\ell} - \theta \frac{b^c}{\theta \left(1 + \mu - \beta\right) + \beta} \right) \left(1 + \mu\right),$$

where the condition $m > 0$ implicitly determines the upper bound value for the security liquidity $\bar{\theta}^{cs}$.

A.4 Proof of Proposition 4

The proof follows similar steps as the ones detailed in appendix section A.3 to prove Proposition 3. However, I start not only from guessing that both banks operate, but also that the liquidity premia in each state are given by

$$\kappa_h = 0 \text{ and } \kappa_\ell = \frac{1 + \mu - \beta}{\pi}.$$

Then the zero profit condition on shadow bankers (25) immediately implies

$$E\left[1 + r^k\right] = \frac{1}{\beta} \Rightarrow q^k = \frac{\beta}{1 - \beta} \bar{A}.$$

Now the price of capital is simply the discounted value of the future expected revenues. The return on capital in the low state pins down the value of commercial bank securities relative to commercial bank capital. Then use the zero profits condition for the commercial banker to recover the default loss rate

$$\chi_\ell = \frac{\beta \tau - \theta \left(1 + \mu - \beta\right)}{\beta \pi \tau - \theta \left(1 + \mu - \beta\right)}.$$

In equilibrium $0 < \chi_\ell < 1$, which gives a first condition on the lower bound for the security liquidity θ

$$\theta > \frac{\beta\tau}{1+\mu-\beta} \tag{30}$$

The second part of the of lower bound is derived from the solution for the capital acquired by the shadow banker. The solution for the default loss rate χ combined with the household's liquidity constraint (1) in the high and low state solves for the shadow bank securities b^s. Then the shadow bank balance sheet constraint (12) solves for the capital owned by the shadow bank K^s. In equilibrium this solution must be feasible, or $0 < K^s < \bar{K}$, which returns

$$\theta > \frac{(1-\beta)(1-\pi)(1+\mu-\beta)}{\bar{K}\left[1+\mu-\beta(1-\pi)\right]\left[\bar{A}(1-\beta\pi)-A_\ell(1-\beta)\pi\right]}. \tag{31}$$

The combination of (30) and (31) defines the equilibrium lower bound $\underline{\theta}^{sc}_\tau$. The remaining part of the model is solved as in appendix section A.3. Here I will only detail the conditions such that the real value of money is positive, or $m > 0$, which holds true when either

$$\tau \leq \frac{\pi(1-\beta)(1+\mu-\beta)}{\bar{A}\bar{K}\left[1+\mu-\beta(1-\pi)\right]} = \bar{\tau}^{sc},$$

or

$$\theta < \frac{\bar{A}\bar{K}\left[1+\mu-\beta(1-\pi)\right]\beta\pi\tau-(1-\beta)(1+\mu-\beta)\left[\beta-\beta(1-\pi)\pi-(1+\mu)(1-\pi\pi)\right]}{(\beta-\mu-1)\left[(1-\beta)(1+\mu-\beta)\pi-\bar{A}\bar{K}(1+\mu-\beta(1-\pi))\tau\right]}\tau = \bar{\theta}^{sc}_\tau.$$

A.5 Proof of Proposition 5

Take the per period expected utility (28) and take the standard first order conditions to recover the following candidate maxima for the liquidity premium in the high state

$$\begin{aligned}
\kappa_{h,1} &= 1+\mu-\beta \\
\kappa_{h,2} &= \frac{1+\mu-\beta+\sqrt{(1+\mu-\beta)^2-4\beta^2(1-\pi)\pi}}{2(1-\pi)} \\
\kappa_{h,3} &= \frac{1+\mu-\beta-\sqrt{(1+\mu-\beta)^2-4\beta^2(1-\pi)\pi}}{2(1-\pi)}
\end{aligned}$$

Looking at existence and feasibility of the solution, $\kappa_{h,2}$ satisfies the constraint (that is $0 \leq \kappa_{h,2} \leq 1+\mu-\beta$) if

$$\sqrt{\frac{(1+\mu)(2\beta-\mu-1)}{\beta^2}} \leq 1-2\pi$$

While $\kappa_{h,3}$ is acceptable if

$$\sqrt{\frac{(1+\mu)(2\beta-\mu-1)}{\beta^2}} \leq 1-2\pi \text{ or } 1+\mu \geq 2\beta$$

Since the second derivative evaluated at $\kappa_{h,1}$ is positive if $1+\mu < 2\beta$, $\kappa_{h,1}$is the unique interior maximizer if $\sqrt{\frac{(1+\mu)(2\beta-\mu-1)}{\beta^2}} > 1-2\pi$. If the latter condition fails, but $1+\mu < 2\beta$, $\kappa_{h,1}$ and $\kappa_{h,3}$ are both local maxima and $\kappa_{h,2}$ is a local minimum. Finally, if $1+\mu > 2\beta$, $\kappa_{h,3}$ is a local maximum and $\kappa_{h,1}$ is a local minimum. An illustration of the possible optimum is given in the picture below. The left panel shows the case where $\kappa_{h,1}$ is the welfare maximizing liquidity premium in the high state (which requires a relatively small value for μ), while the right panel shows the case of an interior maximizer at $\kappa_{h,3}$ (which exists at higher levels of inflation μ).

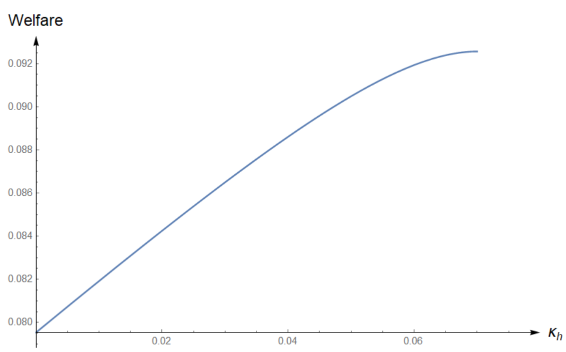


Figure 1: Low μ

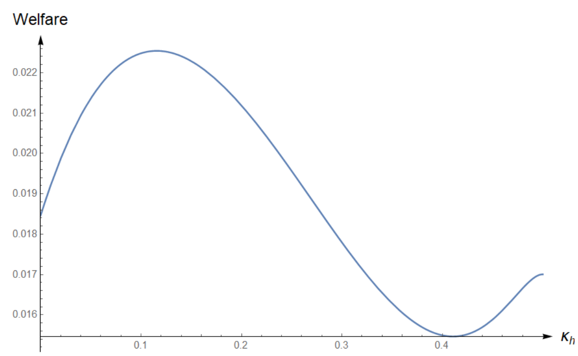


Figure 2: High μ

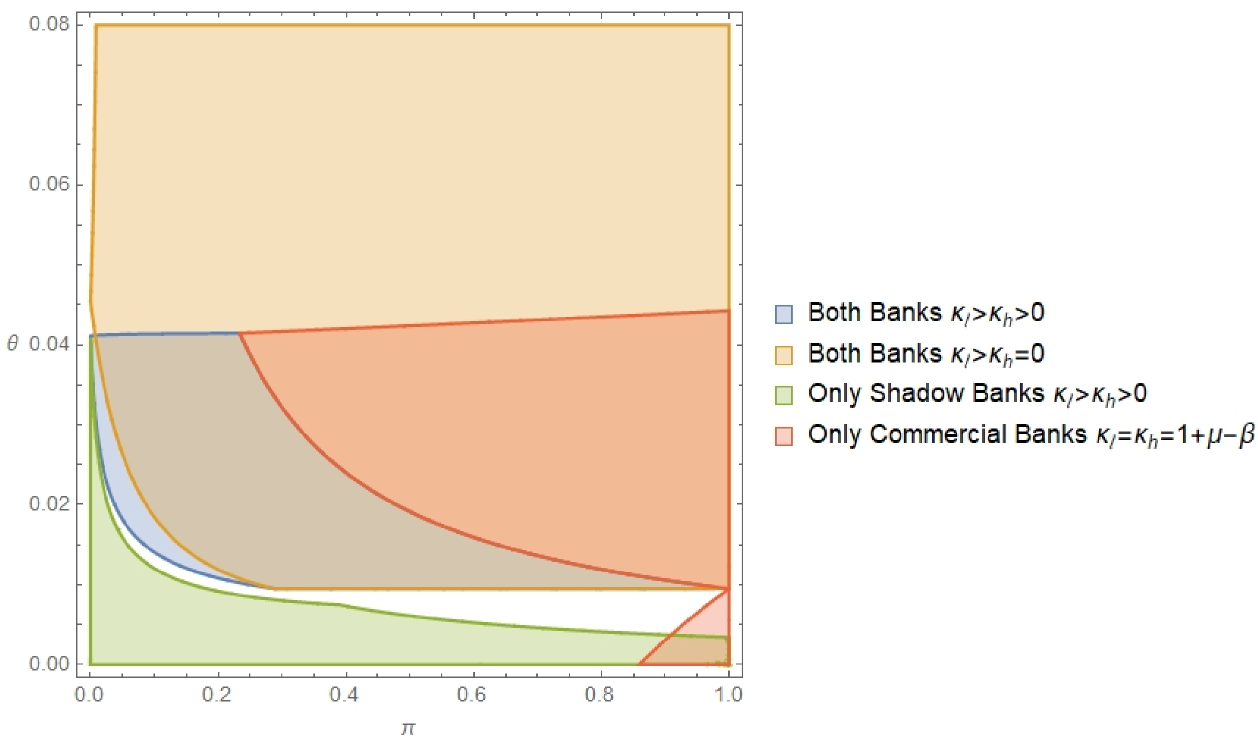
B Numerical Illustrations

B.1 Equilibrium Multiplicity with Positive Balance Sheet Costs

The following figure illustrates the regions where different equilibria exists in the probability of the low state π and security liquidity θ plane. The other parameters are chosen as follows

Parameter	Value
A_ℓ	1.0007
A_h	1.1
$\bar{K}$	1
τ	0.0007
β	0.95
μ	0.02

Note how the region in yellow, which represents the equilibrium where both commercial and shadow bankers exist and liquidity in fully satiated in the good state, partially overlaps with the blue region, where the two bankers still operate, but the liquidity premium is always positive, and the red region, where only the commercial banking is the only profitable type of intermediary.



To highlight the differences between equilibria, consider a parameterization where multiple equilibria are possible. By setting the liquidity value of securities to $\theta = 0.035$ and the probability of a crisis to $\pi = 0.04$, two monetary equilibria exists. In the first one, labeled "Worse Equilibrium", liquidity is organized as in Proposition 4 and consumption is maximized in the high state. In the second one, labeled "Better Equilibrium", both bankers still operate,

Variable	Description	Worse Equilibrium	Better Equilibrium
K^c	Commercial Capital	18%	64.22%
K^s	Shadow Capital	82%	35.78%
m	Real Currency	0.3206	0.1571
c_h^{am}	High State am Consumption	1	0.9474
c_ℓ^{am}	Low State am Consumption	0.3519	0.6627
	Per Period Utility	1.6034	1.8337

Table 5: Multiple Equilibria Illustration

but morning consumption is not maximized in the low state since shadow bankers are constrained. First, the composition of the financial sector is dramatically different between the two states. In the better equilibrium, the commercial banks have a much larger security offering, which means that shadow bankers have less liquidity premium that they can capture. This is a welfare improvement for the household, as consumption is now more stable between the two states. However, none of the equilibria achieves the welfare maximizing morning consumption, hence the choice of labels. Table 5 summarizes these results.

C Data Sources and Aggregation

I recovered the Fr Y-9C data from the Wharton Research Data Services (WRDS), with a focus on the Schedule HC Consolidated Balance Sheet, as measured at the end day of the filing quarter, using data from Q1 2006 to Q3 2017. Q4 2017 was only partially available for the sample of Bank Holding Companies and therefore it was dropped. While the schedules about balance sheet details (such as a detailed decomposition of loans, securities held and deposit liabilities) have changed multiple times across the considered time period, the consolidated balance sheet schedule has not, and therefore data is fully comparable across the entire time series. Total equity capital required some reconstruction, as older reports only include its two components, the total holding company equity capital and the non-controlling interests in consolidated subsidiaries.

With the fully uniformed data, I compute new aggregated variables to look at trends as informed by the model. Specifically, liabilities are separated in deposits and risky liabilities. The deposits aggregate is given by the interest and non-interest bearing deposits in domestic and foreign offices. The risky liabilities aggregate is composed of purchased Federal Funds, reverse repo, trading liabilities, other borrowed money, and subordinated notes. Table 6

Variable	Construction and Reference Codes
Total Assets	BHCK2170
Total Equity	BHDMG105
Deposits	BHDM6631 + BHDM6636 + BHFN6631 + BHFN6636
Risky Liabilities	BHDMB993 + BHCKB995 + BHCK3548 + BHCK3190 + BHCK4062 + BHCK2750 + BHCKC699

Table 6: Variables Construction with Reference Codes

summarizes the full list of relevant variables with the FR Y-9C codes. Then, the relevant ratios are computed to obtain the calibration targets.