Too Big to Rush

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Abstract

A sudden need for liquidity prompts banks to sell their assets at a discount to obtain cash. This sale disturbs the economy and slows down growth because the buyers of the assets reduce their investments in positive NPV projects. Small banks do not internalize their own impact on prices, which encourages them to start a fire sale too early. A (relatively) small probability of a liquidity shock might trigger a fire sale, causing a real crisis. Big banks internalize their own price impact, which reduces the severity of a crisis. Their sale decision is more in line with that of the social planner because they are too big to rush to sell their assets.

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Sell when you can; you are not for all markets.

– William Shakespeare, *As you like it* (Act III.5)

1 Introduction

Banks, at times, can be too prudent. For example, one of the most striking features of the recent financial crisis is the freeze in the credit market: instead of lending, banks built up cash reserves and accumulated safe assets.[1] Brunnermeier [2009] refers to the freeze in the interbank market as a textbook example of precautionary hoarding.

When banks hoard cash, fewer positive NPV projects than are socially optimal are financed. This hampers growth.[2] In this paper, I develop a model in which competition causes a decline in economic output by encouraging banks to hoard cash. In the model, banks face the possibility of an exogenous liquidity shock. Output in the economy declines even before the shock occurs because banks will find it optimal to divert funds from the real assets to satisfy their potential future demand for liquidity.

Hoarding cash today is optimal from the banks' perspective. However, banks' individually optimal decisions yield a socially undesirable outcome: the buyers of banks' assets need to forego their positive NPV projects. As a consequence, while

[1] See, for instance, Ashcraft et al. [2011], Acharya and Merrouche [2012], and Berrospide [2013] for empirical evidence.

[2] Even representatives of two diametrically opposite schools of thought, Hayek [1932] and Keynes [1932], agree that the economy is hurt when agents hoard cash. This opinion has been echoed, among others, by Friedman and Schwartz [1963] and Bernanke [2010].

the aggregate level of liquidity goes up, total investments in the economy go down. Depending on the likelihood of the shock, a real crisis might occur even before the realization of the shock. That is, banks might trigger a crisis endogenously by rushing to obtain liquidity. This would not happen in an economy managed by a social planner who would not demand liquidity before the realization of the shock.

Banks' total demand for cash—and, thus, the severity of the crisis—increases with the likelihood of the shock and decreases with the price of cash. In my model the price of cash depends on the behavior of other banks and is, therefore, endogenously determined. Because the buyers of the financial assets have to be compensated at an increasing rate to forego their existing positive NPV projects, banks need to offer a greater discount if more assets in the market are on sale. Therefore, each bank imposes a negative externality on other banks when it sells its assets to raise liquid funds.

The negative externality in obtaining cash is the main reason for the inefficiency in my model. In particular, acting as price takers, banks in the competitive market do not internalize their own impact on the equilibrium terms of trade when they sell their assets. This encourages them to sell more, which leads to distortions in the allocation of funds in the economy. Big banks, on the other hand, internalize their impact and can better time when to obtain cash. They demand less liquidity and, therefore, divert a smaller amount of funds from the real assets. That is, they play the role of a moderator and facilitate a more efficient allocation.

My paper contributes to the literature studying why banks might abstain from financing the real economy. Diamond and Rajan [2011] explain credit market freezes with a speculative motive. In their model, the possibility of a decrease in asset prices (*i.e.,* the anticipation of a fire sale) in the future incentivizes banks to hoard cash instead of extending new loans. Bebchuk and Goldstein's [2011] explanation of freezes relies on coordination failure. In their model, a bank's payoff from lending increases as other banks lend. An inefficient freeze occurs because of the self-fulfilling expectations that other banks will not be lending. Their result stems from complementarity (*i.e.,* a bank's incentive to keep cash increases as others keep cash) whereas my result is drawn from substitutability (*i.e.,* a bank's incentive to sell assets decreases as others sell).

In perhaps the most closely related article, Gale and Yorulmazer [2013] study freezes in the interbank market. In their model, cash is demanded both for precautionary and speculative reasons: it helps banks satisfy their own needs when a liquidity shock hits, and at the same time allows them to make profit by providing liquidity to other banks hit by the shock. In their model, a bank provides a positive externality on other banks by keeping cash (*i.e.,* the price of cash goes down if the number of banks keeping cash goes up) whereas in my model a bank imposes a negative externality on others by demanding cash (*i.e.,* the price of cash goes up if the number of banks demanding cash goes up). Moreover, it is crucial for their results that the liquidity shock hits in three periods. There would not be any inefficiency in their model if the shock hit in only one (or two) period(s) as in my model.

My paper also contributes to the literature on bank competition. This literature can be divided into two camps: studies that find bank competition beneficial for welfare, and those that do not.[3] My paper falls into the second camp. My results suggest that a concentrated banking system improves welfare by reducing the inefficient demand for liquidity.

2 A Model of Perfect Competition

2.1 Environment

The model has three dates: $t \in \{1, 2, 3\}$. There is a continuum of financiers and a continuum of bankers. I represent the sets of each type of agents by the unit interval $[0, 1]$, where each point in the interval denotes a different agent. I measure the fraction of agents in any subset by its Lebesgue measure. The assumption of a large number of individually insignificant (*i.e.,* atomistic) bankers ensures that none has enough market power to affect the terms of trade in the economy. I will change the perfectly competitive market set-up later in Section 4 and introduce a monopolist with a competitive fringe. I dispense with financial discounting to avoid notational clutter.

Financiers are endowed with W units of a single good which can be used for consumption and investment. I refer to it as cash and count it in dollars. They are risk-neutral and they consume at date $t = 3$. Financiers have access to the following production technology: x dollars invested at date $t = 1$ yield $f(x)$ dollars at date $t = 3$. $f(\cdot)$ satisfies the usual neoclassical properties: $f'(\cdot) > 0$, $f''(\cdot) < 0$,

[3] For example, Jayaratne and Strahan [1998], De Nicole [2000], and Boyd and De Nicole [2005] argue that competition improves bank stability whereas Keeley [1990], Repullo [2004], and Beck, Demirguc-Kunt and Levine [2006] suggest that it promotes bank failures.

and $f(0) = 0$. I further assume that the third derivate of the production function is positive. This assumption is innocent and does not affect my results (see footnote 8). Moreover, commonly used production functions, such as Cobb-Douglas, satisfy this condition.

Investments in the production technology are fully reversible. That is, a financier can liquidate some part of his investment (without any cost) and obtain his cash back. For example, if a financier invests one dollar at date $t = 1$ and removes 0.2 dollars from production at date $t = 2$, he would receive $0.2 + f(0.8)$ dollars at date $t = 3$. I assume that absent an additional incentive, liquidation is never efficient. That is, $f'(W) > 1$. I refer to investments in this production technology as real assets.

Bankers own financial assets which are worth $\tilde{r}$ at date $t = 3$. $\tilde{r}$ is a random variable with $E[\tilde{r}] = r$ and its exact distribution is not important. Financial assets can be interpreted as a pool of many small projects such as a loan portfolio. Bankers are financed with deposits of face value d_0 and they are solvent in the long term: $d_0 < r$.[4]

To simplify the exposition of welfare calculations, I assume that deposit contracts are owned by financiers. It would be possible to include an initial stage (*i.e.,* date $t = 0$) to the model, at which bankers collect deposits from financiers and lend to another type of agents (*i.e.,* entrepreneurs). I abstract away from this additional layer to focus on the effect of liquidity management on welfare. Similar results could also be obtained by introducing entrepreneurs and by assuming that they

[4]Bankers' profits in this competitive market can be justified by introducing a free-entry condition which allows financiers to become bankers at some cost, and by setting this entry cost equal to bankers' equilibrium profits.

can pledge outputs of their projects to banks without any cost.

Bankers' investments in financial assets are not reversible. Bankers cannot liquidate their projects before date $t = 3$ to obtain cash. To ensure that bankers are able to sell their assets to financiers (if needed), I assume that the return on bankers' assets is higher than the return on the financier's production. In particular, I assume that r is greater than $f'(0)$. This assumption is sufficient but not necessary for the results. Bankers consume at date 3.

There also exists a simple storage technology available to both financiers and bankers. It allows agents to transfer their consumption goods (*i.e.,* their cash) to the next period without any loss or gain. Because financiers can liquidate their investments at no cost, they would never invest in the storage technology. As it will be clear, the storage technology might be employed by bankers at date $t = 1$ to keep cash against a possible liquidity shock at date $t = 2$.

If I closed the model at this point, financiers would invest all their endowments and both types of agents would wait until date $t = 3$ to consume the total output in the economy. Welfare, defined as the total consumption at date $t = 3$, could be calculated as follows:

$$
\begin{aligned}
Welfare &= \underbrace{r - d_0}_{\text{bankers' consumption}} + \underbrace{d_0 + f(W)}_{\text{financiers' consumption}} \\
&= r + f(W).
\end{aligned}
$$

Now I include the possibility of a liquidity shock into the model. One way of obtaining a need for liquidity is to introduce uncertainty in the time preferences of depositors (*i.e.,* financiers). For example, many banking models include impatient individuals who, with a positive probability, would like to consume early (*e.g.,* Diamond and Dybvig [1983] and Allen and Gale [2000]). In those models the demand for liquidity by impatient depositors promts the liquidation of illiquid assets. In this paper, to create a need for liquidity, I will assume that bankers, with an exogenously given probability, need additional financing at an interim stage. Additional financing might be interpreted as an unexpected need of cash to cover the operating expenses of the current investments. Such a structure has been employed in some other banking models such as Holmstrom and Tirole [1998]. My results would not change if I introduced uncertainty in the time preferences of depositors as in Diamond and Dybvig [1983].

At date $t = 2$, with probability $p \in [0, 1]$, bankers face a common liquidity shock. In particular, some of their investments require additional financing and each banker needs to invest an additional λ dollars if the liquidity shock hits. Otherwise (*i.e.,* if this additional amount is not paid), his investment yields nothing. I assume that λ is smaller than W. This assumption ensures that there is enough cash in the economy when the liquidity shock hits so that bankers can raise funds from financiers to be able to continue financing their projects. I also assume that sinking λ dollars when the liquidity shock hits has a positive NPV. Since bankers need to offer the return $f'(W - \lambda)$ to get λ dollars from financiers, this assumption can be stated as

$$\lambda f'(W - \lambda) < r - d_0.$$

The left-hand side of the expression above represents bankers' cost of raising λ dollars from financiers. The right-hand side is bankers' (gross) profit if they raise

λ dollars and sink into their investments when the liquidity shock hits. Observe that investments in real assets decrease from $f(W)$ to $f(W - \lambda)$ as a result of the liquidity shock. That is, a liquidity shock causes an economic crisis by reducing output.

I also introduce a government and allow it to intervene by providing liquidity when a shock hits. I assume that it is costly for the government to provide liquidity, perhaps, because in that case it has to divert funds from public projects. The net per dollar cost of the government's intervention on the economy is ψ. For example, if the government provides λ dollars to bankers at date $t = 2$, the welfare loss would be $\lambda\psi$ whereas it would have been $f(W) - f(W - \lambda)$ if financiers had provided λ dollars. ψ can be high or low: $\psi \in \{\psi^l, \psi^h\}$ with $\psi^l < f'(W)$ and $\psi^h > f'(W - \lambda)$. The ex ante probability that ψ is equal to ψ^h is $q \in [0, 1]$.

The government acts as a social planner and intervenes (*i.e.,* provides liquidity) only if its intervention is efficient for the economy (*i.e.,* when ψ is equal to ψ^l). When it intervenes, it sets the price of the liquidity (*e.g.,* interest rate) and bankers choose to obtain cash either from the government or financiers. When the welfare effects of several pricing strategies are the same, the government chooses the strategy that maximizes its payoff. Finally, for simplicity, I do not allow partial intervention. That is, if the liquidity shock hits at date $t = 2$, either the government or financiers provide the liquidity, not both.

2.2 Competitive Banker's Problem

At date $t = 1$, depending on the magnitude of p, bankers might want to keep some cash. Bankers can obtain cash by selling their assets to financiers.[5] Alternatively, they can wait until date $t = 2$ and sell their assets only if the liquidity shock hits. As it will be clear, the reason bankers want to obtain some cash at date $t = 1$ is that it will be more costly to obtain cash at date $t = 2$ if the liquidity shock hits.

Let C_1 be the amount of cash bankers demand at date $t = 1$. In order to obtain C_1 dollars, they sell the fraction μ_1 of their assets. Because financiers have an opportunity to invest in their production technology, bankers have to offer a discount to convince financiers to buy their financial assets at date $t = 1$. Let d_1 be the (gross) return financiers receive at date $t = 3$ when they buy bankers' assets at date $t = 1$. The following equality holds when the market clears:

$$C_1 d_1 = r\mu_1.$$

To obtain C_1 dollars from financiers, bankers sell their assets worth $r\mu_1$ dollars. Bankers cannot sell their assets without offering a discount (*i.e.,* $\frac{1}{d_1}$) because the marginal return of financiers' production technology is greater than one.

Bankers' decision at date $t = 2$ is trivial: if the liquidity shock does not hit, they do not take any action. In that case, their profit at date $t = 3$ becomes $(1 - \mu_1)r + C_1 - d_0$. Otherwise (*i.e.,* if the shock hits at date $t = 2$), each banker

[5] A banker can also raise funds in other forms, for instance by collecting deposits. My results would not change if the banker obtained cash by issuing new debt contracts. In general, because the new debt will be subordinated relative to the existing debt, such a form of financing will not be possible because of the debt overhang problem. See Stein [2012] for an explanation why an asset sale is an unavoidable consequence of a liquidity shock.

sells fraction μ_2 of their assets to obtain $C_2 = \lambda - C_1$ dollars.[6] If the government's cost of intervention is too high, financiers buy bankers' assets. For financiers to be willing to liquidate their investments to pay $\lambda - C_1$ dollars to bankers, the marginal return of their remaining investments should be equal to the return they obtain from financial assets. The following equation pins down the return of the assets sold to financiers in the fire sale at date $t = 2$:

$$\begin{aligned} d_2 &= f'(W - C_1 - C_2) \\ &= f'(W - \lambda). \end{aligned} \tag{1}$$

Thus, the price of bankers' assets at date $t = 2$ is $\frac{1}{f'(W-\lambda)}$. Note that the government, when it intervenes, buys bankers' assets at the same price. To see that, first observe that the price cannot be lower than $\frac{1}{f'(W-\lambda)}$; otherwise bankers would prefer obtaining cash from financiers instead of the government. The price cannot be higher than $\frac{1}{f'(W-\lambda)}$ either because in that case the government could increase its payoff, without affecting welfare, by reducing price to $\frac{1}{f'(W-\lambda)}$. Therefore, regardless of who the provider of the liquidity is, d_2, the return from the fire sale at date $t = 2$, would be the same.

Now I can derive the return of the assets sold at date $t = 1$ (*i.e.*, d_1). At date $t = 1$, a financier should be indifferent between buying the banker's assets with his last penny and investing in his production technology. With probability $1 - p + p(1 - q) = 1 - pq$, the financier will not provide additional financing to the banker at date $t = 2$. It could be either because the liquidity shock does not hit or the liquidity

[6] I define μ_2 as the ratio of the value of assets sold at date $t = 2$ to r. Note also that it is not optimal to obtain more than λ dollars at any date.

shock hits but the government intervenes. In both cases, the marginal return of the financier's investment is $f'(W - C_1)$. With the complementary probability pq, the financier provides additional financing and earns d_2 on each dollar he gives to the banker at date $t = 2$. The indifference condition gives

$$d_1 = (1 - pq)f'(W - C_1) + pqf'(W - \lambda). \tag{2}$$

The inefficiency of a fire sale is evident from the expression above. A fire sale at date $t = 1$ reduces investments in real assets from $f(W)$ to $f(W - C_1)$, which is inefficient in expectation because there is a possibility that the liquidity shock will not hit.

Bankers' expected profit at date $t = 1$ is

$$(1 - p)\Big[(1 - \mu_1)r + C_1 - d_0\Big] + p\Big[(1 - \mu_1 - \mu_2)r - d_0\Big],$$

which on substituting for $\mu_1 = \frac{C_1 d_1}{r}$ and $\mu_2 = \frac{(\lambda - C_1)d_2}{r}$ simplifies to

$$r - d_0 - p\lambda d_2 + \Delta C_1,$$

where I define

$$\Delta := 1 - p - d_1 + pd_2. \tag{3}$$

Observe that bankers' profit function is linear in C_1. Therefore, the amount of cash demanded at date $t = 1$ depends on Δ, the coefficient of C_1. For example, if Δ is negative, none of the bankers will demand cash. Also observe that each competitive banker takes Δ as given. That is, an individual banker has no power to affect d_1 or d_2, and thus Δ. The following lemma uses these observations to derive the equilibrium. The proofs of the lemmas are in the Appendix.

Lemma 1. *Define* $p_1 = \frac{f'(W)-1}{qf'(W)+(1-q)f'(W-\lambda)-1}$. *If the probability of the liquidity shock is smaller than* p_1*, bankers do not demand cash at date* $t = 1$*. Otherwise, bankers' total demand for cash,* C_1*, at date* $t = 1$ *can be calculated from the equation below:*

$$f'(W - C_1) = \frac{1}{1 - pq}\Big[1 - p + p(1 - q)f'(W - \lambda)\Big].$$

Bankers in a competitive market have an incentive to obtain some cash at date $t = 1$ against the potential liquidity shock at date $t = 2$. They demand cash because if a liquidity shock hits, their cost of obtaining cash will be higher: $f'(W - C_1) < f'(W - \lambda)$. Their rush to raise funds at date $t = 1$, before a liquidity shock hits, causes misallocation of funds in the economy by reducing investments in real assets. Welfare in the economy is given below:

$$r+(1-p)\Big[f(W-C_1)+C_1\Big]+p\Big[qf(W-\lambda)+(1-q)\big(f(W-C_1)-\psi^l(\lambda-C_1)\big)\Big], \quad (4)$$

where C_1 can be obtained from Lemma 1. The analysis so far is summarized in the proposition below and illustrated in Figure 1.

Proposition 1. *Competitive bankers' demand for cash at date* $t = 1$ *increases with the probability of the liquidity shock.*

Proof. Competitive bankers' demand for cash at date $t = 1$ is zero when the probability of the liquidity shock is smaller than p_1. When this probability is greater than p_1, the first-order condition of the bankers' problem requires Δ to

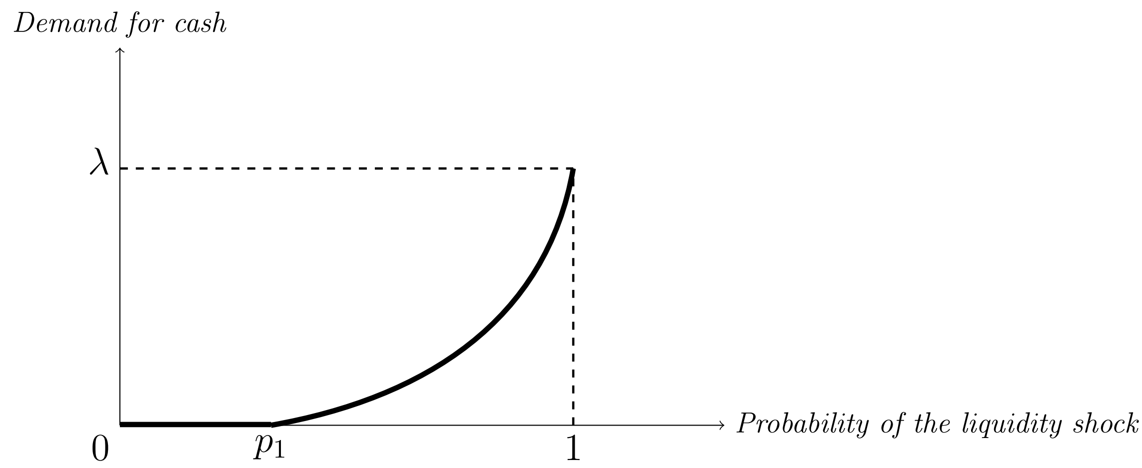

Figure 1: Competitive bankers' demand for cash at date $t = 1$ increases with the probability of the liquidity shock.

be zero. The partial derivative of C_1 with respect to p at the optimum can be obtained by applying the implicit function theorem to the first-order condition:

$$\frac{\partial C_1}{\partial p} = -\frac{(1-q)\big(f'(W-\lambda)-1\big)}{(1-pq)^2 f''(W-C_1)}.$$

Because the expression above is positive, bankers' demand for cash increases with the likelihood of the liquidity shock. □

With his demand at date $t = 1$, a banker contributes to the increase in the price of cash. That is, each banker imposes a negative externality on other bankers. One might ask why this externality is not corrected by the price system. After all, pecuniary externalities do not result in misallocation of resources.[7] A pecuniary externality causes an inefficiency in my model because the government cannot commit not to provide liquidity. In particular, the government cannot commit at

[7]See, for example, Shubik [1971] and Laffont [1987] for explanations why pecuniary externalities do not lead to violations of the standard welfare theorems.

date $t = 1$ not to intervene when its intervention is beneficial for the economy at date $t = 2$.

To see why the possibility of a government intervention creates inefficiency, assume that the probability of government intervention is zero in the model. Observe that when $1 - q$ is zero, bankers do not demand cash at date $t = 1$ because the threshold p_1 in Lemma 1 becomes one. Intuitively, the possibility of the government intervention at date $t = 2$ softens bankers' budget constraint at date $t = 1$ by preventing the fire-sale price to fall too much. In particular, when government intervention is not a possibility, the buyers of financial assets at date $t = 1$ demand a higher discount in the fire sale because they know that if the liquidity shock hits they will make a higher profit with their cash in hand. The subsequent increase in the fire-sale price deters bankers from selling their assets at date $t = 1$, and the inefficiency disappears.

3 Social Planner's Problem

The social planner's objective function at date $t = 1$ is given below:

$$(1-p)\Big[r + f(W - C_1) + C_1\Big] + p\Big[r + qf(W-\lambda) + (1-q)\big(f(W-C_1) - (\lambda - C_1)\psi^l\big)\Big].$$

The first part represents the output in the economy if the liquidity shock does not hit. The second part corresponds to the output if the shock hits. In the latter case the social planner also recognizes the loss in welfare due to the government's intervention. Asset sales and interest payments are transfers between agents and, thus, they do not show up in the social planner's objective function. The social planner cares only about the total production in the economy. The following proposition derives the social planner's strategy.

Proposition 2. *The social planner does not sell any of the banker's assets at date $t = 1$.*

Proof. The partial derivative of the objective function with respect to C_1 is negative:

$$(1-p)\big(-f'(W-C_1)+1\big)+p(1-q)\big(-f'(W-C_1)+\psi^l\big)<0.$$

Thus the objective function is maximized by setting C_1 to zero. □

The intuition behind this result is clear: the sale of the financial assets at date $t = 1$ reduces the investment in the real assets, which will be inefficient if the liquidity shock does not hit. The welfare under the social planner's management can be written as

$$r+(1-p)f(W)+pqf(W-\lambda)+p(1-q)\big(f(W)-\lambda\psi^l\big). \tag{5}$$

The loss in welfare (due to the rush to obtain liquidity) in the perfectly competitive market can be calculated by subtracting (5) from (4):

$$(1-p)\Big[f(W)-f(W-C_1)-C_1\Big]+p\Big[(1-q)\big(f(W)-f(W-C_1)+C_1\psi^l\big)\Big]. \tag{6}$$

For small values of p, competition does not result in any output loss. In particular, when the probability of the liquidity shock is smaller than p_1, bankers do not sell any asset at date $t = 1$ (*i.e.,* $C_1 = 0$) and the expression (6) becomes zero. When the probability of the shock is greater than p_1, bankers would like to sell their assets before others do (*i.e.,* $C_1 > 0$). Such a rush for liquidity adversely affects the output by prompting financiers to reduce their investments in real assets even before the liquidity shock hits.

4 Monopolist with a Competitive Fringe

The analysis in the previous sections shows that populating the economy with many small banks causes a welfare loss. The main reason for this inefficiency is that small banks do not recognize their own impact on the equilibrium terms of trade (*i.e.,* fire-sale prices at date $t = 1$). Their individually optimal decisions become suboptimal from the social planner's perspective. In this section I introduce into the model one big bank which internalizes its impact on the prices. In particular, I assume that some bankers with the total measure of $\phi > 0$ merge and establish one big bank. The rest of the banking system consists of many small bankers which, in aggregate, occupy $1 - \phi$ of the banking system.

I will denote the big bank and the small competitive bankers with superscripts "b" and "s" respectively. The total demand for cash in the economy at date $t = 1$ is the sum of demands of both types of bankers:

$$C_1 = \phi C_1^b + (1 - \phi) C_1^s.$$

Fire-sale prices at dates $t = 1$ and $t = 2$ can be obtained from (1) and (2). The big bank's expected profit at date $t = 1$ is

$$(1-p)\Big[(1-\mu_1^b)r + C_1^b - d_0\Big] + p\Big[(1-\mu_1^b-\mu_2^b)r - d_0\Big],$$

which on substituting for $\mu_1^b = \frac{C_1^b d_1}{r}$ and $\mu_2^b = \frac{(\lambda - C_1^b)d_2}{r}$ simplifies to

$$r - d_0 - p\lambda d_2 + \Delta C_1^b.$$

Contrary to small bankers' objective, the big bank's objective function is not linear in its demand for cash (*i.e.,* in C_1^b) because d_1 in Δ depends on C_1^b. That is, by demanding cash the big bank affects the fire-sale price at date $t = 1$. The partial derivative of the big bank's profit function with respect to C_1^b gives

$$\Delta + \phi C_1^b f''\big(W - \phi C_1^b + (1-\phi)C_1^s\big).$$

The second term above represents the big bank's own influence on the fire-sale price at date $t = 1$. Because $f''(\cdot)$ is negative, the big bank has less incentive to sell its assets compared to small bankers which take the fire-sale price as given.[8] The following lemma derives the equilibrium at date $t = 1$.

Lemma 2. *Define* $p_1 = \frac{f'(W)-1}{qf'(W)+(1-q)f'(W-\lambda)-1}$ *and*

$p_2 = \frac{f'(W)-1}{qf'(W)+(1-q)f'\big(W-(1-\phi)\lambda\big)-1}$.

[8]Note that the second-order condition of the maximization problem is satisfied because the third derivative of the production function is positive. If the third derivative were negative, the big bank would demand even less cash. For example, in the extreme, if the third derivative were negative infinity, the big bank would not demand any cash at all.

The equilibrium at date $t = 1$ can be characterized by three regions:

- $p \in [0, p_1)$*: There is no demand for cash and, thus, no fire sale.*
- $p \in [p_1, p_2)$*: Small bankers demand some cash, but the big bank does not. The total demand for cash in the economy,* $C_1 = (1-\phi)C^s$*, can be computed from the expression below:*

$$f'\big(W - C_1\big) = \frac{1}{1-pq}\Big[1 + p + (1-q)f'(W-\lambda)\Big].$$

- $p \in (p_2, 1]$*: Each small banker demands λ dollars while the big bank demands* C_1^b *dollars. The total demand for cash in the economy,* $C_1 = \phi C^b + (1-\phi)\lambda$*, can be computed from the expression below:*

$$f'(W-C_1) = \frac{1}{1-pq}\Big[1+p+(1-q)f'(W-\lambda)\Big] + \frac{C_1 - (1-\phi)\lambda}{1-pq} f''(W-C_1). \tag{7}$$

Note that the upper threshold p_2 approaches p_1 as ϕ decreases. When the market becomes perfectly competitive (*i.e.*, $\phi = 0$), Lemma 2 is equivalent to Lemma 1.

When the probability of the liquidity shock is between p_1 and p_2, the total demand for cash in the economy is the same in both market structures. Although the big bank does not sell its assets, each small banker sells more than the amount they would sell in a perfectly competitive market. When the probability of a shock is greater than p_2, the total demand for cash is higher in the competitive market because the last term in expression (7) is negative. The following proposition summarizes the analysis and Figure 2 illustrates the result by comparing the demand

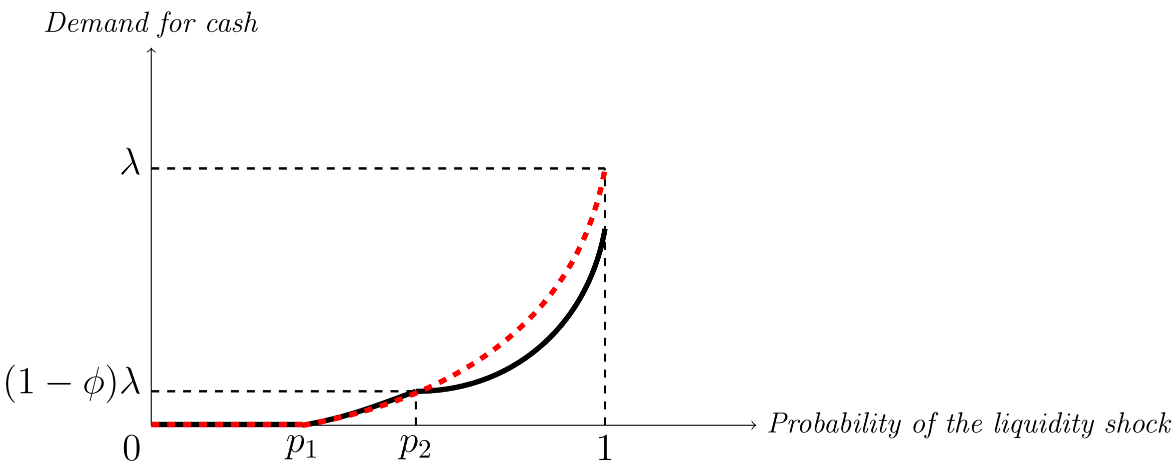

Figure 2: The black curve shows the aggregate demand for cash in the monopolistic banking structure with a competitive fringe. The red dashed curve represents the demand for cash in the competitive market.

for cash in the two market structures.

Proposition 3. *Introduction of a big bank into a perfectly competitive market improves efficiency by reducing the demand for cash in the economy at date* $t = 1$.

Proof. The welfare loss can be calculated by inserting the total demand for cash C_1 into expression (6). The partial derivative of (6) with respect to C_1 is

$$(1-p)\big[f'(W-C_1)-1\big]+p(1-q)\big[f'(W-C_1)+\psi^l\big],$$

which is positive. A decrease in the total demand for cash at date $t=1$ increases welfare. $\square$

5 Conclusion

Big banks tend to take excessive risks, because they expect to receive assistance from the government if their bets go bad. The possibility of a government bailout weakens the market discipline by reducing depositors' incentives to monitor.

In this paper, I identify a market failure and argue that big banks are helpful in correcting this failure. In particular, I show that when faced with the possibility of a liquidity shock, competitive banks cause a decline in the output by removing funds from the real assets through a fire sale. They have an additional incentive to trigger a fire sale because they do not internalize their own impact on prices.

Big banks can improve welfare. They play the role of a moderator in fire sales because they internalize the effect of their own actions. Their timely sales improve efficiency by decreasing the amount of funds removed from the real assets. These results suggest that instead of blindly penalizing banks for being too big, regulators should balance the benefits and costs of having big banks.

Appendix

This appendix contains the proofs of the lemmas.

Proof of Lemma 1

Proof. The representative competitive banker's problem is given below:

$$\max_{C_1} r - d_0 - p\lambda d_2 + \Delta C_1,$$

where

$$d_1 = (1 - pq)f'(W - C_1) + pqf'(W - \lambda)$$

$$d_2 = f'(W - \lambda)$$

$$\Delta = 1 - p - d_1 + pd_2.$$

The objective function is linear in C_1. Therefore, each banker's demand for cash at date $t = 1$ depends on the sign of Δ. There are three cases to consider:

- $\Delta < 0$. In this case, none of the bankers demands any cash before a liquidity shock hits. They set $C_1 = 0$. The return on assets sold in the fire sale at date $t = 1$ becomes

$$d_1 = (1 - pq)f'(W - \lambda) + qf'(W).$$

  The condition $\Delta < 0$ can be written as

$$p < \frac{f'(W) - 1}{qf'(W) + (1 - q)f'(W - \lambda) - 1}.$$

The expression on the right-hand side of the inequality above defines p_1, the threshold probability that bankers start selling their assets.

- $\Delta = 0$. In this case, at date $t = 1$ each banker is indifferent between obtaining some cash and not. In the symmetrical equilibrium each banker demands the same amount of cash C_1, which can be calculated from the equation below:

$$\Delta = 0 \Leftrightarrow f'(W - C_1) = \frac{1}{1 - pq}\Big[1 - p + p(1 - q)f'(W - \lambda)\Big].$$

  Note that because $C_1 \in [0, \lambda]$, this equilibrium is possible only if $p \geq p_1$.

- $\Delta > 0$. In this case, each bank demands $C_1 = \lambda$, the maximum amount of cash they will need if a liquidity shock hits. The returns on assets bought in the fire sales at date $t = 1$ and $t = 2$ (*i.e.,* d_1 and d_2) become equal:

$$d_1 = d_2 = f'(W - \lambda).$$

  If at date $t = 1$ a banker knows that the cost of obtaining cash will be the same at date $t = 2$, he optimally postpones his selling decision. By postponing he can be sure that he obtains cash only when he needs it. Therefore, bankers will not sell any of their assets at date $t = 1$ if d_1 is equal to d_2. This contradicts with the claim that Δ is positive in the equilibrium.

□

Proof of Lemma 2

Proof. The representative small banker's problem is given below:

$$\max_{C_1^b} r - d_0 - p\lambda d_2 + \Delta C_1^b,$$

where

$$d_1 = (1-pq)f'(W - \phi C_1^b - (1-\phi)C_1^s) + pqf'(W-\lambda)$$

$$d_2 = f'(W-\lambda)$$

$$\Delta = 1 - p - d_1 + pd_2.$$

As in the competitive market, the small banker's problem is linear in his demand for cash (*i.e.,* C_1^b). That is, small bankers base their decisions on the sign of Δ. The big bank's profit function is given below:

$$r - d_0 - p\lambda d_2 + \Delta C_1^b.$$

The partial derivative of the big bank's profit function with respect to C_1^b gives

$$\Delta - \phi C_1^b f''\big(W - \phi C_1^b - (1-\phi)C_1^s\big), \tag{8}$$

which is not linear. Observe that because $f''(\cdot)$ is negative, the big bank's incentive to obtain cash is smaller than that of the small banker. I use the same technique I invoke in the proof of Lemma 1 and derive the equilibrium considering the three different cases with respect to Δ.

- $\Delta < 0$. In this case, none of the bankers demands cash. They set $C_1^b = C_1^s =$

0. The return on the assets sold in the fire sale at date $t = 1$ becomes

$$d_1 = (1 - pq)f'(W - \lambda) + qf'(W).$$

The condition $\Delta < 0$ can be written as

$$p < \frac{f'(W) - 1}{qf'(W) + (1 - q)f'(W - \lambda) - 1}.$$

As in Lemma 1, the expression on the right-hand side of the inequality above defines p_1, the threshold probability that bankers start selling their assets.

- $\Delta = 0$. In this case, at date $t = 1$ each small banker is indifferent between obtaining some cash and not. The big bank does not demand any cash (i.e., $C_1^b = 0$). In the symmetrical equilibrium each small banker demands the same amount of cash C_1^s. The total demand for cash in the economy becomes $C_1 = (1 - \phi)C_1^s$, which can be calculated from the equation below:

$$\Delta = 0 \Leftrightarrow f'(W - C_1) = \frac{1}{1 - pq}\Big[1 - p + p(1 - q)f'(W - \lambda)\Big].$$

Applying the implicit function theorem to the expression above yields:

$$\frac{\partial C_1}{\partial p} = -\frac{(1 - q)\big(f'(W - \lambda) - 1\big)}{(1 - \phi)(1 - pq)^2 f''(W - C_1)},$$

which is positive. That is, as p increases, small bankers' demand for cash increases. When their demand reaches λ dollars (i.e., when they are fully insured at date $t = 1$ against a liquidity shock), Δ becomes positive. This observation determines p_2, the upper threshold of the interval in which such an equilibrium is possible:

$$p_2 = \frac{f'(W) - 1}{qf'(W) + (1-q)f'(W - (1-\phi)\lambda) - 1}.$$

- $\Delta > 0$. In this case, small bankers demand $C_1^s = \lambda$, the entire amount of cash they will need at date $t = 2$ if a liquidity shock hits. The big bank also demands some cash: to satisfy the first-order condition (*i.e.,* to set the expression (8) to zero), C^b must be positive. The first-order condition yields

$$(1-pq)f'\big(W - \phi C^b - (1-\phi)\lambda\big) - \phi C_1^b f''\big(W - \phi C_1^b - (1-\phi)\lambda\big) = 1 - p + (1-q)f'(W-\lambda)$$

or

$$f'(W - C_1) = \frac{1}{1-pq}\Big[1 + p + (1-q)f'(W-\lambda)\Big] + \frac{C_1 - (1-\phi)\lambda}{1-pq} f''(W - C_1),$$

where $C_1 = \phi C^b + (1-\phi)\lambda$ is the total demand for cash in the economy.

$\square$

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