What Do CDO Tranche Spreads Tell Us About Credit Availability and Credit Rating Standards?

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Abstract

We introduce a Credit Rating Agency (CRA) to a consumption-based model to explain the time-series variation of credit spreads on CDO tranches. Practising Bayesian persuasion, the CRA controls the type-II error of ratings to maximize the proportion of investment-grade rated firms while attempting to maintain their probability of default below a threshold. The model's type-II error strongly predict a measure of rating standards from an ordered probit model. Our analysis suggests the importance of credit rating standards in explaining the clustering of defaults of investment-grade firms. Out-of-sample, it implies moderate tranche spreads during the pandemic despite disastrous economic growth.

1 Introduction

The collapse of the senior collateralized debt obligation (CDO) market was at the epicentre of the 2008-2009 financial crisis leading to the collapse of the largest and most prestigious US banks.[1] The technique of tranching of the claims made it possible to repackage credit risks and produce senior claims with significantly lower default probabilities and higher credit ratings than the average assets in the underlying pool, meeting investors' demand for Aaa-rated securities. The structured finance market demonstrated spectacular growth during the half decade before the financial crisis, but almost dried up following massive downgrades and defaults of highly-rated structured products during the crisis (see Coval, Jurek, and Stafford (2009b)). Somewhat puzzlingly, even though economic growth was significantly worse in the first quarter of 2020, senior tranche spreads did not rise to the extent that they did in the financial crisis.

One important narrative that has repeatedly been brought up in the academic literature as well as the financial press, is that there was a role played by credit rating agencies (CRAs), which handed out too many high credit ratings prior to the financial crisis. Indeed the tranche spreads that we study in this paper are constructed from the CDX index by *Markit*, which is comprised of investment-grade firms at the inception of a new series (cohort). Therefore, the credit rating standards at the time of the creation of a new series will affect the spreads of the index and its tranches. However, the asset pricing literature in pricing CDO tranches does not explicitly model a CRA or study its role in influencing investors' expectation on the value of these securities. Recent papers (most notably Seo and Wachter (2018)) do point to the role of the consumption disaster during the crisis in leading to the collapse of the CDO market, and understanding some of the dynamics of spreads during the crisis period. We build on this consumption-based framework, modelling both long-run risk and economic disasters, but introducing a new feature, that is, the presence of a stylized credit rating agency (CRA), which rates firms in the economy using the principle of Bayesian persuasion. We show in the paper that the time-variations in credit rating standards used by the CRA have an impact on the clustering of defaults of investment-grade products.

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[1]A typical structured debt product such as a collateralized debt obligation (CDO) is a large pool of economic assets with a *prioritized* structure of claims (tranches) against this collateral.

In our model there are good and bad quality firms, with the latter being more exposed to hidden jump risks. The CRA conducts research on firms' credit quality, and is able to partially determine their quality. We assume that consumers have a preference for investing in highly-rated securities and that the CRA chooses the intensity of its research to maximize the proportion of firms with investment-grade ratings, subject to a default probability constraint of the investment-grade rating class.[2] [3] In periods of strong fundamentals, the default probabilities of all firms are lower, so the CRA gives some more bad firms high ratings to get the default probability up to its limit, but also conditionally exposes investors to more jump risk.

Another major theme that has arisen among academics and the financial press, is the role of the collapse of credit after the economic collapse in 2008-2009 in the pricing of CDO tranches. We in fact uncover some new evidence in this paper, that credit growth does play a big role in the pricing of alternative CDO tranches, although its impact is different for different tranches. We study the time series of spreads on tranches on the Dow Jones North American Investment Grade Index of credit default swaps, which are shown in Figure 1.[4] The "equity" tranche (top-left panel) represents the 0 to 3 percent loss attachment points (these securities suffer losses if the loss on the entire collateral pool is between 0 and 3 percent of the

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[2]In the seminal framework of Bayesian persuasion proposed in Kamenica and Gentzkow (2011), the signal sender (a prosecutor) has a bias in her objective function (she only gets paid for convictions). She then changes the quality of her investigation process to change the type-I and type-II errors of the evidence to maximize the likelihood of the signal receiver (the judge) to convict. We model a similar communication game, with the signal sender being the CRA, and the signal receiver being the market.

[3]There are several institutional reasons for investors to prefer investment-grade securities (see, for example, Boot, Milbourn, and Schmeits (2006) and Coval, Jurek, and Stafford (2009b)). In addition, the issuer-pays business model adopted by most credit rating agencies gives the rating agencies the incentive to issue inflated ratings; for discussion, see e.g., Bar-Isaac and Shapiro (2011), Bolton, Freixas, and Shapiro (2012), Fulghieri, Strobl, and Xia (2014), Harris, Opp, and Opp (2013), Cohn, Rajan, and Strobl (2013), Kartasheva and Yilmaz (2012), and Goldstein and Huang (2020).

[4]It is important to note that contractual features of payment for protection in credit default swaps can be specified as either upfront payments with fixed quarterly spread payments, or as zero upfront payments with variable spread payments. The payment features have changed substantially over time making intertemporal comparisons of the raw data difficult. We summarize these changes in Appendix 2 of this paper. Here using our model, we present approximated spreads that would result with zero upfront payment and fully variable quarterly spreads. In Section 3.3, we examine the ability of our theoretical model to fit the raw data.

underlying capital, are wiped out if the losses exceed 3 percent), while the "senior" tranche (top-right panel) represents the 15 to 100 loss attachment points. While both spread series rose rapidly during the financial crisis, the rise in the senior tranche spread was more spectacular, from only about 3.5 basis points (b.p.) before the crisis, to above 130 b.p. at its peak. The equity tranche by comparison, only rose by a factor of five from about 1135 b.p. in the second quarter of 2007 to 6097 b.p. at its peak in the second quarter of 2009. Post-crisis, the initial recovery of the senior spread was notably faster: by the first quarter of 2010, the senior spread was down to 30 b.p., while the equity spread remained substantially above pre-crisis levels until 2012.

Next, we examine the relationship between the tranche spreads and macroeconomic fundamentals. The bottom-left panel of Figure 1 shows that the 4-quarter lagging moving average of real consumption growth bottomed out in the middle of the great recession, and resumed at a more normal pace by mid-2009 right after the recession. The bottom-right panel shows that the 4-quarter moving average of the ratio of credit growth at nonfinancial companies to GDP bottomed out at the end of the recession, but returned to normal levels only in mid-2011, substantially later than the normal resumption of consumption. These observations suggest that the senior tranche is more correlated with economic growth, while the junior tranche is more correlated with credit availability. In Table 1, we regress the spread on the entire pool (CDX) as well as the equity and senior tranches on the two fundamentals. For each of the spread series, it is noteworthy that despite the presence of a macroeconomic factor, credit growth additionally impacts tranche spreads. However, the relative importance of the two fundamentals for equity and senior spreads is quite different. In lines 4 to 6, we see that credit growth explains nearly 64 percent of the variation in the equity spread, while consumption growth only explains only about 22 percent of its variation. In contrast, in lines 7 to 9, we see that credit growth explains 42 percent of the variation in the senior spreads, while consumption growth explains 33 percent.

Motivated by this evidence, our model has both credit growth and economic growth as state variables. While the role of economic growth in affecting the prices of these securities is now well established, we assume that credit growth affects the parameters that determine the jump (catastrophic) risks of firms' cash flows. One possibility (consistent with the view of

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Minsky (1986)) is that during periods of easy credit, credit is provided to weaker borrowers, who are more susceptible to complete failures. In such a situation, increased credit would predict future declines in economic growth. Alternatively, credit could respond to macroeconomic growth with lenders getting more cautious and cutting credit following periods of weak growth, thus propagating the effects of weak growth further. In this situation, weak growth would predict future declines in credit availability.

We perform a two-stage structural estimation of the parameters of our model to understand the role of alternative fundamentals and credit rating standards in driving the dynamics of CDO tranche spreads. At the first stage, we estimate the parameters of credit growth, consumption growth, and aggregate earnings growth of US firms in a regime-switching framework for the period 1952 to 2017. With these estimates, in the second stage, we estimate sets of jump risk parameters of low quality firms in our model, which depend on the growth and credit states, over the sample from the third quarter of 2004 to 2017. Our parameter estimates suggest that the intensity of destructive shocks to bad firms is higher during periods of lower economic growth as well as lower credit growth, i.e., credit quality is procyclical.

The spreads in the model for structured products that are comprised entirely of investment-grade firms are a function of both the quality of the good and bad firms as well as the rating standards (type-II error) determined by the CRA. The optimal type-II error changes over time as the CRA commits to a maximum default probability over a 5-year horizon conditional on the fundamental state. In periods, of weak fundamentals, the CRA mixes fewer bad firms into the investment-grade class, however, the default probability may remain above the target despite higher credit rating standards. Conversely, in periods of strong fundamentals, the CRA is able to provide high ratings to most (all) firms and the default probability may remain below the target. In the intermediate range, the fundamentals are just sufficiently strong so that with a chosen level of type-II error by the CRA, the proportion of bad firms in the investment-grade category leads to a default probability is at the target level.

The results from the structural estimation are quite illuminating on the determinants of tranche spreads. Using a standard simulated method of moments (SMM) framework, the tranche spread implications of our model with the CRA has a p-value of only 2 percent if all the spread data are used; however, the p-value increases above 10 percent if we exclude the

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senior (15,100) tranche prior to the financial crisis. The model fits the junior tranche spreads in the data quite well through the entire sample, but it has trouble fitting the senior (15-100) tranche prior to the financial crisis (data spread is lower than model spread in this period).[5] The model fits the senior tranches well during and after the financial crisis. The delay in recovery of the equity tranche after the end of a macroeconomic recession occurs because even though firms' solvency ratios improve, the jump risks of firms remain high amidst weak credit growth. On balance, the losses at such time are large enough to impact junior tranches, but not senior tranches. Our evidence is supportive of the view that credit availability remains weak after bad economic shocks, as opposed to the reverse. We back up these estimates by impulse response functions and Granger causality tests that suggest that growth causes weak credit, but not the reverse. Over time, our model replicates the feature in the data that senior tranche spreads are relatively more exposed to macroeconomic growth shocks, while junior spreads are more exposed to credit availability shocks.

We view the main contribution of our model is to shed light on the way that time-varying credit rating standards affect the clustering of defaults of investment-grade firms, which affects spreads on alternative CDO tranches. To provide plausibility for the rating standards in our model (type-II error), we compare them to the credit rating standards implied from the framework of Blume, Lim, and Mackinlay (1998) and Alp (2013), who use an ordered probit model of firms credit ratings to measure time variation in credit rating standards. The independent variables used are firm level indicators of credit quality such as leverage ratios, interest coverage ratios, as well as measures of firms' risk, such as their idiosyncratic volatility and exposures to standard systematic risk factors. The time fixed-effect, α_t, is a measure of *credit rating standards* at time t. We find that there is a correlation of about 0.66 between the six-quarter lagged model type-II error and the intercept from the ordered probit model, i.e., historical credit rating standards are lagging relative to measures of rating standards embedded in CDO prices. This 'slowness' of ratings is well known (see, e.g, Altman and Rijken (2004)).

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[5]The overpriced senior tranche prior to the financial crisis is consistent with anecdotal evidence in the financial press as well as the movie "The Big Short".

In addition to generating an empirically plausible credit ratings standards process, our model also generates an empirically plausible process of average credit ratings. We compare the detrended time-series of the proportion of US firms in Compustat with investment-grade ratings, with the proportion firms at each date that have investment-grade ratings in our model. Even though, we do not use this proportion as a moment in our estimation procedure, we find that the six-quarter lagged model and data proportions are 65 percent correlated, and display many of the same trends.

Finally, we test the out-of-sample performance of our model. Our model's parameters are estimated using data until 2017, and we test its ability to fit fundamentals and tranche spreads in the period from 2018 to 2020:Q2. The most successful aspect of this exercise from our model's point of view is that it predicts that during the pandemic-induced recession of 2020, tranche spreads increased, but nowhere near the levels attained during the financial crisis. This is true in the data as well. Despite even worse economic growth, the model's spreads increase only modestly because credit growth in this period is massive, and due to weak growth, our model predicts that the CRA would not mix bad firms into the investment-grade class. Overall, the out-of-sample performance provides further support to the model's mechanism of tranche spreads being determined by credit growth and rating standards, in addition to economic growth.

Relation to the Literature

Our paper contributes to the growing literature on the pricing of CDX tranches. In their seminal paper, Coval, Jurek, and Stafford (2009a) investigate whether CDX tranches were mispriced prior to the crisis. Using a CAPM-based pricing kernel, they extract state prices from the prices of 5-year index options using the methodology of Breeden and Litzenberger (1978), and use these state prices to determine fair CDO spreads in the years before the financial crisis. Their estimation results suggest that senior CDO tranches were mispriced prior to the financial crisis, with spreads being too low by a factor of 3 to 4.[6] Collin-Dufresne, Goldstein, and

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[6]These authors also point to the importance of considering the systematic risk component of the default losses of CDO tranches, as opposed to just the expected default losses, a theme that is reiterated by Brennan, Hein, and Poon (2009).

Yang (2012) extend the methodology of Coval, Jurek, and Stafford (2009a) to allow for the possibility of early default such as in Black and Cox (1976). Under this alternative assumption they find a smaller mispricing of the senior tranches. However, these authors indicate that a dramatic widening of the senior spread in the crisis can only be reconciled with the possibility of a catastrophic jump, which they use as a "free parameter" in fitting spreads. Perhaps, our paper is closest to Seo and Wachter (2018) who develop an equilibrium model with a risk of a catastrophic economy-wide losses. They show that such a model is able to generate a pricing kernel required to price senior CDX tranches in the sample from 2004 to 2008. These authors do acknowledge the role of the financial sector in the propagation of the economic disaster, but they do not explore this direction. As mentioned, based on our evidence, there is a role of credit provided by the financial sector in the pricing of CDO tranches, which we attempt to model, albeit in a simple way. Fostel and Geanakopolos (2012) argue that CDX spread dynamics were affected by the timing of innovations such as the tranching.

The major focus of our paper is on how time-varying credit rating standards affect CDO tranche spreads. The literature puts forward several reasons for time-varying quality of ratings; specifically lower quality of ratings in the times of economic expansion. For example, Bar-Isaac and Shapiro (2013) show that changing cost of analysts over the business cycle affect the accuracy of credit ratings; Fulghieri, Strobl, and Xia (2014) suggest that the rating precision is cyclical based on a model of credit agency reputation. Cornaggia, Cornaggia, and Hund (2017) find that ratings were considerably less accurate before the financial crisis in the areas with more complexity. Skreta and Veldkamp (2009) argue that increased complexity of assets creates an incentive to rating shop and can lead to higher rating inflation. Goldstein and Huang (2020) show that a CRA can affect real decisions despite some ratings inflation. In contrast to these papers, we propose a model of time-varying rating standards based on Bayesian persuasion practised by the rating agencies that interacts with time-varying fundamental credit quality. We show that our model's ratings are strongly related to those in the data.

There is also a growing literature on the role of credit ratings on alternative asset-backed securities, which we contribute to. Benmelech and Dlugosz (2009) document a major disconnect between the credit ratings of Collateralized Loan Obligations (CLO) securities and the credit quality of the underlying collateral. Griffin and Tang (2012) show that credit ratings

of CDOs were inflated prior to the financial crisis, and that CDOs with more inflated ratings suffered more frequent downgrades subsequently. Griffin (2020) provides evidence that credit ratings on Residential Mortgage Backed Securities (RMBS) were inflated due to the conflict of interest in the rating industry and show that the prices of these securities indicate that the marginal investor was unaware of rating inflation. In contrast, Ospina and Uhlig (2018) examine realized losses on non-agency residential mortgage-backed securities (RMBS) over the 2007–2013 period and question the conventional narrative, that improper ratings of RMBS were a major factor in the financial crisis of 2008. It is important to note that in this paper, we consider rating standards on corporate entities as opposed to ratings on structured products.

Our model of credit ratings contributes to a broader literatures on Bayesian persuasion and the role of beliefs in asset pricing. This framework of information design using Bayesian persuasion was first developed in the seminal paper of Kamenica and Gentzkow (2011).[7] In models where beliefs matter, Hong and Kubik (2003) analyze promotions and demotions of analysts and find that optimism counts more than accuracy in determining career shifts. Malmendier and Shanthikumar (2007) point out that security analysts tend to bias stock recommendations upward, particularly if they are affiliated with the underwriter. They find strong evidence that sophisticated investors take into account incentives of the information provider and update their beliefs as rational Bayesians. In a survey on Bayesian persuasion, DellaVigna and Gentzkow (2010) argue that the data supports belief-based models with rational receivers and persuasive communication is effective in shaping investors' beliefs.

Finally, our paper contributes to the growing literature on the role of credit in business cycles. One strand of this literature seeks to explore how investors expectations and credit growth relate to economic fundamentals (see e.g. Jorda, Schularick, and Taylor (2011), Schularick and Taylor (2012), He and Krishnamurthy (2013), Baron and Xiong (2017), Krishnamurthy and Muir (2017), and Bordalo, Gennaioli, and Shleifer (2018)). In this paper, we

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[7]There is a rapidly growing literature on information design and Bayesian persuasion. Kolotilin, Mylovanov, Zapechelnyuk, and Li (2017) propose a more general model of public persuasion with a privately informed receiver. Bergemann and Morris (2016) and Alonso and Zachariadis (2021) analyze an equilibrium in a model with public persuasion of multiple privately informed receivers. Goldstein and Huang (2016) consider a Bayesian persuasion in a coordination game where the sender can announce whether or not the fundamentals exceed a given threshold. Inostroza and Pavan (2021) consider Bayesian persuasion in the global games framework.

reexamine some of the channels in these papers using data on fundamentals and CDO tranche spreads by structurally estimating the hidden catastrophic risks of firms in alternative states of economic and credit growth.

2 A Model For Pricing Tranches on Collateralized Debt Obligations

As a starting point, we use a consumption-based asset pricing framework incorporating both long-run risk and disasters, each of which have been used in the credit risk literature (see e.g. Bhamra, Kuehn, and Strebulaev (2009), Chen (2010), and Seo and Wachter (2018)) for pricing tranches on collateralized debt obligations (CDOs). On this model structure, we build in two new features: First, we assume that the jump risks in the cash flows of firms depend not only on economic growth, but also on credit growth in the economy. Second, we model a stylized credit rating agency (CRA), which rates each firm in the economy as being either investment-grade or speculative-grade using the principle of Bayesian persuasion. The CDO products that we price are based on investment-grade firms, and the actions of the rating agency cause time-variation in the credit quality of firms that are rated investment-grade, thus affecting the pricing of CDO tranches.

2.1 Preferences and the Pricing Kernel

The representative consumer has stochastic differential utility of Duffie and Epstein (1992), which is a continuous-time version of the recursive preferences of Kreps and Porteus (1978), Epstein and Zin (1989), and Weil (1990). The utility at time t for a consumption process c is

$$U_t = E_t \left[\int_t^\infty f(c_s, U_s) \, ds \right]. \tag{1}$$

The normalized aggregator function $f(c, U)$ is given by

$$f(c, U) = \frac{\phi}{1 - \delta} \frac{c^{1-\delta} - ((1-\gamma)U)^{(1-\delta)(1-\gamma)^{-1}}}{((1-\gamma)U)^{(1-\delta)(1-\gamma)^{-1}-1}}, \tag{2}$$

where γ is the coefficient of relative risk aversion, ϕ is the rate of time preference, ψ is the elasticity of intertemporal substitution and $\delta = \psi^{-1}$.

We start with the real side of the economy. Aggregate consumption, C_t, follows the process:

$$\frac{dC_t}{C_t} = \mu_C(s_t)dt + \sigma_C dW_t, \tag{3}$$

where $\mu_C(s_t)$ is the state-dependent drift of the growth rate of output, σ_C is a 1×3 vector of constant volatilities, and where $W_t = (W_{1t}, W_{2t}, W_{3t})'$ is a 3×1 standard Brownian motion process. In our empirical section, we will assume that $\mu_C \in \{\mu_C^H, \mu_C^L, \mu_C^D\}$, where the first two rates are regular high and low growth states, while the last one is a 'disaster' growth state. Aggregate earnings growth in the economy, Y_t, follows

$$\frac{dY_t}{Y_t} = \mu_Y(s_t)\,dt \; + \; \sigma_Y\,dW_t, \tag{4}$$

where the drift $\mu_Y(s_t)$ is once again in one of three possible states $\{\mu_Y^H, \mu_Y^L, \mu_Y^D\}$, and is perfectly correlated with the drift of consumption, and σ_Y is a 1×3 vector of constant volatilities.[8] We next specify credit growth in the economy and its relation to economic growth. The ratio of credit growth-to-GDP (in short credit growth) follows:

$$dG_t \; = \; \mu_G(s_t)\,dt \; + \; \sigma_G\,dW_t, \tag{5}$$

where credit growth $\mu_G \in \{\mu_G^H, \mu_G^L, \mu_G^D\}$, that is credit growth can be in either high, low, or disaster states, the last one capturing sharp credit crunches in the economy, and σ_G is a 1×3 vector of constant volatilities.

There are overall 9 composite states of real growth and credit growth, however, in our empirical section, when we estimate our model, we find that only 6 of these states have positive probabilities of occurring in our sample. For example, during the financial crisis, disastrous credit and growth did not occur simultaneously. We number these 6 states as

$$\{(\mu_G^H, \mu_C^H, \mu_Y^H), (\mu_G^H, \mu_C^L, \mu_Y^L), (\mu_G^L, \mu_C^H, \mu_Y^H), (\mu_G^L, \mu_C^L, \mu_Y^L), (\mu_G^L, \mu_C^D, \mu_Y^D), (\mu_G^D, \mu_C^H, \mu_Y^H)\}.$$

The first four states, are simply the cross products of high and low credit, and high and low growth. The 5th state has low credit, and disastrous growth, while the 6th state has disastrous credit and high growth.

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[8]Even though consumption and aggregate earnings drifts move together, we will see below that the parameters of consumption growth determine the discount rates in the model, while the aggregate earnings growth parameters determine the cash flow.

The objective transition probabilities in continuous-time between the 6 states are determined by the generator matrix Λ, with elements λ_{ij} that determine the probability of switching from state i to state j in an interval of size dt as approximately $\lambda_{ij}dt$, and $\lambda_{ii} = -\sum_{j \neq i} \lambda_{ij}$. Notice that the transition probabilities are defined over the composite economic growth and credit availability states, so for example, the state of credit growth may affect the transition probability from a high to a low economic growth state (or vice versa).

For pricing assets, we need to specify all probabilities under the risk-neutral measure, which are determined from the objective probabilities and the pricing kernel of the economy. As shown in Bhamra, Kuehn, and Strebulaev (2009) and Chen (2010), the stochastic discount factor (SDF) follows the Markov-modulated jump-diffusion process:

$$\frac{dm_t}{m_t} = -r(s_t)dt - \sigma_m \, dW_t + \sum_{s_t \neq s_{t-}} \left(e^{\beta\left(s_{t-},s_t\right)} - 1 \right) dM_t^{(s_{t-},s_t)}, \tag{6}$$

where $r(s)$ is the real risk-free rate, $\sigma_m = \gamma \sigma_C$ are the market prices of risk associated with the Brownian motions W_t, and $\beta^{(s,s')}$ determines the risk premium for switches in state from state s to state s'. M_t is a matrix of compensated processes with element (s, s') following

$$dM_t^{(s,s')} = dN_t^{(s,s')} - \lambda_{ss'}dt, \quad s \neq s', \tag{7}$$

where $N_t^{(s,s')}$ is a Poisson counting process with intensity $\lambda_{ss'}$, i.e, $dN_t^{(s,s')} = 1$ with probability $\lambda_{ss'} \, dt + o(dt)$, and 0 otherwise. The jump intensity between states s and $s' \neq s$ under the risk-neutral (Q) measure is

$$\lambda_{ss'}^Q = \exp\left(\beta\left(s, s'\right)\right) \lambda_{ss'}. \tag{8}$$

Under the Q-measure we can write the compensated processes as

$$dM_t^{*\,(s,s')} = dN_t^{(s,s')} - \lambda_{ss'}^Q dt, \quad s \neq s'. \tag{9}$$

The risk-free rate, and risk-adjustment factors, $\beta(s, s')$ are given by

$$r(s) = -\frac{\phi(1 - \gamma)}{1 - \delta}\left[\left(\frac{\delta - \gamma}{1 - \gamma}\right) h(s)^{\delta-1} - 1\right] + \gamma\mu_C(s) - \frac{1}{2}\gamma(1 + \gamma)\sigma_C\sigma_C' - \sum_{s'} \lambda_{ss'}e^{\beta(s,s')}$$

$$\beta(s, s') = (\delta - \gamma) \log\left(\frac{h(s')}{h(s)}\right),$$

where $h(s)$ is determined by the solution to the system of equations (A.4) in Chen (2010). It is useful to note, that we are assuming that the mean economic growth rates affect the pricing

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kernel of the economy, however the mean credit growth rates do not. However, the joint transition matrix over composite states of economic and credit growth implies the pricing kernel is affected by the state of credit growth as well as economic growth. In our calibrated model for example, we find that strong earnings growth is more persistent with high credit growth relative to low credit growth.

2.2 The Firms

There are two types of firms: good and bad. The cash flow of a good firm, $X_t^{i,g}$, follows

$$\frac{dX_t^{i,g}}{X_t^{i,g}} = \mu_Y(s_t)\,dt + \sigma_X\left(\sqrt{\rho_i}\,dW_{2,t} + \sqrt{1-\rho_i}\,dW_t^i\right) \tag{10}$$

where we assume that the drift of each firm's cash flow growth is the same as the aggregate cash flow drift, the volatility of each firm's cash flow growth is σ_X, W_t^i is a firm-specific standard Brownian motion process, and the correlation between the growths of firms i and j is ρ. The cash flow growth of a bad firm, $X_t^{i,b}$, follows

$$\frac{dX_t^{i,b}}{X_{t_-}^{i,b}} = (\mu_Y(s_t) + \kappa(s))\,dt + \sigma_X\left(\sqrt{\rho_i}\,dW_{2,t} + \sqrt{1-\rho_i}\,dW_t^i\right) - dL_t^{s,i}, \tag{11}$$

which is identical to the cash flow process of a good firm, except for the last term. In it, $L_t^{s,i}$ is the counter of a Poisson jump process for firm i in state s. The jump occurs with a probability $\kappa(s)\,dt$ in an interval of length dt, and in case of the jump, the cash flow becomes zero forever. Merton (1976) calls this a "destructive" shock. It is useful to note, that aggregate consumption and earnings do not have jump risk components, so that these idiosyncratic destructive shocks to firms' cash flows are uncorrelated with the pricing kernel of the economy, and as such, do not carry risk premiums. Implicitly we are assuming that the idiosyncratic destructive shocks to the cash flows of the 125 firms in the Markit CDX indices at any point of time are a 'small' component of aggregate consumption shocks, while the continuous components of these cash flows could be a non-negligble part of aggregate consumption.

The continuous components of bad firms' are each mutually correlated with correlation coefficient, ρ_i, the same as for good firms. In addition, we assume that their destructive jump shocks are correlated. In particular, we assume that the joint distribution of jumps follow

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the correlated binomial (CB) distribution first applied in the finance literature in Cifuentes and Pagnoncelli (2014).[9] For this distribution, the probability of having K jump shocks among N bad firms is

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\begin{aligned}
\text{CB}(K; p, N, \varphi) &= \binom{N}{K} p^K (1-p)^{N-K} (1-\varphi) \quad \text{for} \quad K = 1, \cdots, N-1, \quad (12) \\
&= (1-p)^N (1-\varphi) + (1-p)\varphi \quad \text{for} \quad K = 0, \quad (13) \\
&= p^N (1-\varphi) + p\varphi \quad \text{for} \quad K = N, \quad (14)
\end{aligned}
$$

where the probability of each firm having a shock is p, and the correlation among shocks is φ. Relative to the standard binomial distribution, the CB distribution shifts some probability mass from the interior to the end points of the support, i.e., it has a greater probability of 0 shocks, and N shocks. There are two special cases of the CB distribution worth mentioning: if $\varphi = 1$, then it two-point distribution with parameter p, and if $\varphi = 0$, it is a standard binomial distribution.

The unlevered value of a firm of uncertain quality with current cash flow of X_t is $V_t^\alpha(s) = v^\alpha(s) X_t$. The following Lemma provides expressions for vectors of the valuation ratios of firms in the S states.

Lemma 1 *For a firm whose quality is uncertain, and the investor has a probability that the firm is good is α, the vector of V/X ratios is*

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v^\alpha = \left(Diag(r + (1-\alpha)\kappa - \mu_y^Q) - \Lambda^Q \right)^{-1} \mathbf{1}, \quad (15)
$$

in which the risk-neutral drift of the cash flow in state s is $\mu_Y^Q(s) = \mu_Y(s) - \gamma \sqrt{\rho_i} \sigma_X' \sigma_C$, $Diag(v)$ denotes the diagonal matrix created from a vector v, and $\mathbf{1}$ is the $S \times 1$ unitary vector.

The proof is in the appendix. It is interesting that similar to Merton (1976), in the presence of destructive shocks we can think of the discount rate increasing from r to $r + (1-\alpha)\kappa$ for valuing assets.

Each firm maintains a static capital structure that consists of equity and a zero-coupon bond with face value D_t^i. We follow Black and Cox (1976) and David (2008) and

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[9]Longstaff and Rajan (2008) provide some evidence that defaults of firms are correlated.

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assume that the asset value of the firm is a traded security, and ¡the face value of debt

$$D_t^i = D_0^t \exp \left[\int_{h=0}^t r_h - \frac{1}{v_h^\alpha} dh \right]. \qquad (16)$$

These assumptions ensure that under the risk-neutral measure, there is no trend in the distance between the firm value and the face value of outstanding debt. The following lemma shows that for each type of firm, the solvency ratio under the risk-neutral measure has zero drift (is a local martingale).

Lemma 2 *For a firm whose quality is uncertain, and the investor has a probability that the firm is good is α, the solvency ratio under the risk-neutral measure follows:*

$$\frac{dZ_t^{i,\alpha}}{Zt_-^{i,\alpha}} = \sigma_X \left(\sqrt{\rho_i} \, dW_{2t}^* + \sqrt{1 - \rho_i} \, dW_t^i \right)$$

$$+ \sum_{s_t \neq s_{t-}} \left(\frac{v^\alpha(s_t)}{v^\alpha(s_{t-})} - 1 \right) dM_t^{*(s_{t-}, s_t)} - (1-\alpha) dL^{i,s}, \quad (17)$$

where $dW_t^* = dW_t - \sigma_m \, dt$, and $dM_t^{*(s_{t-}, s_t)}$ is in (9).

As seen, the solvency ratios are subject to three types of shocks: (a) the continuous Brownian shocks to aggregate cash flow growth and idiosyncratic cash flow growth; (b) macroeconomic regime changes that cause jumps in the asset valuations; and (c) destructive shocks for bad firms.

Similarly, under the objective measure the solvency ratio of a firm of unknown quality follows:

$$\frac{dZ_t^{i,\alpha}}{Zt_-^{i,\alpha}} = \sigma_m \sqrt{\rho_i} \sigma_X + \sigma_X \left(\sqrt{\rho_i} \, dW_{2t} + \sqrt{1 - \rho_i} \, dW_t^i \right)$$

$$+ \sum_{s_t \neq s_{t-}} \left(\frac{v^\alpha(s_t)}{v^\alpha(s_{t-})} - 1 \right) dM_t^{(s_{t-}, s_t)} - (1-\alpha) dL^{i,s}. \quad (18)$$

We adopt the assumption by Black and Cox (1976), and more recently, Seo and Wachter (2018), that a firm defaults at time τ at the first time the market value of the firm's asset hits the face value of its debt, i.e., its solvency ratio either hits or falls below one:

$$\tau_t^i = \inf \left\{ \tau > t : Z_t^{i,\alpha} \leq 1 \right\}. \qquad (19)$$

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In a continuous setting, Black and Cox (1976) justify this assumption as a safety covenant for bondholders. Notice though, that due to the jumps in asset valuations and cash flow, the solvency ratio can fall below 1. Using (18), for any finite time horizon, T, we can calculate the default probability of a typical firm within a pool of firms with a fraction α of good firms by simulation as

$$\Pi^D(s_t, \alpha, Z_t, T) = \text{Prob}(\tau_t^\alpha \le t + T | s_t, \alpha, Z_t^\alpha = Z_t). \tag{20}$$

Since good and bad firms' cash flows are identical, except for the possibility of the destructive jump shocks to the latter, ceteris paribus, the default probability decreases in α. It is also useful to note that the default probability of the typical firm does not depend on either the continuous correlation, ρ, or the jump correlation, φ.

At the default time τ, the debt holders recover a fraction Φ of the unlevered firm value, while the complementary fraction $1 - \Phi$ is lost due to the deadweight costs of bankruptcy.

2.3   The Credit Rating Agency (CRA)

In this section, we build a model of a CRA based on the principle of Bayesian persuasion introduced to the literature in Kamenica and Gentzkow (2011). Bayesian persuasion is a game of the choice of information precision, where an information sender optimally controls the precision of a signal to induce a particular action by an information receiver. In our case, the information sender is the CRA, and the information receiver is the representative consumer/investor in the economy.

We assume that firms are either good or bad types as defined in Section 2.2. Firm types are unobservable, but by conducting research on them, the CRA can partially determine the type. The CRA then issues a rating of either G or B for each firm, which is a signal of the firm's quality. The investor has prior beliefs about the quality of each firm, and after observing the rating of each firm, as well as the precision of the ratings, the investor updates her probability of each firm's type conditional on its rating using Bayes' rule. We assume that the consumer has a preference for investing in G-rated securities and that the CRA chooses the intensity of its research to maximize the proportion of firms with G ratings, subject to a default probability constraint of the G-rating class to be discussed below. As in Lizzeri (1999) and

Kartasheva and Yilmaz (2012), we assume that the CRA commits to this precision structure of ratings.

More precisely, we assume that the proportion of good firms in the population of firms, α_0, is constant over time, and known by both the CRA and investors. Following, Kamenica and Gentzkow (2011), we assume that by adjusting its research prodedure, the CRA can costlessly control the type I and type II rating errors, i.e., $\pi_t^I(Z_t, s_t) = \mathbb{P}[B|\text{good}; Z_t, s_t]$ and $\pi_t^{II}(Z_t, s_t) = \mathbb{P}[G|\text{bad}; Z_t, s_t]$; these errors are chosen conditional on the solvency ratio of firms, Z_t, and the state of the economy, which affects the jump risk parameters of bad firms.[10] We assume that persuasion at date t determines the cross-sectional distribution of investment-grade firms that are used in CDO pools created at time t. The composition of these pools then remains fixed over the tenor of the CDOs, although some firms are lost from the pools as they default. By choosing $\pi^I(Z_t, s_t)$ and $\pi^{II}(Z_t, s_t)$, the CRA maximizes

$$\mathbb{P}_t[G] = \alpha_0 \left(1 - \pi^I(Z_t, s_t)\right) + (1 - \alpha_0) \pi^{II}(Z_t, s_t), \tag{21}$$

which is the probability of assigning a G-rating to each firm subject to the constraint

$$\Pi^D(s_t, \alpha_1(t), Z_t, T) \leq \Pi^G, \tag{22}$$

i.e. the default probability at the T-period horizon has to be below a fixed number Π^G, where $\Pi^D(s_t, \alpha_1(t), Z_t, T)$ is computed using (20), and $\alpha_1(t) = \mathbb{P}(g|G)$ is the posterior probability of a G-rated firm being good. Bayes' law implies that after observing the ratings assigned to each firm, the investor's posterior probability is

$$\alpha_1(t) = \mathbb{P}[\text{good}|G] = \frac{\alpha_0 \left(1 - \pi^I\right)}{\alpha_0 \left(1 - \pi^I\right) + (1 - \alpha_0) \pi^{II}}. \tag{23}$$

Credit rating agencies do not clearly define the meaning of a particular rating. They do publish estimates of default probabilities of each rating category, which actually show significant time variation. A good working definition of a rating is that it is a measure of the "through-the-cycle" default probability of a firm (see e.g. Carey and Treacy (1998) and David (2008)). Altman and Rijken (2004) provide evidence that the CRAs base their ratings

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[10]Later in this section, we add a constraints on the ability of the CRA to influence investors' posterior belief of firms' quality.

on longer-term (5 years) default probabilities as opposed to 1-year default probabilities. We use this interpretation of the rating, and in particular assume that the CRA targets a rating to have a 5-year default probability below a critical threshold. This definition, along with the time variation in firms' fundamentals, gives the rating agency some flexibility in changing the composition of firms in any rating category. The CRA attempts to issue as many G-ratings as possible while ensuring that the average probability of default of firms in that rating category does not exceed the rating's threshold. In our empirical section, we assume that the CRA attempts to ensure that the 5-year default probability of investment-grade firms is below 2.8 percent, which is the historical average of the default probability of Baa3 (the lowest investment-grade rating) firms.

A couple of points are worth nothing: First, $\alpha_1(t)$ will be higher, for a lower type-I error, as fewer good companies are given B ratings, and hence the proportion of good companies among those rated G is higher. Second, $\alpha_1(t)$ will be higher for a lower type-II error, as fewer bad companies are given G ratings, once again increasing the quality of the G-rated pool. Therefore, α_1 can be increased by lowering both type of errors.

The intuition above helps understand the optimal rating process by the CRA. We first notice in the objective function in (21), that the number of G ratings is higher, for a lower type-I error. So, the CRA should lower it as much as possible. Lemma 3 below shows that the CRA will optimally set $\pi^I = 0$. The optimal type-II error, however, is not equal to zero. As seen in (21), the number of G ratings increases in the type-II error (the probability of assigning G ratings to bad firms). The CRA plays a balancing act between increasing π^{II}, and increasing α_1. Why would the CRA want to increase α_1? It might, if given the solvency ratio Z_t of firms in the economy, and the jump risks in the current state, its default probability at the prior belief, α_0, is above its committed level. As discussed below (20), the default probability is decreasing in α, so the CRA must increase α_1 until this default probability constraint is met.

At this point, we make the further assumption that $\alpha_1(t) < \bar{\alpha} < 1$, i.e. the CRA is unable to influence investor's belief of the quality of G-rated firms beyond an upper bound. This could arise from technological limitations of the CRA being able to perfectly learn firms' type with their research. We now provide an algorithm that the CRA can use to determine the optimal type-II error and hence induce the posterior belief $\alpha_1(t)$.

1. Given the prior belief, α_0 of investors, and the current solvency ratio of firms, Z_t and the state s_t if $\Pi^D(s_t, \alpha_0, Z_t, T) \leq \Pi^G$, then all firms are given the G rating, i.e. $\pi^{II} = 1$. In such periods, the default probability of the G-rated category is below its target, and $\alpha_1(t) = \alpha_0$.

2. If $\Pi^D(s_t, \bar{\alpha}_1, Z_t, T) > \Pi^G$, then the CRA does its best possible. It improves investors' posterior to its maximum possible, i.e. $\alpha_1(t) = \bar{\alpha}$.

3. If neither of the above conditions hold, then by continuity of Bayes' law, there is an $\alpha_0 < \alpha_1(t) < \bar{\alpha}$, such that $\Pi^D(s_t, \alpha_1(t), Z_t, T) = \Pi^G$. In this case, π^{II} can be backed out from (23) after setting $\pi^I = 0$. In this case, the default probability of the G-rated category is exactly at its target.

2.4 CDX Tranche Pricing

We use our model to price tranches based on the Dow Jones CDX North American Investment Grade Index (CDX.NA.IG). The underlying portfolio is an equally weighted basket of $N = 125$ investment-grade single-name credit default swaps (CDS) (see e.g Duffie and Singleton (2003) for definitions of CDS). CDX tranches are derivative contracts written on the CDX, which provide the protection buyer with different layers of protection on default events of the underlying firms. Following prior research, we determine implied payments (made as a combination of quarterly spreads with fixed and variable components, or as fixed spreads with upfront payments) on the index as well as its tranches from the no-arbitrage condition that discounted cash flows to the protection buyer (the "protection leg") and protection seller (the "premium leg") be equal in value under the risk-neutral measure (see e.g. Collin-Dufresne, Goldstein, and Yang (2012) and Seo and Wachter (2018)).

Each tranche j is defined by its lower and upper attachment points, which we denote as A_L^j, and A_U^j, respectively. The lower attachment points refers to the level of subordination of the tranche, and the upper attachment point to the level at which the losses in the pool wipe out the entire tranche notional. For example, a tranche with attachment points 3-7 (henceforth, following market parlance, we will state the attachment points in percentage points) starts losing value only if the loss on CDX exceeds 3%. If the loss on the full portfolio exceeds

7%, the notional amount of the tranche is completely exhausted. Note that the entire CDX index can be considered as a tranche with the lower attachment point equal to 0 and the upper attachment point equal to 100.

We let $L_{t,s}$ denote the cumulative fractional loss at time s on the CDX originated at time t given by

$$L_{t,s} = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}_{\{t<\tau_i\leq s\}} \left(1 - R_{i,\tau_i}^*\right),$$

where $R_{i,\tau}^*$ is the recovery rate for a firm i defaulting at τ_i. The fractional loss on tranche j with attachment points A_L^j and A_H^j is given by

$$L_{t,s}^j = \frac{\min\left(L_{t,s}, A_H^j\right) - \min\left(L_{t,s}, A_L^j\right)}{A_H^j - A_L^j}.$$

When the cumulative loss increases, the protection seller pays the protection buyer the amount of the loss. The no-arbitrage value of the protection leg for tranche j (including the whole CDX index) is given by

$$\text{Prot}_{j,t} = E_t^Q \left[\int_t^T \exp\left(-\int_t^s r_u du\right) dL_{t,s}^j \right]. \tag{24}$$

The protection buyer of each tranche makes quarterly premium payments to the protection seller. The amount of the payment depends on the tranche notional. By convention, in case of a default, tranche notionals are adjusted by loss net recovery starting from the most junior tranche, and recovery starting from the most senior tranche. For example, the first default reduces the notional of the most junior tranche, 0-3, by $\frac{1}{N}(1 - R_{i,\tau_i}^*)$, and reduces the notional of the most senior tranche, 15-100, by $\frac{1}{N}R_{i,\tau_i}^*$. The recovery on tranche j is

$$R_{t,s}^j = \frac{\min\left(R_{t,s}, 1 - A_L^j\right) - \min\left(R_{t,s}, 1 - A_H^j\right)}{A_H^j - A_L^j},$$

where $R_{t,s}$ is the cumulative fractional recovery on the CDX given by

$$R_{t,s} = \frac{1}{N} \sum \mathbf{1}_{\{t<\tau_i\leq s\}} R_{i,\tau_i}^*.$$

The premium leg for tranche j is given by

$$\text{Prem}_{j,t} = U + S\,E^Q \left[0.25 \sum_{k=1}^{4T} \exp\left(-\int_t^{t+0.25k} r_u du\right) \int_{t+0.25(k-1)}^{t+0.25k} \left(1 - L_{t,s}^j - R_{t,s}^j\right) ds \right], \tag{25}$$

where S is the running spread, which is a quarterly payment from the insurance buyer to the insurance seller, and U is a fixed upfront payment that the seller pays to the buyer. The expectations for the protection and premium legs are each calculated using Monte Carlo simulations of firms' asset value processes in (17), the default assumption in (19), and the mix of good and bad firms in the G-rating category by the CRA as described in Section 2.3.

As we will see, the convention for market quotes has changed over time, and is different for different tranches. Until about 2009 (the switching date varies by tranche), all tranches were quoted as the running spread defined here, except the 0-3 tranche, which was quoted as an upfront, with a fixed spread $\bar{S}$. Therefore $S(A_L, A_U)$ is implied using $U(A_L, A_U) = 0$ and equating the values of the protection and premium legs for all but the 0-3 tranche. For the (0,3) tranche, $S = \bar{S} = 500$ b.p and the value of $U(0, 3)$ is adjusted so that the two leg values are equated. Note that the implied upfront can be negative. Subsequently (post 2009), the quotes have evolved to fixed spreads and upfronts are calculated as above for all the tranches. However, the fixed spreads have not been constant over time, making intertemporal comparisons of upfronts difficult.

Due to changes in quotation specifications, we are unable to structurally estimate our model's parameters using the raw data. In particular, to identify all parameters, we need second moments (volatilities and correlations) of spreads, but some of the variations in the spreads are spurious, due to changes in quotation specifications. To create homogeneous series, we construct "zero upfront variable (ZUV) spreads", which are spreads that are implied when upfronts and fixed spreads are set to zero. We proceed in two stages: first, we estimate the parameters of our model using these ZUV spreads, then, we convert them back to either

upfronts or running spreads using the data quotation specifications.[11] These are then compared
to the unadjusted raw data.

As we describe in Appendix 2, we attempt to create time-series of spreads with 5-
year tenors; however, since some of the series do not trade at some dates, traded tranches at
different dates have different ages. For example, we use series 9 from 2007 Q4 to 2010 Q3
because it was the most liquid series in that period. For this reason, CDX pools have different
remaining tenor at different dates in our sample. When we calculate spreads on a "seasoned"
pool at date t of age a quarters, then it has remaining tenor of $(20 - a)$ quarters. To calculate
the expectations in the protection and premium legs we must simulate prior macroeconomic
states conditional on the current state for which we the smoothing formula using the quarterly
backward transition matrix $P^Q[s_{t-1} = i | s_t = j] = \lambda_{ij}^Q / \sum_k \lambda_{kj}^Q$ (see Hamilton (1994)). With
the 'smoothed' probabilities of prior states, we calculate the expected tranche values over
pools whose optimal mix of good firms is determined at $t - a$ in each state as described in
Section 2.3.

3 Empirical Analysis

In this section, we structurally estimate our model and evaluate its implications for the pricing
of CDO tranches. Our empirical estimation is implemented in two stages. At the first stage,
we use standard maximum likelihood of regime switching models (see Hamilton (1994)) to

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[11]The expectation (rate), $E^Q \left[0.25 \sum_{k=1}^{4T} \exp\left(- \int_t^{t+0.25k} r_u du \right) \int_{t+0.25(k-1)}^{t+0.25k} \left(1 - L_{t,s}^j - R_{t,s}^j \right) ds \right]$ in equa-
tion (25) is referred to in industry parlance as the "RPV". The RPV is not observable, as it is not a rate on
traded security. The RPV is needed to covert quarterly spread payments to upfronts, and vice versa, and is
a combination of riskless rates and portfolio losses (and recoveries). In the existing literature, authors have
calculated it using specific assumptions, such as a CAPM-based pricing kernel, conditional log-normal port-
folio distributions, and linearly declining notionals over time (e.g. Coval, Jurek, and Stafford (2009a)). We
calculate it using our model with regime switches in states and notional declines that are state-dependent. In
addition, our assumption has non-zero covariance between riskless rates and losses, while this covariance is
often assumed to be zero. We plot ZUV spreads for the (0,3) tranche for the period between 2004:Q3 and
2007:Q3 approximated by our model, and by Coval, Jurek, and Stafford (2009a) in Figure 12 in the online
appendix. As can be seen, that albeit some differences at some dates, the two approximations are highly
correlated.

estimate the cycles in credit availability and macroeconomic growth. The sample for the first stage is 1952 to 2017. We assume that the regimes are observed by the agents in the model, but are unobserved by the econometrician. The estimation provides us with the beliefs of the econometrician about the fundamental states over time. In the second stage, using the beliefs and estimated parameters in the first stage, we use the simulated method of moments (SMM) to estimate the parameters of firms' projects that fit tranche spreads for the sample from 2004:Q3 to 2017. In particular, the spreads are calculated in each state, and then averaged using the estimated beliefs.

3.1 First Stage Maximum Likelihood Estimation of Regime Switching Model

The first step in estimating a regime switching model is the choice of the number of regimes (we use the term 'regime' and 'state' interchangeably). As is well known, formal tests on the number of regimes are difficult and lack power (see e.g. discussion in Garcia (1998) and Hamilton (2008)). We follow the practical approach taken by Gray (1996) and Bansal and Zhou (2002) and make use of the GMM-based χ^2 criterion to determine the number of regimes for each of the fundamental variables, using the scores of the likelihood function as moments. For each of the fundamental variables, consumption, and the ratio of credit growth to GDP, we find that the two regime specification is rejected, while the three regime specification is not rejected. For each variable there are normal high and low states, as well as a disaster state. We therefore model 'disasters' as states of extreme growth (as in Rietz (1988)) rather than jumps (as in say Seo and Wachter (2018)). We further assume that the regimes of earnings growth at firms are perfectly correlated with the regimes of consumption growth.

We then estimate a specification with *composite* regimes of the two fundamentals. Unconstrained, we would have nine regimes, however, in our estimation, we find that three of these regimes have zero probability of occurring in our sample, so we only use six regimes. This procedure of estimation of composite regimes is developed in David and Veronesi (2013).

The states are ordered as (High Credit-High Growth), (High Credit-Low Growth), (Low Credit-High Growth), (Low Credit-Low Growth), (Low Credit-Disaster Growth), and (Disaster Credit-High Growth). It is also worth noting that since the probabilities of the disaster states are very low, we make the simplifying assumption that the probability of entering disaster states from any state are equal, and the probability of transitioning to any state from a disaster state are equal as well, leading to a smaller number of parameters to be estimated. The parameter estimates are shown in Table 3, and it is worth noting that the annualized disaster states of the credit growth to GDP ratio, consumption growth, and earnings growth, are -4 %, 0 %, and -88%, respectively.[12] Also, as noted earlier, the state of credit growth affects the transition from high to low growth states: with High Credit, the transition probability to Low Growth is in a quarter is 4.4 percent, however, in the Low Credit state, the transition probability increases to 11.1 percent. Therefore, low credit signals growth instability.

In Figure 2, we look at the econometrician's probability of the six states, which are observed by the consumer/investor in the economy. The first four states are regular states, and there are frequent episodes of filtered probabilities of these states being quite high. The probabilities of states 2 and 4, increased in most recessions since 1952. Also, importantly, the probabilities of these low growth states increased substantially in 2012 and 2016, even though these two periods were not classified as economic recessions by the NBER. As we will see, the increase in the likelihoods of these low growth states contributed to higher spreads in these periods.

The historical and expected fundamentals from the regime switching model are shown in Figure 3. The model explains 64% of the variation in credit growth, and 18% and 40% of the variation in consumption growth and aggregate earnings growth, respectively. In addition, during the financial crisis, it fitted the low expected growths well.

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[12]While it may seem that 0% consumption growth is hardly disastrous, as seen in Figure 3, at the worst point of the great recession, consumption growth was about -3.2 percent (at an annual rate) in 2008:Q4, and in other quarters was larger than that. The lowest consumption growth in our entire sample from 1952 to 2017 is about -10 percent at an annual rate in 1980:2, but in the following two quarters it exceeded 4 percent at an annual rate.

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3.2 Second Stage SMM Estimation For Model With CRA

In the second stage of our estimation, we take the filtered probabilities of the states from the first stage and use data on ZUV spreads to estimate the parameters of firms' cash flows using the Simulated Method of Moments (SMM). We use data on reported spreads on four tranches with attachment points 0-3, 3-7, 7-15 and 15-100, and the whole CDX index 0-100.[13] We have daily spread data and create time series at a quarterly frequency by taking the averages of the daily spreads in each quarter. Overall, we have 5 time-series of spreads, and we fit these conditionally. In addition, we use the unconditional volatilities of the spreads, and their correlations in our SMM procedure. In addition, we use the unconditional 5-year probability of default of investment-grade firms as an additional moment, overall, giving us 21 moments to fit.

For fitting these moments, we estimate α_0, the proportion of good firms in the population of all firms; $\bar{\alpha}$, the highest possible belief that the CRA can induce; σ_X, the volatility of firms' cash flows; ρ, the correlation of the continuous component of the cash flows; Φ, the proportional bankruptcy cost parameter; and, φ, the correlation of destructive shocks. For fitting CDO tranche spreads, we find that we need time-variation in the intensities of destructive shocks as defined in (12) – (14). We assume that the jump intensities of shocks are state-dependent, which provide 6 parameters to estimate. Overall, we have 21 moments, and 12 parameters to estimate, and hence our SMM procedure has 9 degrees of freedom.

The estimated parameters from the second stage are in Table 4. We estimate that only about 26 percent of the firms in the full sample are of good quality, which is the representative investor's prior about firm quality. The CRA's rating process is quite informative for investors as we estimate that it can increase the investor's posterior of good quality firms in G-rated pools to about 55 percent. We estimate the bankruptcy cost parameter is about 29 percent, which is quite similar to the estimates of a number of papers, and likely includes indirect costs

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[13]As described in Appendix 2, the attachment points are slightly different for two of the tranches prior to 2010. When we estimate our model, we change the attachment points in our model to match those in the data; in particular, prior to 2010, we use the 7-10 attachment points instead of 7-15, and the 15-30 attachment points rather than 15-100. The differences between the last two would be very small if portfolio losses in excess of 30 percent are highly unlikely, as is true for our estimated model.

of bankruptcy. The average volatility of firms' cash flow of 21 percent, with a systematic component, measured as the correlation of the continuous component, of 76 percent. It is useful to note that we model unsystematic destructive shocks to firms' cash flows, so that the the overall systematic component is lower. Finally, the correlation of destructive shocks to bad firms is about 39 percent.

The estimated jump intensities of bad firms' destructive shocks in the 6 states have intuitive patterns: the risk of destructive shocks is decreasing in the level of credit availability as well as on economic growth. In periods of high credit and high growth, the classic boom periods, the risk of disastrous shocks is is below 1 percent, at its lowest among states. In periods of low credit, the risk of such shocks increases to a medium level of about 4 percent in high growth states, but to a high 10 percent in low growth states. In each of the pathological states, disaster credit (state 5) or disaster growth (state 6), the jump intensities are above 15 percent. We will discuss the implications for alternative views on the relationship between growth and credit cycles separately in Section 4 below.

In Figure 4, we plot the data and model ZUV spreads on the tranches, and measures of the goodness of fit are in Table 5. Our model fits are good in several dimensions. In particular, the 0-3 (equity), 3-7, and 7-15, and the 0-100 (CDX) are priced quite closely before, during, and after the financial crisis. More formally, as seen in the top panel of Table 5, the model spreads explain between 72 and 83 percent of the variation in the data spreads, the α coefficients are all statistically not different from zero, and the β coefficients are all not different from 1. The model spread for the (3,7) tranche does overestimate the data spread either side of the financial crisis, but successful replicates other fluctuations in other periods. The model has the hardest time in explaining the 15-100 (senior tranche) spread variations. In particular, consistent with Coval, Jurek, and Stafford (2009b), the spread was too low by a factor of 3 or more prior the financial crisis. After the crisis, our model spread follows the data quite closely. The low model (15,100) spreads before the crisis occur despite our modeling time varying credit rating standards, but the model does correctly price all other tranches in this period.

The remaining panels of Figure 4 display important components of the model, which we discuss here. The middle-right panel shows the investor's prior and posterior probabilities

of firms in the G-rating category being good. As seen, until about 2007, the posterior was close to the prior. As explained at the end of Section 2.3, this occurs when fundamentals are strong to the extent that at the investor's prior belief, the default probability of the G-rating class is below its target. In this case, the CRAs issues highly uninformative ratings, with type-II error of close to 1, and hence, does not influence the investor's beliefs. In the midst of the financial crisis, the posterior hit its maximum of around 0.55. This occurs when at the maximum level of persuasion, fundamentals are bad enough that the default probability is above the CRA's target. In this case, the CRA does its best to give bad firms B ratings and hence improve investor's posterior beliefs about the quality of the G-rating class. Quite strikingly, the posterior remained close to its maximum level until about 2010, even though the NBER-dated recession ended in the second quarter of 2009. The next two panels shed further light on this issue. The bottom-left panel shows the 5-year default probability of the G-rated set of firms peaked in the midst of the financial crisis at over 7 percent, but remained elevated substantially above its target level until 2010. Therefore, the model implies that the CRA would have to keep the ratings informative for this period, even though fundamentals improved. The bottom-middle panel shows the average log-solvency ratio of investment-grade firms, and it is substantially negatively correlated with the default probability. However, it does not fully explain the default probability and posterior dynamics, as the state of economic and credit growth, which affect the intensity of destructive shocks to firms, also impact the default probability and the amount of persuasion by the CRA. Indeed, partly, the default probability remained elevated as credit growth remained weak until 2010 (see Figures 2 and 3).

Besides, the financial crisis period and its aftermath, the model also explains a large amount of the variation in spreads in the period between 2010 and 2017. In particular, there were two bouts of weak growth, which were not characterized by the NBER as recessions, in 2011-12, and again in 2016-17 (see Figures 2 and 3) when CDO spreads rose substantially. In both cases, credit remained strong initially, but later weakened. The growth declines were nowhere near of the magnitude of previous two recessions, and according to our model, investors would believe that these periods had weak growth of the kind modelled in states 2 and 4, rather that the disaster states. In Figure 4 we see that the model's spreads for all the tranches were close to that in the data in both episodes. Note, that senior tranche spreads

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are elevated when investors fear that there are potential losses for a substantial proportion for firms, and hence our model predicts that such systemic losses in these periods were unlikely with the moderately weak growth during these periods. The model does imply that the CRA would have to improve the informative of its ratings as seen by the higher posterior (Figure 4 middle-right panel) as the default probability of the G-rating class rose above its target level in these episodes (bottom-left panel), again not to the extent of the previous two recessions.

The middle and bottom panels of Table 5 show the other ability of our model to fit the other moments in the SMM procedure. The default probability in the model is close to its target, as are the volatilities of the spreads. The bottom panel shows that the model-correlation of ZUV spreads are also very close to the data.

Overall, as noted, the model does well in explaining fluctuations in all tranche spreads, except the senior (15,100) spread in the years before the financial crisis. Using the chi-squared statistic from the objective value of the SMM procedure for the full sample, we see that the p-value for model is only 2 percent. However, if we exclude the errors of the (15,100) spread from 2014:4 to 2017, the p-value rises above 10 percent. An important caveat is that, in our model specification here, we have used a constant correlation of firms' destructive shocks. In an earlier version of this paper, we assume that the correlation of these shocks increases proportionately in the amount of persuasion (posterior minus prior probability of firm being a good type). With a time-varying correlation, the model prices the (15,100 tranche well prior the financial crisis, and fits in all other periods are better as well. The sensitivity of the senior tranche price to the correlation has been well established in the literature (see e.g. Duffie and Garleanu (2001) and Gibson (2005)). However, practitioners have complained that that default correlation is very hard to measure since defaults are rare events, but changing it freely can justify any pattern of observed spreads (see, e.g. Mackenzie and Spears (2014)).

3.3   Ability of The Model To Explain the Raw Data

As discussed in Appendix 2, the contractual features of the different tranches have changed substantially over time. To estimate our model, we convert the raw data into ZUV spreads. Here, using the estimated structural parameters, we study the ability of our model with the

CRA to explain the raw data. We show the raw data and our model fits of it in Figure 5, and provide goodness-of-fit statistics in Table 6.

The left panels of the figure show the fixed spread component of the tranche spreads over time, the middle panels show the variable spreads, and the right panels show the upfront payments. For example, for the 0-3 spread, the fixed spread was 500 b.p. until 2015, and 100 b.p. thereafter. This means, that the raw data for the 0-3 tranche was quoted as an upfront payment, as the quarterly payment was fixed. For the 3-7 tranche, the fixed spread was 0 until the first quarter of 2009, it was then 500 b.p. until 2010, and 100 b.p. thereafter. Therefore, until 2009, the raw data was quoted as a running spread, and thereafter, it is quoted as an upfront, with the qualification, that the quarterly fixed payment changed after 2010. Therefore, when we look at the time series of upfronts in the right panel, we must keep in mind that some of the variation in the quoted upfront arises due to the change in the contracted fixed spread. For the (0,100) spread, the fixed component was zero for the full sample, and the raw date was quoted as a variable spread.

In the middle and right panels, along with the raw data, we also display the model's spreads/upfronts in the raw data format using the conversion process in Section 2.4. As seen the fits of the variable spreads are really close to the data (with the exception of the 15-100 spread before the financial crisis), with R^2s of the model fits of between 76 and 96 percent. The right panels show that the fits of the upfronts are not as good (in the 33 - 63 percent range) with the model still capturing most major fluctuations in the data. It must be noted that some of the fluctuation in the upfronts are due to changes in the fixed spread components, which are exogenous to our model.

3.4   The Credit Spreads Puzzle

For several years it was widely acknowledged that it was hard to reconcile relatively low historical low default probabilities along with relatively high credit spreads using structural-form credit risk models with alternative assumptions (see e.g. Huang and Huang (2012)). This was called the "Credit Spreads Puzzle". In the late 2000s, several papers were able to reconcile this issue. The key intuition that developed was that spreads measure expected

default losses under the risk-neutral measure, while historical losses are under the observed measure, so that high prices of risk for bearing macroeconomic shocks along with the greater incidence of default during periods of low consumption growth, would be consistent with the facts. In particular, Bhamra, Kuehn, and Strebulaev (2009) and Chen (2010) use long-run risk models, which imply high prices of risk for consumption shocks.[14] In this paper, we build on this literature by studying spreads on CDO tranches in addition to the standard spread (CDX spread), and show that the high level of spreads on all the tranches can be justified, while maintaining default probabilities at empirically observed levels.

Our model has both long-run risk and disasters, each of which raises the price of risk of consumption shocks. We use standard parameters for the elasticity-of-intertemporal substitution (1.1) and risk-aversion (10), as in these above papers. In addition to the insights of these above papers, for fitting tranche spreads, we need additional assumptions on jump risk intensities of bad firms and their correlations, which have different impacts on junior and senior spreads. Besides fitting the level of spreads for the CDX and the tranches, our structural-form estimation also helps understand the dynamics of spreads and the underlying risks of firms' cash flows. As we build in the Sections 4, our structural estimation shows the importance of building in changes in shocks to credit as well as credit rating standards in understanding the dynamics of spreads.

4 Understanding the Relations Between Economic Growth, Credit Growth, and Credit Rating Standards

In the introduction, we noted that the senior tranche spread is relatively more exposed to economic growth shocks, while the equity tranche spread is relatively more exposed to credit

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[14]Chen, Collin-Dufresne, and Goldstein (2009) show that firms' default boundaries increase in bad times, and hence the increased likelihood of defaults in bad times increases the credit spread. David (2008) showed that a substantial proportion of the spread arises from a convexity effect, which arises as credit spreads are non-linear (convex) functions of relevant firm's fundamentals. Bhamra, Kuehn, and Strebulaev (2009) also use this convexity effect.

growth shocks. In particular, the equity tranche spread remains elevated after the end of recessions, until credit growth recovers. In this section, we will shed some light on these relationships, as well as the role of changes in credit rating standards in determining the different exposures.

4.1 Economic Growth and Credit Growth Cycles

As a first step, we examine the relationship between economic and credit growth. We estimate a standard Vector Auto Regression (VAR) between the ratio of credit growth-to-GDP and consumption growth for quarterly data from 1952 to 2017. The generalized impulses from the estimated parameters are shown in Figure 6. The top panel shows that credit growth increases as a response to economic growth, and the relation is statistically significant up to 10 quarters. The bottom panel shows that the response of economic growth to credit shocks is insignficantly different from zero, that is, credit growth does not appear to cause economic growth. We confirm these relationships with Granger causality tests: the p-value for the null hypothesis that credit growth does not cause economic growth is 0.85, while that of the reverse hypothesis that economic growth does not cause credit growth is smaller that 10^{-4}.

These relationships cast doubt on the narrative that excessive credit causes crisis and weak economic growth, but are supportive of the view, that the financial sector curtails credit after a negative growth shock. In fact, our 2nd stage estimation results in Table 4 show that the jump intensities of bad firms' destructive shocks are lower during periods of high growth and high credit. Our results are consistent with the recent evidence on the reverse causality from economic growth to credit markets in Boons, Ottonello, and Valkanov (2022).

4.2 Why Does Junior Tranche Correlate More with Credit Growth and Senior Tranche More with Economic Growth

As we see in the bottom-middle panel of Figure 4, the average solvency ratio of investment grade firms recovered to a fairly strong level at the end of the recession in 2009. The solvency ratio affects all firms and is the largest component of correlation for firms' default in our structural credit risk framework. However, credit growth remained extremely weak until 2010,

and our structurally estimated parameters show that the jump intensities of destructive shocks are very high (0.16) remain high in periods of disastrous credit growth. Due to the resulting relatively high default rate, our model implies that credit rating standards remain strong in such periods, so that there is a smaller proportion of bad firms in investment-grade pools initiated. Overall, with better solvency ratios, and a lower proportion of firms with potentially destructive shocks, losses are able to affect the junior tranche, but not the senior tranche, which only experiences losses if more than 15 percent of the capital is destroyed. Supporting this view, the solvency ratio is 80 percent correlated with the senior spread, but only 60 percent with the equity spread. More generally, the solvency ratio is highly correlated with economic growth, and hence the senior spread is relatively more exposed to economic growth shocks.

5 Credit Rating Standards

Following Blume, Lim, and Mackinlay (1998) and Alp (2013), we use an ordered probit model of firms credit ratings to measure time variation in credit rating standards. We obtain long-term issuer credit ratings from Standard and Poor's Capital IQ database. We assign successive integer ratings from 10 to 17 for investment-grade bonds rated BBB, A, $\cdots$, AAA.

In particular we use the above authors' specification:

$$R_{it} = 17 \qquad\qquad \text{if} \quad Z_{it} \in [\mu_{16}, \infty], \qquad (26)$$

$$R_{it} = 16 \qquad\qquad \text{if} \quad Z_{it} \in [\mu_{15}, \mu_{16}], \qquad (27)$$

$$.. \qquad\qquad (28)$$

$$R_{it} = 10 \qquad\qquad \text{if} \quad Z_{it} \in [\mu_9, \mu_{10}], \qquad (29)$$

in which, $Z_{it} = \alpha_t + \beta' \cdot X_{it} + \epsilon_{it}$, is the latent variable that affects the rating with partition points μ_i, the independent variables, X_{it}, are firm level indicators of credit quality such as leverage ratios, and $E[\epsilon_{it}|X_{it}] = 0$. The intercept, α_t, is a measure of *credit rating standards* at time t. While Alp (2013) uses annual intercepts, we use quarterly ones to compare these credit standards with those implied by our CDO pricing model. Also, notably, while Alp (2013) fits her model until 2007, our sample ends in the third quarter of 2016. We use almost the same list of independent variables as Alp (2013), whose exact definitions of these variables are in

Table 7. In addition to the variables in her paper, we add firm's exposure to the momentum and HML factors. We do not include the SMB coefficient since firms' size is one of the explanatory variables.

The parameter estimates of the betas in the the ordered probit regression are in Table 8, while the time-varying intercepts are plotted in Figure 8. We first make some brief comments on the beta estimates. All our coefficients have the same sign as in Alp (2013). In particular, higher operating margins, dividend payouts, R&D expenses, tangibility, firm size, and retained earnings lead to higher ratings, while higher leverage ratios and capital expenditures, lead to lower ratings. In addition, all the measures of firms' risk – idiosyncratic volatility, and loadings on market, HML, and momentum lead to lower ratings. Finally, firm's book-to-market ratio and their cash-to-asset ratios do not have significant affects on their credit ratings. The insignificance on cash is consistent with the evidence in Acharya, Davydenko, and Strebulaev (2012). For our fitted ratings, we excluded the insignificant variables. The overall specification for the ratings is highly significant with a p-value smaller than 10^{-5}, and its pseudo-R^2 is about 0.104.

The quarterly time series of the intercept from 1985 to 2016 is shown in the top panel of Figure 8. A higher intercept means lower credit standards (higher unexplained ratings). Our estimated series has very similar properties to the annual series in Alp (2013), with investment-grade ratings generally getting more stringent until about 2006 (with an exception of a deterioration in standards between 1996 and 2001). Rating standards deteriorated quite sharply from 2006 to 2008, and then tightened sharply after the financial crisis until 2011. Standards then moderated, however, there was one additional bout of tightening standards in 2014 – albeit far less than that in the financial crisis.

We compare the credit rating standards from the ordered probit model, and from our CDO pricing model. In our model, a measure of rating standards is the type-II error (probability of bad firms getting G ratings) as developed in Section 2.3. We extract our model's implied π_t^{II} from (23), where we put $\pi_t^I = 0$ at each date. Using the optimal $\alpha_1(t)$ set by the CRA as described at the end of Section 2.3 and displayed in the middle-right panel of Figure 4, we get

$$\pi_t^{II} = \frac{1 - \alpha_1(t)}{\alpha_1(t)} \frac{\alpha_0}{1 - \alpha_0}. \tag{30}$$

We find that there is a correlation of about 0.66 between the six-quarter lagged type-II error and the intercept from the ordered probit model, i.e., credit rating standards are lagging relative to measures of rating standards embedded in CDO prices. This 'slowness' of ratings has been recognized by several authors in academia as well as industry (see, e.g., Altman and Rijken (2004)). For the period from 2004:Q4 to 2016:Q3, the sample for our estimated model, we show the intercept as well as the six-quarter lagged type-II error in the bottom panel of Figure 8. As seen, the lagged model type-II error comoves with the intercept for all the major fluctuations in the sample. The lag implies, for example, that the market's expectations of tightening rating standards rose at the start of the great recession, rather than towards its end. As discussed in Section 3.2, our model implies that credit ratings were very uninformative in the years before the financial crisis, and here we see that the model's type-II error was close to 1 in this period. Our model implies that the type-II error declined during the crisis, but since then has fluctuated substantially, approaching 1 again in 2011 and 2015, preceding minor economic corrections in these periods noted earlier.

We finally investigate the relationship between the model's type-II error and fundamentals. We estimate the relationship:

$$
\begin{aligned}
\log(\text{Type-II Error})(t) &= -0.891 + 0.047 \frac{\text{Credit Growth(t)}}{\text{GDP(t)}} + 0.272 \, \text{Consumption Growth}(t) + \epsilon(t). \\
&= [-16.889][5.623] \qquad\qquad\qquad [3.165]
\end{aligned}
$$

The R^2 for the relationship is 0.496; T-statistics are in parenthesis and are adjusted by Newey-West's procedure for autocorrelation and heteroskedasticity. As seen, the type-II error is higher in periods of higher credit growth, as well as economic growth. Combining the information on the jump intensities of bad firms in the fundamental states (Table 4), we see that in the model during periods of high credit and high economic growth, the jump intensities of each firm are lower, but there is a greater proportion of bad firms in the investment-grade category. Looking again in Figure 4, we see that spreads on the junior (0,3) and (3,7) spreads rose during periods of higher type-II errors in 2011-12 and 2015-16, although the senior spreads rose less. This relative pattern arises due to a combination of more bad firms with less frequent jump shocks.

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5.1 Does a Bayesian Persuasion Based CRA Explain Ratings in the Data?

As discussed in the section 3.2, besides the two fundamental variables (credit growth and economic growth), tranche spreads in our model are affected by time-varying credit rating standards, which affect the proportion of bad firms in investment grade pools. In this section, we discuss further how credit rating standards evolve in our model, and provide evidence that our model's implications are supported in the data.

To determine whether the model's credit ratings are close to those in the data, we first calculate the proportion of US firms in the Compustat database with investment-grade ratings. The time-series of this proportion is displayed in the top panel of Figure 7. The series has a downward trend, which has also been noted by other authors (e.g. Alp (2013)). We therefore, detrend this series using the Hodrick-Prescott filter, and use this as our targeted endogenous variable to be explained.

We next construct the proportion of firms that are assigned investment-grade ratings in our model. Using the law of total probability, the conditional probability of a firm having a G-rating in our model is

$$P_t(G) = \alpha_0 + (1 - \alpha_0) \pi_t^{II}, \tag{31}$$

where we use our result that the optimal type-I error of the CRA in our model is 0.

Finally, using (31) and the type-II error process, we plot $P_t(G)$ and the detrended proportion of investment-grade firms in the bottom panel of Figure 7. As seen, the two series, have a correlation of 65 percent, which is remarkable, given that we did not target the rating proportion in our structural estimation. It is also useful to note, that the investment-grade proportion, both in the data and the model, fell after the financial crisis, and did not fully recover until about 2016. While there have been other theoretical models of the credit rating process (see the references in the introduction), to the best of our knowledge, ours is the first to generate an empirically plausible times-series of ratings.

6 Out-of-Sample Performance of The Model

Our model parameters are estimated using data until 2017. In this section, we briefly discuss the ability of our model to fit fundamentals and tranche spreads in the period from 2018 to 2020:Q2. This out-of-sample period has the pandemic-caused recession, when economic growth as well as credit growth took unprecedented magnitudes, so it is a difficult test of our model's predictions. The figures showing the performance are in the online appendix, and here we make the main points.

As seen in Figure 9, fundamentals exhibited normal fluctuations until 2020, and then consumption growth fell about 10 percent in the second quarter of 2020. Earnings fell by 8.5 and 12.9 percent in the first and second quarters, while credit grew by 28.1 and 18.3 percent in these two quarters. Clearly, the consumption growth and credit growth outcomes were outside the possibilities of our estimated model. In addition, in our estimated model with 6 states, there is no possibility of simultaneously having disastrous growth and such rapid expansion of credit. As seen in Figure 10 however, the econometrician's probability of state 2 (high-credit and low growth) reached 1 in the first quarter of 2020, and then, in the second quarter, it fell to about 0.4, as the probability of state 5 (low credit-disastrous growth) increased to about 0.6.

Figure 11 shows the ability of our model to fit the tranche spreads in the out-of-sample period. As seen, the model's (0,3), (3,7), and (0,100) spreads were quite accurate through this entire period, although the model overestimated the spreads in the first quarter of 2020. The model's (7,15) and (15,100) spreads were too high through this entire period, but most significantly, in the pandemic period, they increased, but not to the magnitudes of the financial crisis. For example, the model's (15,100) spread rose to 135 b.p. in the financial crisis, but only about 40 b.p. in the pandemic. This was despite economic growth being far lower in the pandemic. The major difference in the two episodes is the level of credit growth, which was at its lowest during the financial crisis, and its highest during the pandemic quarters. Our estimated parameters suggest that the risks of catastrophic defaults are lower in periods of such high credit growth, and due to the low quality of fundamentals, the CRA does not mix low grade firms into the investment-grade class. Both factors lead to only a moderate increase in senior tranche spreads. This successful prediction by our model, lends further

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support to the model's mechanism of tranche spreads being determined by credit growth and rating standards, in addition to economic growth.

As a final point, the observed tranche spread increases during the first two quarters of 2020 were likely moderated by the setting up of the Secondary Market Corporate Credit Facility (SMCCF) by the Federal Reserve Board. The Board's actions were taken "to support credit to employers by providing liquidity to the market for outstanding corporate bonds." (see `https://www.federalreserve.gov/monetarypolicy/smccf.htm`). Our model's predicted spreads would serve as a good counterfactual to assess the impact of the Board's actions.

7   Conclusion

We present a new model to shed light on the time-variation of credit spreads on CDO tranches both before and after the financial crisis of 2008. Building on a consumption-based framework with both long-run risk and economic disasters, the model's new feature is the presence of a stylized credit rating agency (CRA), which rates firms in the economy using the principle of Bayesian persuasion. The CRA optimally changes the type-II error (probability of giving bad firms high ratings) over time, depending on the fundamental credit quality of firms, thus affecting spreads on investment-grade products. Our model also assumes that the fundamental credit quality of firm depends on economic growth as well as credit growth in the economy.

We structurally estimate our model, and show that with over 15 years of tranche spreads data, the model is not rejected as long as we exclude the data on the senior-most (15,100) tranche before the financial crisis. Over time, our model replicates the feature in the data that senior tranche spreads are relatively more exposed to macroeconomic growth shocks, while junior spreads are more exposed to credit availability shocks. The changing credit rating standards are an important component of our model in explaining the dynamics of the CDO spread tranches. Indeed, we view the main contribution of our model is to shed light on the way that time-varying credit rating standards affect the clustering of defaults of investment-grade firms, which affects spreads on alternative CDO tranches. We provide evidence that the rating standards in our model (type-II error), are strongly related to credit rating standards

implied from the framework of Blume, Lim, and Mackinlay (1998) and Alp (2013) although the latter are lagging six-quarters relative to measures of rating standards embedded in CDO prices. We provide evidence that the average ratings of firms in our model are highly correlated with average ratings in the data. The model correctly predicts that tranche spreads during the pandemic of 2020 were only moderately high, despite disastrous economic growth, due to high credit growth and relatively tight credit rating standards.

Appendix 1

Proof of Lemma 1 The valuation of a firm of uncertain quality satisfies the system of ODEs

$$r(s)\,V(X,s) = \mu_Y^Q(s)X\frac{\partial V}{\partial X} + \frac{1}{2}\left(\sigma_X X\right)^2 \frac{\partial^2 V(X,s)}{(\partial X)^2} + X + \sum_{s'=1}^{S}\lambda_{ss'}^Q V(X,s') - (1-\alpha)\kappa(s)\,V(X,s),$$

$$(32)$$

whose solution is $V^\alpha\left(X,s\right) = X\,v^\alpha(s)$ where v^α is in (15). ∎

Proof of Lemma 2

Using Ito's lemma for jump diffusion processes (see, e.g. Duffie), we can see that the value of a firm of uncertain quality as

$$\frac{dV_t^{i,\alpha}}{Vt_-^{i,\alpha}} = \mu_Y(s_t)\,dt\,dt + \sigma_X(\sqrt{\rho_i}\,dW_{2t} + \sqrt{1-\rho_i}\,dW_t^i) - (1-\alpha)dL^{s,i}$$

$$+ \sum_{s_t \neq s_{t-}}\left(\frac{v^\alpha(s_t)}{v^\alpha(s_{t-})} - 1\right)dM_t^{(s_{t-},s_t)}. \quad (33)$$

Using the expression for $v^\alpha(s_t)$ from Lemma 1, the drift simplifes to $r(s_t) - 1/v^\alpha(s_t) - \sigma_m\sqrt{\rho_i}\sigma_X$, and hence under the Q-measure as

$$\frac{dV_t^{i,\alpha}}{Vt_-^{i,\alpha}} = (r(s_t) - 1/v^\alpha(s_t))\,dt + \sigma_X(\sqrt{\rho_i}\,dW_{2t}^* + \sqrt{1-\rho_i}\,dW_t^i) - (1-\alpha)dL^{s,i}$$

$$+ \sum_{s_t \neq s_{t-}}\left(\frac{v^\alpha(s_t)}{v^\alpha(s_{t-})} - 1\right)dM_t^{*(s_{t-},s_t)}. \quad (34)$$

Now using the debt growth assumption in (16), gives the solvency ratio dynamics. ∎

Lemma 3 *The CRA never assigns B-ratings to good firms, i.e. $\pi^I = \mathbb{P}[B|good] = 0$.*

Proof of Lemma 3 To simplify notation, we omit the arguments of the type-I and type-II errors. The CRA seeks to maximize $\mathbb{P}[G]$ in (21), subject to the default probability constraint (22), and the upper bound on the posterior probability achivable, $\bar{\alpha}$. Since (21) is a weighted sum of π^I and π^{II}, to maximize the probability $\mathbb{P}[G]$ the CRA chooses π^I as low as possible and π^{II} as high as possible subject to the constraints. Now it is easy to see that a low type-I error increases the objective value, and lowers the default probability, so the CRA will choose the lowest value possible, which is 0. ∎

Appendix 2: Data Description

As mentioned in Section 2.4, payments from insurance buyers to insurance sellers are made as a combination of quarterly spreads with fixed and variable components, or as fixed spreads with upfront payments. In this appendix, we provide details of the quotation of spreads on the alternative CDO tranches from September 2004 to July 2020.

The CDX indices roll every six months. In particular, each year on March 20th and September 20th, new series of the index with updated constituents are introduced. While previous series are traded for some time after new series are created, liquidity is usually concentrated on the on-the-run series.[15] We do not have data on Series 1 and 2. The data from September 2004 to September 2007 are obtained from the dataset supplemented to Coval et al. (2009). This subsample contains the on-the-run series numbers 3 to 8. The data from September 2007 to July 2020 are obtained from Bloomberg (CMA New York). Starting from series 15 and onward, only series with odd numbers have been traded with tranches. The CDX indices have 3, 5, 7 and 10 year tenors. We use 5-year CDX indices which are most liquid for most series.

We summarize all the changes in quotations in Table 2 and in addition, we provide time-series plots of each component in Figure 5. The fixed component of spreads, the variable component of spreads, and the upfront payment, are shown in the left, middle, and right panels, respectively. We summarize some of the main points on the changes in quotations from these exhibits here:

1. All tranches of series 3-8 except the 0-3 tranche, are quoted as fully variable spreads (zero fixed spreads). The 0-3 tranche of these series are quoted as an upfront payment with a 500 b.p. fixed spread.

2. The data from September 2007 to September 2010 includes series 9 only (therefore skips series 10-14). All tranches of series 9 except the 0-3 tranche are initially quoted as spreads, and later as upfront payments with fixed spreads. The quote switch for tranche 3-7 occurred in November 2008, for tranche 7-10 in February 2009, and for tranches 10-15, 15-30, and 30-100 in May 2009. After these switches, the fixed spreads are 500

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[15]An exception is series 9 introduced in September 2007 and traded until December 2012, which has been the most liquid series from September 2007 to September 2010.

b.p. for tranches 3-7, and 7-10, and 100 b.p. for tranches 10-15, 15-30, and 30-100. The 0-3 tranche of series 9 is always quoted as an upfront payment with a fixed spread of 500 b.p. and a zero variable spread component.

3. The period from September 2010 to July 2020 contains series with odd numbers in the range 15-33 except series 23. Due to inconsistencies in reported prices, we have replaced series 17 (on-the-run from September 2011 to September 2012) and 23 (on-the-run from September 2014 to September 2015) with off-the-run series 15 and 21, respectively.

4. Before series 15 is introduced in September 2010, the CDX index trades with tranches 0-3, 3-7, 7-10, 10-15, 15-30 and 30-100. Starting from series 15, the structure of tranches changes to 0-3, 3-7, 7-15 and 15-100.

5. Starting from series 15, all tranches are quoted as upfront payments with fixed spreads. Series 15, 17, 19, 21, and 23 have fixed spreads of 500, 100, 100, and 25 b.p. for tranches 0-3, 3-7, 7-15, and 15-100, respectively.

Table 1: What Explains CDO Tranche Spreads? (2004:3 – 2017)

No.	α	β_1	β_2	R^2
CDX Spread				
1.	120.013	-6.547		0.538
	[10.016]	[-4.817]		
2.	115.038		-53.948	0.265
	[7.321]		[-3.560]	
3.	133.868	-5.746	-36.676	0.653
	[15.664]	[-5.997]	[-5.608]	
Spread (0-3)				
4.	3261.805	-209.586		0.636
	[10.73]	[-2.50]		
5.	2970.136		-1451.504	0.222
	[6.166]		[-3.165]	
6.	3593.951	-190.367	-879.284	0.712
	[15.776]	[-7.111]	[-5.089]	
Spread (15-100)				
7.	49.687	-3.640		0.416
	[5.242]	[-3.439]		
8.	50.800		-38.054	0.331
	[5.690]		[-4.220]	
9.	60.650	-3.006	-29.019	0.596
	[9.056]	[-4.161]	[-6.360]	

Tranche spreads are on the Dow Jones North American Investment Grade Index, which are reported by Credit Market Analysis (CMA) and obtained from Bloomberg (see Data Appendix for construction of our time series). CDX represents the full CDO. Spread (AL,AU) represents the spread on a trance with loss attachment points AL and AU in percentage points. For example, the "senior" spread represents the 15 to 100 percent loss attachment points, while the "equity" tranche represents the 0 to 3 loss attachment points. We report the coefficients of the fitted regression:

$$\text{Tranche Spread}(t) = \alpha + \beta_1 \frac{\text{Credit Growth(t)}}{\text{GDP(t)}} + \beta_2 \text{Consumption Growth} + \epsilon(t)$$

for alternative tranches. T-statistics are in parenthesis and are adjusted by Newey-West's procedure for autocorrelation and heteroskedasticity.

Table 2: Fixed spreads of tranches

Start	Series	Tranches' Tickers	0-3%	3-7%	7-10%	10-15%	15-30%	30-100%
Sep. 2004	S3		500	-	-	-	-	-
Mar. 2005	S4		500	-	-	-	-	-
Sep. 2005	S5		500	-	-	-	-	-
Mar. 2006	S6		500	-	-	-	-	-
Sep. 2006	S7		500	-	-	-	-	-
Mar. 2007	S8		500	-	-	-	-	-
Sep. 2007	S9	CT753589 CT753593 CT753597 CT753601 CT753605 CT753609	500	-	-	-	-	-
Sep. 2007	S9	CT753589 CT753593 CT753597 CT753601 CT753605 CT753609	500	500	500	100	100	100

			0-3%	3-7%	7-15%	15-100%
Sep. 2010	S15	CY071225 CY071229 CY071233 CY071237	500	100	100	25
Sep. 2011	S17	CY087579 CY087583 CY087587 CY087591	500	100	100	25
Sep. 2012	S19	CY125375 CY125380 CY125385 CY125390	500	100	100	25
Sep. 2013	S21	CY181667 CY181672 CY181677 CY181682	500	100	100	25
Sep. 2014	S23	CY233259 CY233265 CY233271 CY233277	500	100	100	25
Sep. 2015	S25	CY295911 CY295917 CY295923 CY295929	100	100	100	100
Sep. 2016	S27	CY328667 CY328673 CY328679 CY328685	100	100	100	100
Sep. 2017	S29	CY344120 CY344126 CY344132 CY344138	100	100	100	100
Sep. 2018	S31	CY372924 CY372930 CY372936 CY372942	100	100	100	100
Sep. 2019	S33	CY437839 CY437845 CY437851 CY437857	100	100	100	100

Tranche attachment points are stated in percentage points. Data from September 2004 to September 2007, which traded series 3 to 8, is obtained from the *American Economic Review* webpage for Coval, Jurek, and Stafford (2009a). All tranches of series 9 except the 0-3% tranche were initially quoted as spreads (the first line with S9 series) and later as upfront payments with fixed spreads (the second line with S9 series). The quote switch for tranche 3-7% occurred in November 2008, for tranche 7-10% in February 2009, and for tranches 10-15%, 15-30%, and 30-100% in May 2009.

Table 3: Maximum Likelihood Estimates of 6-Regime Markov Switching Model for Ratio of Credit Growth at Nonfinancial Firms to GDP, Real Consumption Growth, and Real S&P 500 Earnings Growth (2004:3 – 2017)

Ratio of Credit Growth-to-GDP Parameters (%)		
μ_G^H	μ_G^L	μ_G^D
0.835	0.300	-4.086
(0.005)	(0.006)	(0.003)

Consumption Growth Parameters (%)		
μ_C^H	μ_C^L	μ_C^D
5.08	2.00	0.00
(.003)	(0.002)	(0.000)

Aggregate Earnings Growth Parameters (%)		
μ_Y^H	μ_Y^L	μ_Y^D
13.03	-9.76	-88.1
(0.131)	(2.691)	(2.898)

Volatilities (%)		
$\sigma_{G,1}$	$\sigma_{G,2}$	$\sigma_{G,3}$
5.78	0.948	0.218
(0.070)	(0.022)	(0.005)
$\sigma_{C,1}$	$\sigma_{C,2}$	$\sigma_{C,3}$
0.000	1.300	0.000
(0.001)	(0.120)	(0.001)
$\sigma_{Y,1}$	$\sigma_{Y,2}$	$\sigma_{Y,3}$
0.000	0.000	8.762
(0.001)	(0.04)	(0.009)

Quarterly Transition Probability Matrix

	HC-HG	HC-LG	LC-HG	LC-LG	LC-DG	DC-HG
HC-HG	0.899	0.0440	0.054	0.000	0.001	0.001
HC-LG	0.181	0.817	0.000	0.000	0.001	0.001
LC-HG	0.000	0.000	0.887	0.111	0.001	0.001
LC-LG	0.019	0.101	0.081	0.795	0.001	0.001
LC-DG	0.098	0.098	0.098	0.098	0.509	0.098
DC-HG	0.098	0.098	0.098	0.098	0.098	0.509

Transition Matrix (Standard Errors)

	HC-HG	HC-LG	LC-HG	LC-LG	LC-DG	DC-HG
HC-HG		0.006	0.003	0.000	0.000	0.000
HC-LG	0.002		0.001	0.001	0.000	0.000
LC-HG	0.000	0.000		0.002	0.000	0.000
LC-LG	0.002	0.000	0.001		0.000	0.000
LC-DG	0.004	0.004	0.004	0.004		0.004
DC-HG	0.004	0.004	0.004	0.004	0.004	

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Log Likelihood =-401.725

Table 4: Second Stage SMM Estimated Parameters of Firms' Cash Flow Processes for Model With the CRA (2004:3 – 2017)

Prior Belief, Posterior Belief, Bankruptcy Costs					
α_0	0.256	$\bar{\alpha}$	0.552	Φ	0.287
	(0.004)		(0.002)		(0.003)

Cash Flow Volatility and Correlation			
σ_X	0.212	ρ_i	0.760
	(0.001)		(0.004)

Jump Intensities					
$\kappa(1)$	0.009	$\kappa(2)$	0.033	$\kappa(3)$	0.042
	(0.000)		(0.000)		(0.000)
$\kappa(4)$	0.104	$\kappa(5)$	0.184	$\kappa(6)$	0.157
	(0.000)		(0.000)		(0.000)

Jump Correlation	
φ	0.385
	(0.045)

SMM Error Value for full sample: $\chi^2(9) = 19.746$; p-value = 0.020. SMM Error Value for full sample excluding spread(15,100) from 2004:4 – 2007: $\chi^2(9) = 14.488$; p-value = 0.106. Standard errors are in parenthesis.

Table 5: Measures of Goodness-of-Fit of ZUV Spreads for Model With the CRA (2004:3 – 2017)

Regression of Data on Model ZUV Spreads (in Basis Points)

	α	β	R^2
spread(0,3)	203.279	0.944	0.806
	(0.891)	(7.263)	
spread(3,7)	84.188	0.679	0.766
	(1.781)	(6.975	
spread(7,15)	24.181	0.927	0.820
	(1.411)	(8.982)	
spread(15,100)	4.169	0.734	0.645
	(0.710)	(10.434)	
spread(0,100)	-7.055	1.144	0.823
	(-0.690)	(8.448)	

Data and Model ZUV Spread Volatilities in Basis Points

	Data	Model
spread(0,3)	854.590	771.692
spread(3,7)	403.900	527.834
spread(7,15)	202.670	212.95
spread(15,100)	31.355	37.949
spread(0,100)	48.818	40.474

Data and Model 5-Year Target Default Probability

Data	Model
0.028	0.029

Data Correlation of ZUV Spreads

	spread(0,3)	spread(3,7)	spread(7,15)	spread(15,100)	spread(0,100)
spread(0,3)	1.000	0.961	0.889	0.674	0.985
spread(3,7)	0.961	1.000	0.970	0.759	0.985
spread(7,15)	0.889	0.970	1.000	0.863	0.943
spread(15,100)	0.674	0.759	0.863	1.000	0.742
spread(0,100)	0.985	0.985	0.943	0.742	1.000

Model Correlation of ZUV Spreads

	spread(0,3)	spread(3,7)	spread(7,15)	spread(15,100)	spread(0,100)
spread(0,3)	1.000	0.880	0.891	0.885	0.907
spread(3,7)	0.880	1.000	0.977	0.925	0.976
spread(7,15)	0.891	0.977	1.000	0.955	0.968
spread(15,100)	0.885	0.925	0.955	1.000	0.968
spread(0,100)	0.907	0.976	0.968	0.968	1.000

In the top panel, we present results of the regressions of the form: Data Variable $= \alpha + \beta$ Model Variable. Newey-West T.statistics are in parenthesis.

Table 6: Measures of Goodness-of-Fit of Raw Data for Model (2004:3 – 2017)

Regression of Data on Model Variable Spreads (in Basis Points)

	α	β	R^2	Sample
spread(3,7)	13.597	0.622	0.771	2004:Q3 – 2008
	(0.443)	(6.186)		
spread(7,15)	-22.764	0.933	0.80	2004:Q3 – 2009:Q1
	(-1.266)	(7.164)		
spread(15,100)	-27.516	0.925	0.959	2004:Q3 – 2009:Q2
	(-6.590)	(29.944)		
spread(0,100)	-4.262	1.082	0.805	2004:Q3 – 2017
	(-0.389)	(7.224)		

Regression of Data on Model Upfronts (in Basis Points)

	α	β	R^2	Sample
upfront(0,3)	11.101	0.845	0.632	2004:Q3 – 2017
	(1.853)	(5.304)		
upfront(3,7)	6.486	0.463	0.491	2009:Q1 – 2017
	(2.598)	(2.217)		
upfront(7,15)	0.56	0.679	0.337	2009:Q2 – 2017
	(1.057)	(3.612)		
upfront(15,100)	-0.567	0.649	0.397	2009:Q3 – 2017
	(-1.362)	(2.228)		

We present results of the regressions of the form: Data Variable$_t = \alpha + \beta$ Model Variable$_t + \epsilon_t$. Newey-West T.statistics are in parenthesis.

Table 7: Definition of Variables Used for Estimation of Creding Rating Standards

Variable	Definition
Operating Margin	Operating Income Before Depreciation (oibdp) to Sales (sale)
Interest Coverage	Operating Income Before Depreciation plus interest expense (xint) divided by inteerst expense
Total Debt to Assets	Long-term debt (dltt) + short-term debt (dlc) divided by assets (att)
Cash to Assets	Cash and short-term investments (che) to assets (att)
Research and Development to Assets	R&D expenses (xrd) to assets (att). Missings set to zero
Tangibility	Property plant and equipment (ppent) to assets (att)
Retained Earnings to Assets	Retaained aarnings (re) to assets
Capital Expenditures to Assets	Capital expenditures (capx) to assets (att)
Size (NYSE) Percentile	NYSE market capitalization percentile
Idiosyncratic Volatility	Standard deviation of residuals from rolling one-year regression
	of daily stock market returns on four factors (market, SMB, HML¡ and momentum);
	factors are btained from the website of Ken French
Market Beta	Firm's coefficient on market from four-factor model above
Momentum	Firm's coefficient on momentum factor from four-factor model above
HML	Firm's coefficient on HML factor from four-factor regression above

The Computstat data items are in parenthesis.

Table 8: Estimation of Determinants of Credit Ratings and Credit Rating Standadars from Ordered Probit model (1985:4 – 2016:3)

| | | | Number of Observations: 23,142 |
| | | | Pseudo R^2: 0.1083 |
Independent Variable	Coefficient	Standard Error	P-Value
Operating Margin	0.868	0.044	0.000
Interest Coverage	0.182	0.008	0.000
Total Debt to Assets	-2.329	0.066	0.000
Divident Payer	0.441	0.019	0.000
Cash to Assets	-0.145	0.087	0.092
Research and Development to Assets	10.912	0.789	0.000
Tangibility	0.567	0.038	0.000
Retained Earnings to Assets	0.754	0.033	0.000
Capital Expenditures to Assets	-1.335	0.238	0.000
Size (NYSE) Percentile	0.975	0.062	0.000
Market/Book	0.011	0.008	0.164
Idiosyncratic Volatility	-1.689	0.078	0.000
Market Beta	-42.349	2.218	0.000
Momentum	-19.624	2.253	0.000
HML	-11.24	1.368	0.000

We provide estimates of an ordered probit model of firms credit ratings to measure time variation in credit rating standards. Long-term issuer credit ratings of issuers are assigned succesive integer ratings from 10 to 17 for investment-grade bonds rated BBB, A, $\cdots$, AAA. We estimate the specfication:

$$
\begin{aligned}
R_{it} &= 17 & &\text{if} \quad Z_{it} \in [\mu_{16}, \infty], \\
R_{it} &= 16 & &\text{if} \quad Z_{it} \in [\mu_{15}, \mu_{16}], \\
& \;.. \\
R_{it} &= 10 & &\text{if} \quad Z_{it} \in [\mu_9, \mu_{10}],
\end{aligned}
$$

in which, $Z_{it} = \alpha_t + \beta' \cdot X_{it} + \epsilon_{it}$, is the latent variable that affects the rating with partition points μ_i, the independent variables, X_{it}, are the firm level indicators of credit quality listed in each row, and $E[\epsilon_{it}|X_{it}] = 0$. The intercept, α_t, is a measure of *credit rating standards* at time t. The full-timeseries of α_t is shown in Figure 8.

Figure 1: Tranche Spreads, Economic Growth, and Credit Availability
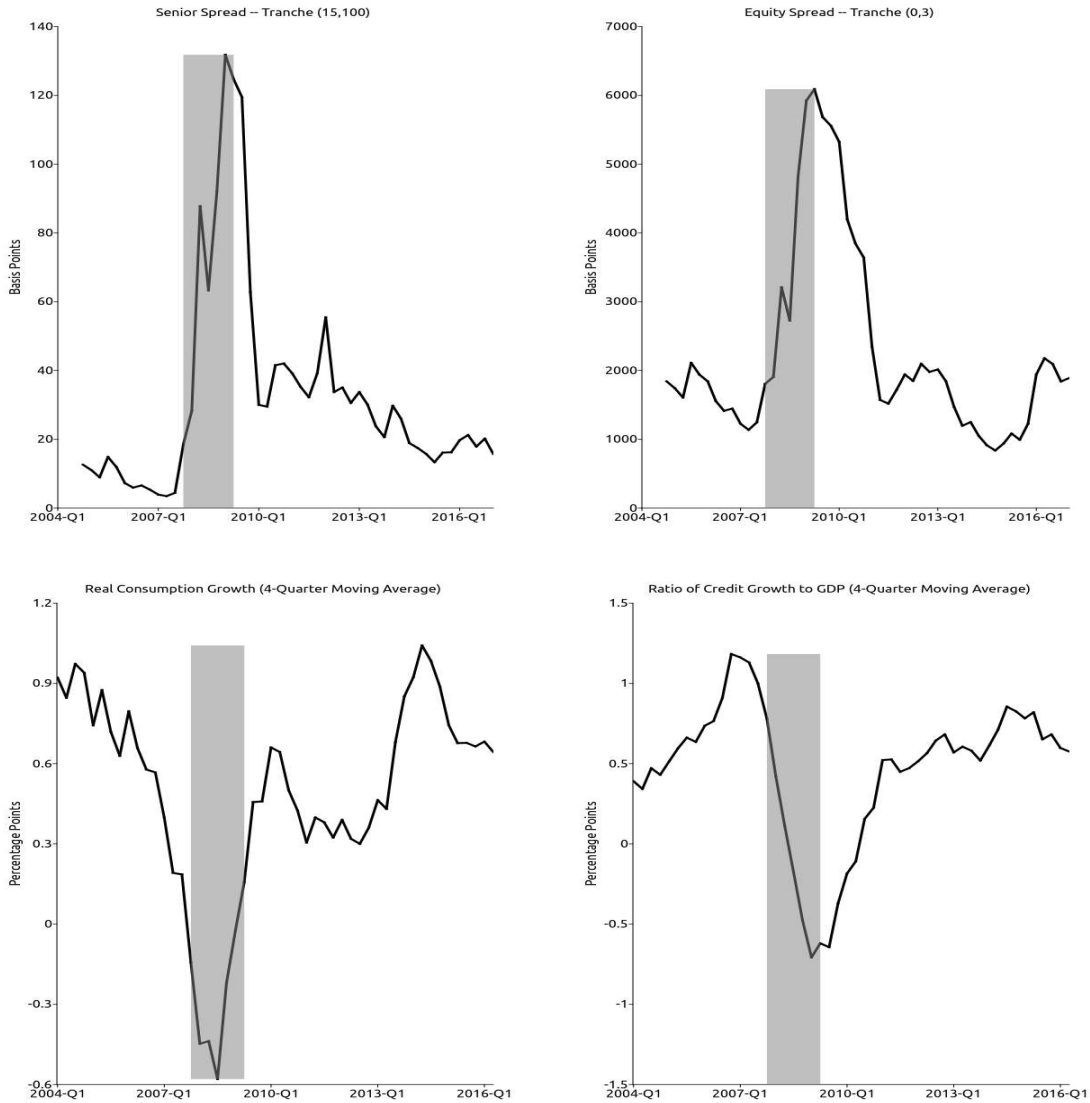
Tranche spreads are on the Dow Jones North American Investment Grade Index, which are reported by Credit Market Analysis (CMA) and obtained from Bloomberg (see Data Appendix for construction of our time series). The "senior" spread represents the 15 to 100 percent loss attachment points, while the "equity" tranche represents the 0 to 3 loss attachment points.
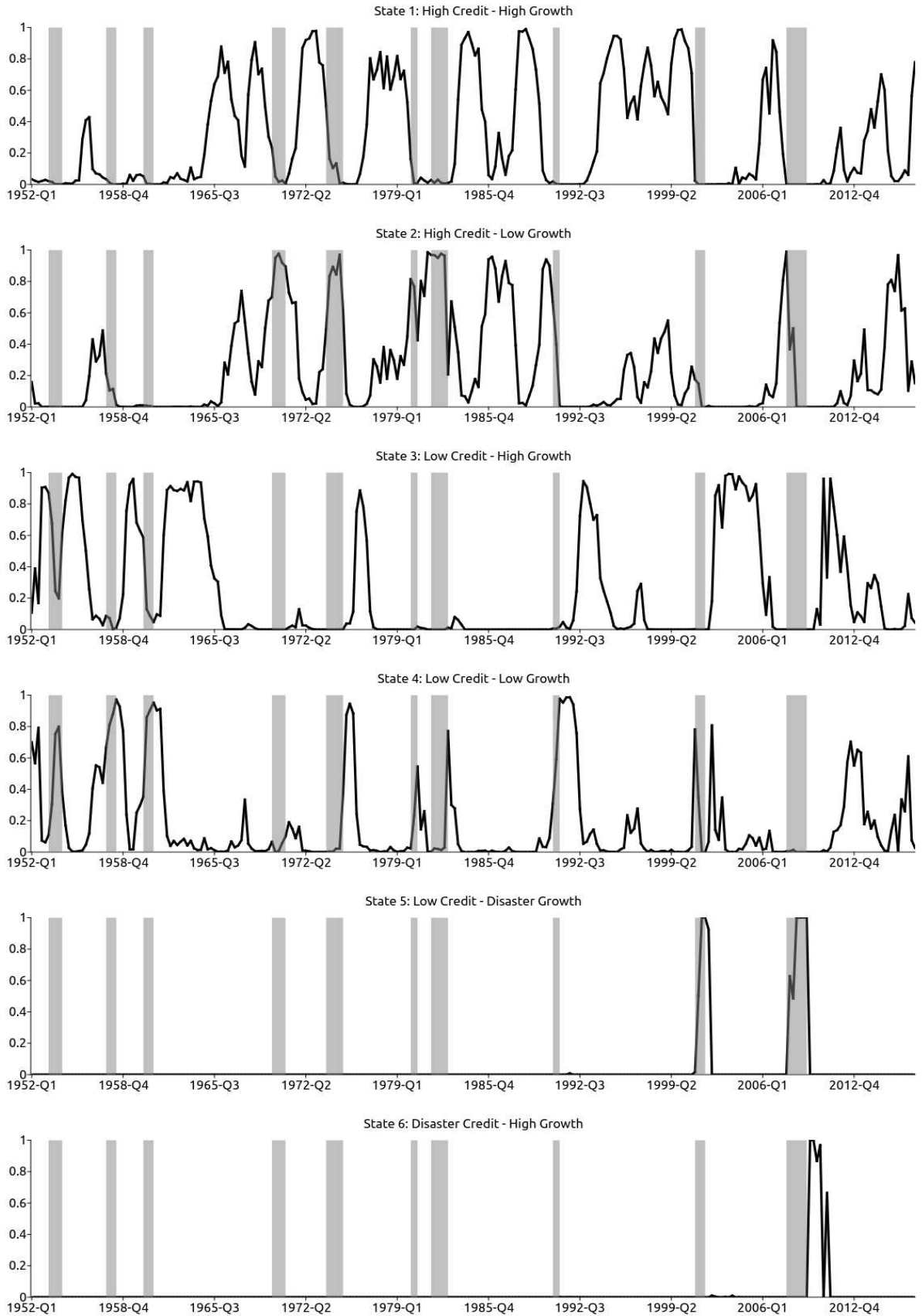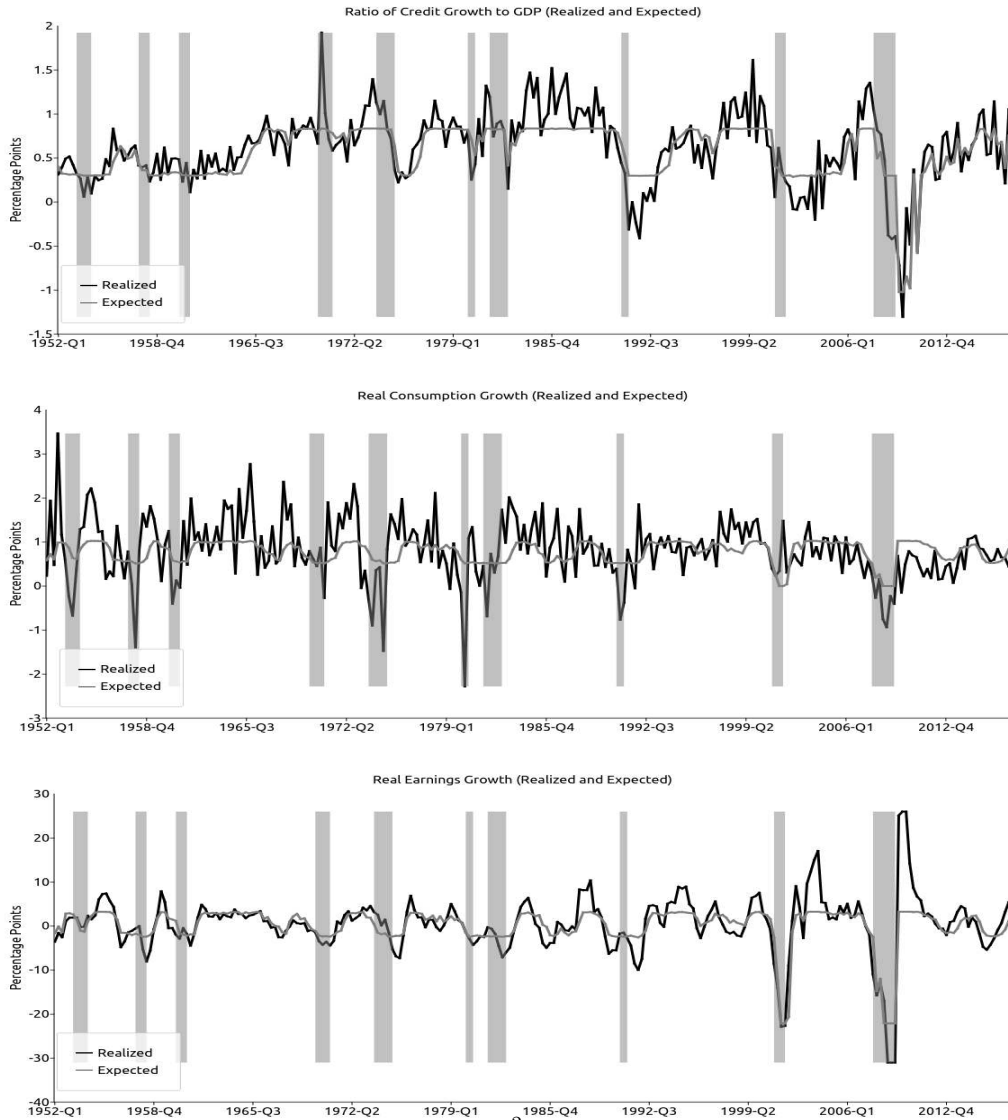
Figure 2: Probabilities of the States From Regime Switching Model (1952 – 2017)

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Figure 3: Fundamentals: Data and Fitted From Regime Switching Model (1952 – 2017)

The expected growth explains realized growth with R^2s of 63.9%, 18.6%, and 40.1%, for credit growth, consumption growth, and aggregate earnings growth, respectively.

Figure 4: Data Approximated and Model Zero Upfront Variable Spreads on CDO Tranches (2004:4 – 2017)
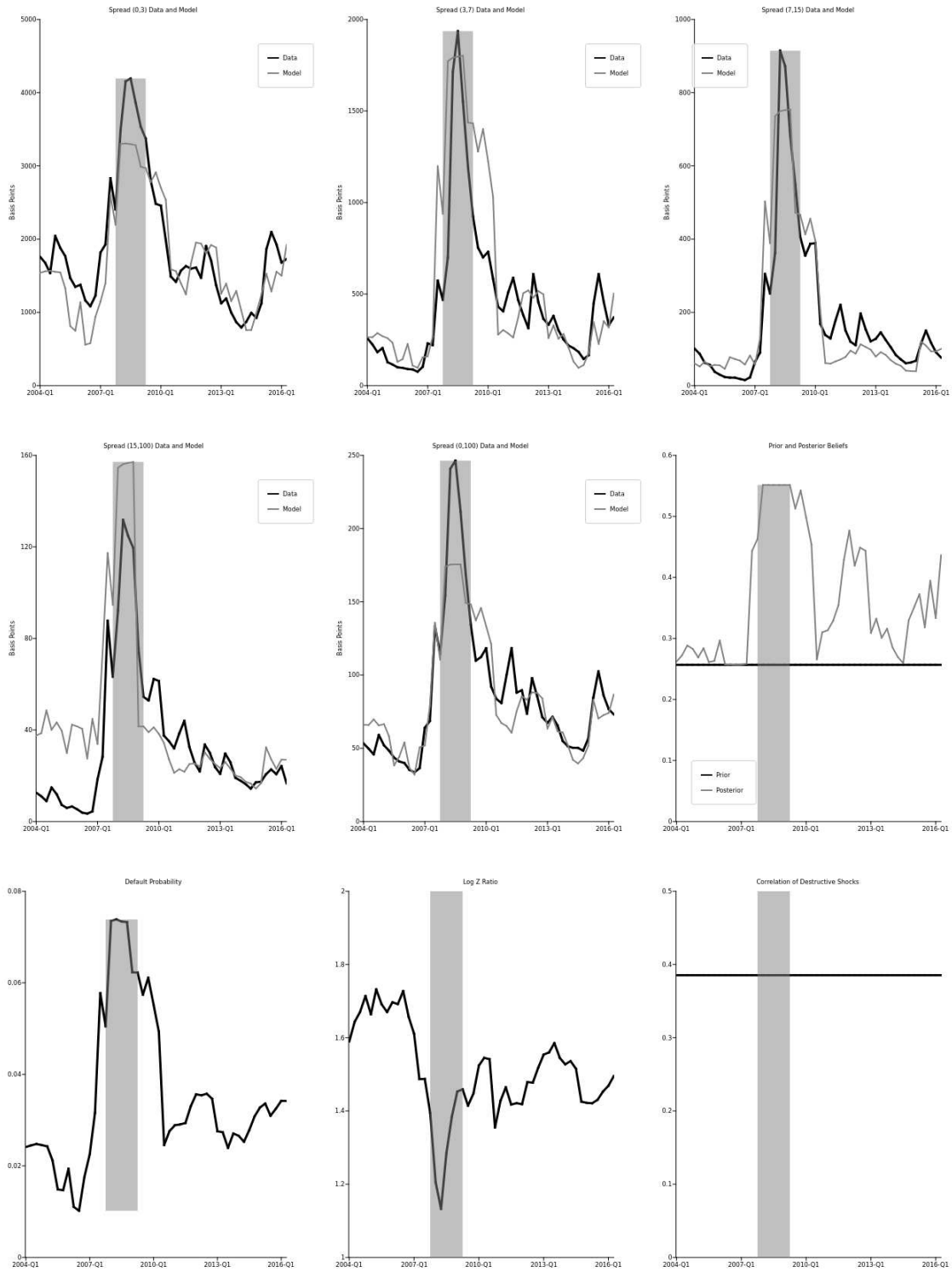
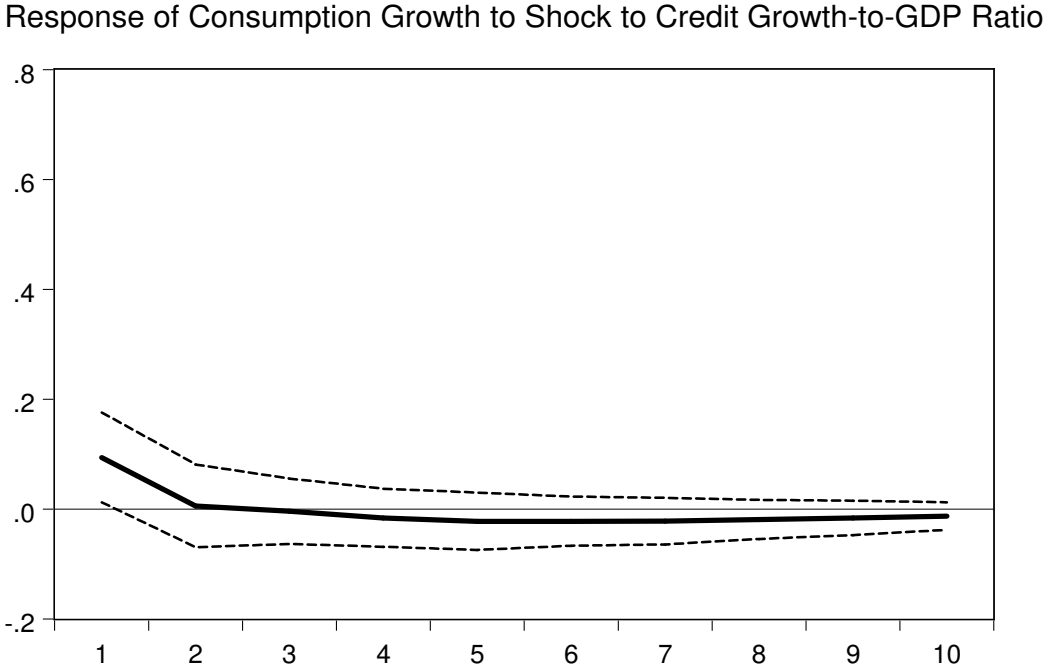Figure 5: Raw Data and Model Upfronts and Variable and Fixed Spreads on CDO Tranches (2004:4 – 2017)
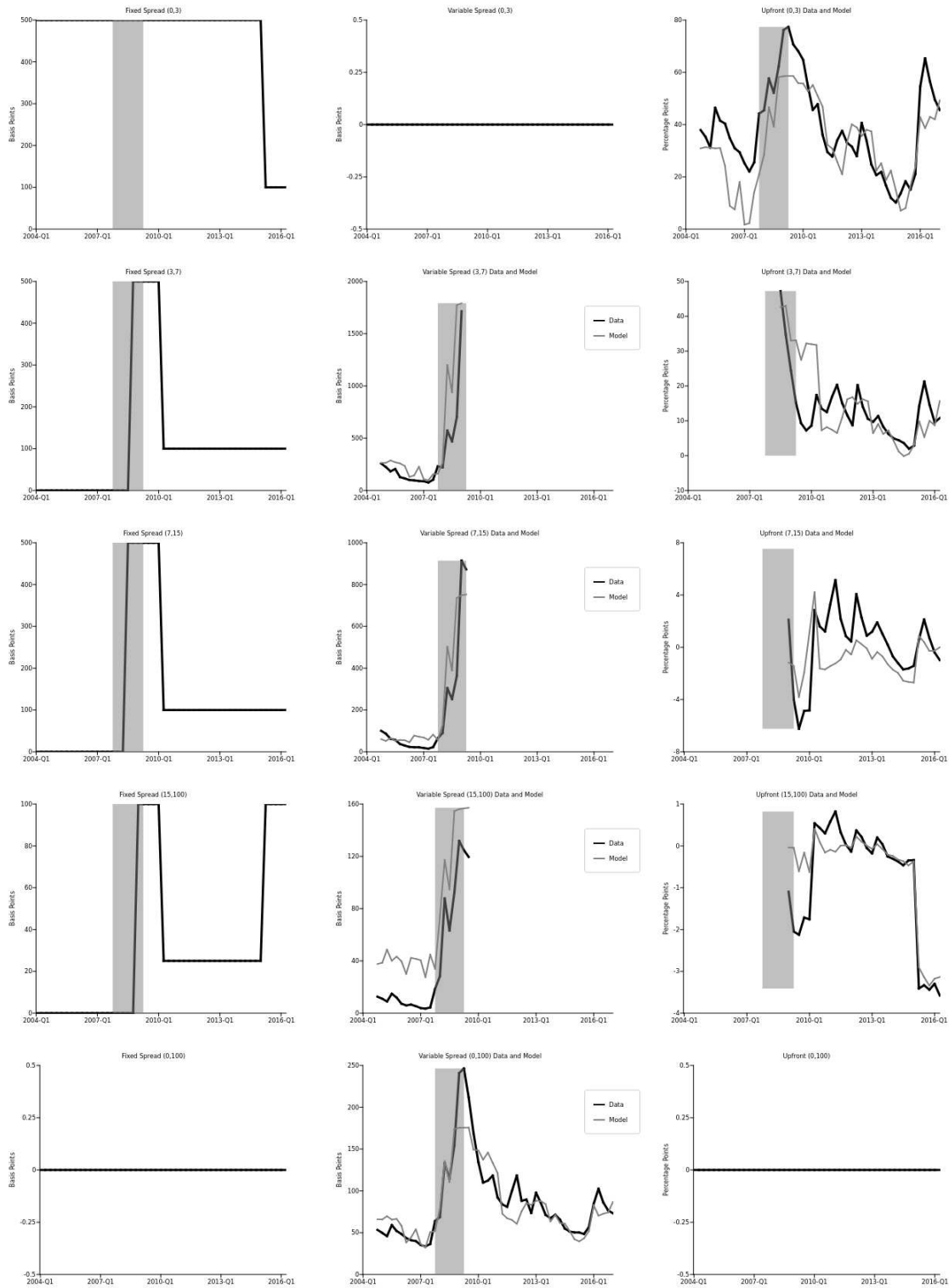
Figure 6: Impulse Responses of Credit Availability and Consumption Growth (1952 – 2017)

Response of Credit Growth-to-GDP Ratio to Shock to Consumption Growth

Response of Consumption Growth to Shock to Credit Growth-to-GDP Ratio
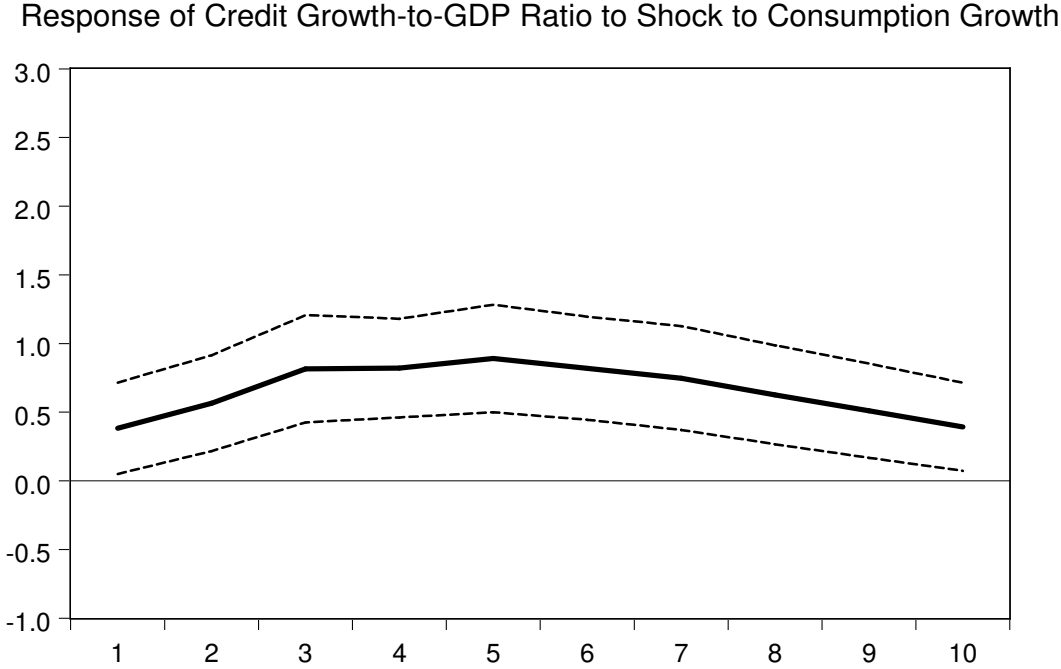
We display the generalized impulse response functions of Pesaran and Shin (1998). 95% confidence bands are in dotted lines.

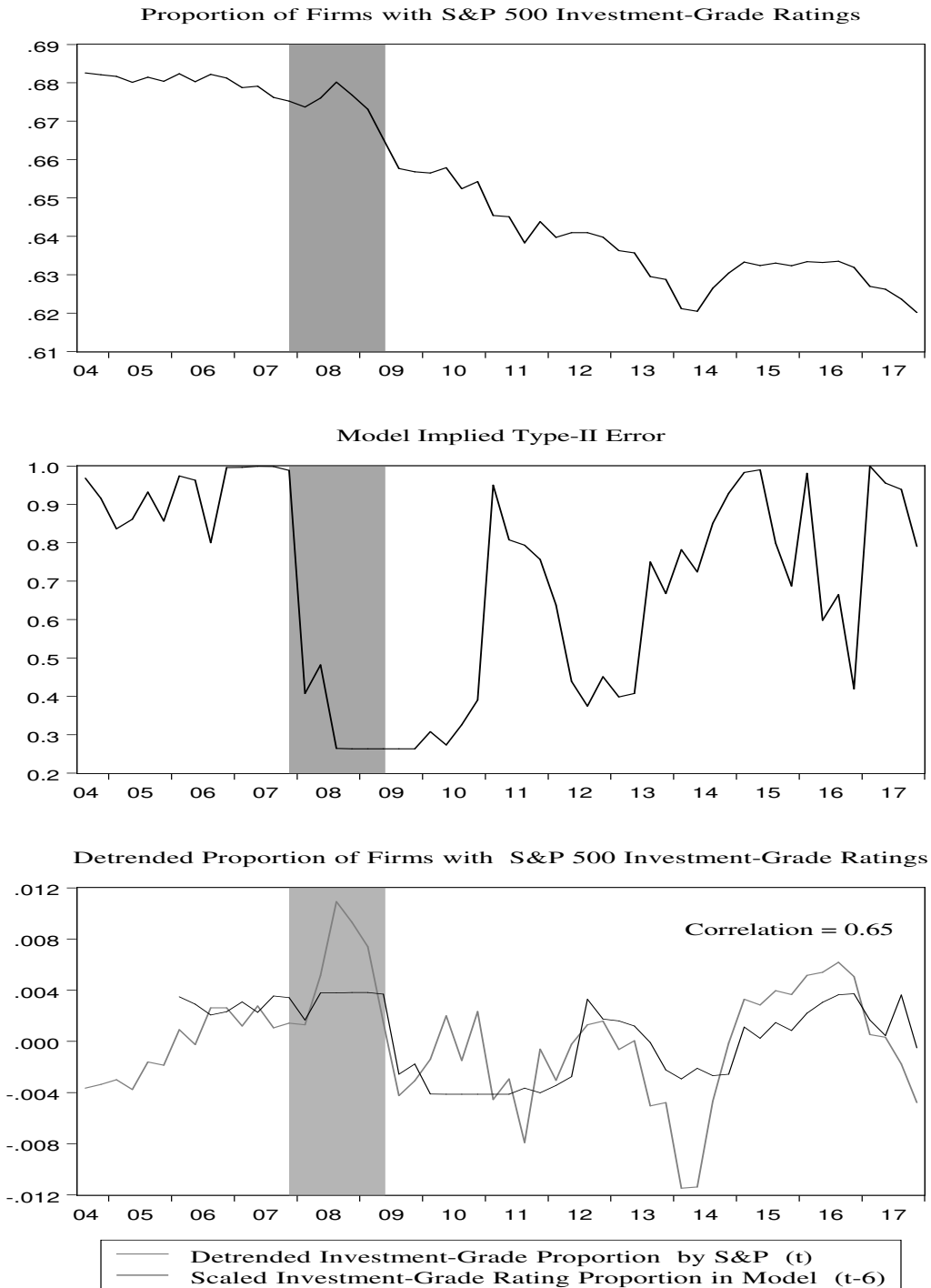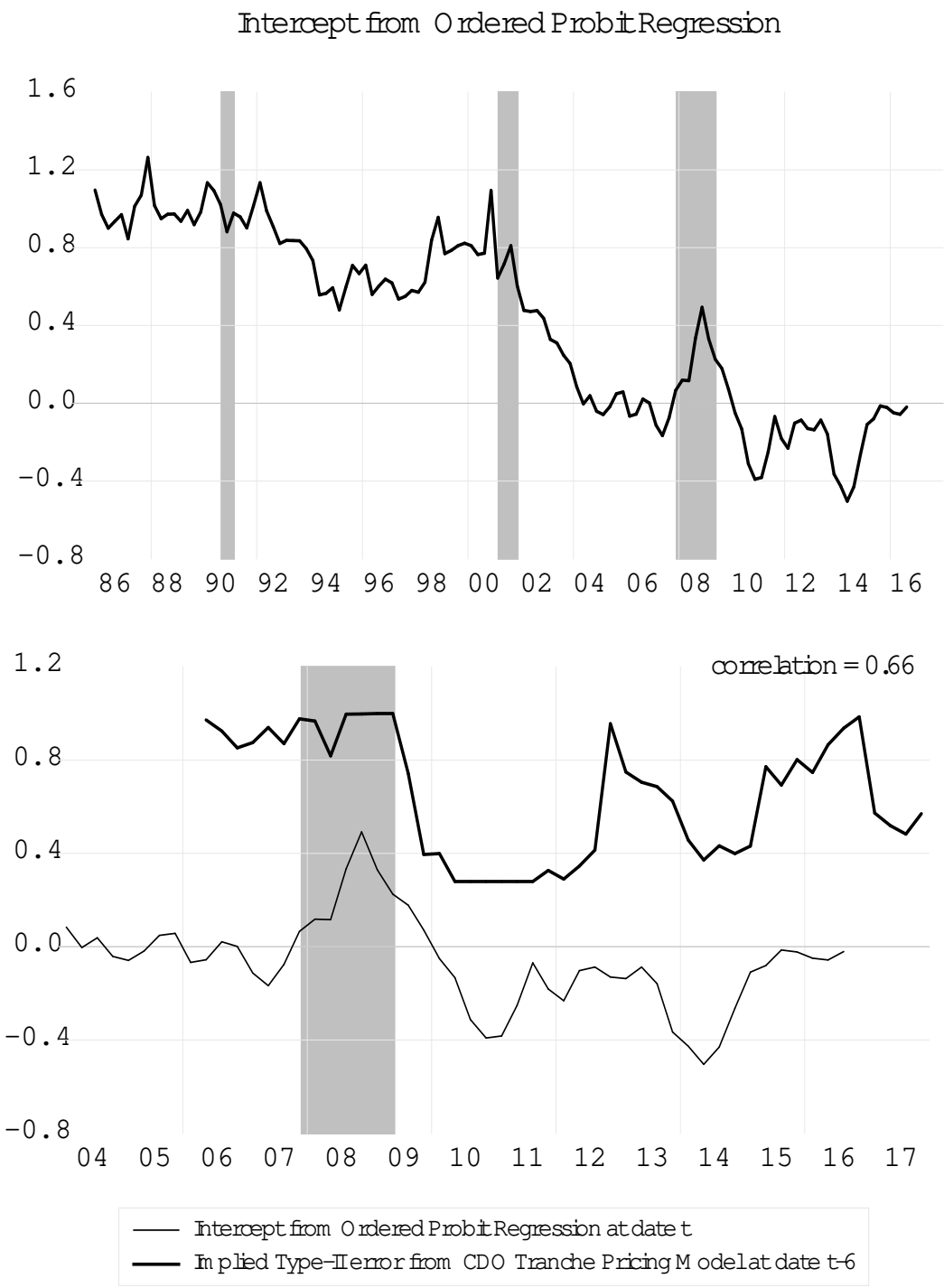Figure 7: Proportion of Firms with Investment-Grade Ratings (2004:4 – 2017)

Proportion of Firms with S&P 500 Investment-Grade Ratings

Model Implied Type-II Error

Detrended Proportion of Firms with S&P 500 Investment-Grade Ratings

Correlation = 0.65

—— Detrended Investment-Grade Proportion by S&P (t)
—— Scaled Investment-Grade Rating Proportion in Model (t-6)

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Figure 8: Crediit Rating Standards for Investment-Grade Ratings (2004:4 – 2017)

Intercept from Ordered Probit Regression

correlation = 0.66

— Intercept from Ordered Probit Regression at date t
— Implied Type-II error from CDO Tranche Pricing Model at date t-6

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8 Online Appendix

In this appendix we present results for the out-of-sample performance for the period 2018 - 2020:2 based on the parameters of our model, which is estimated from 1952 - 2017.

Figure 9: Fundamentals: Data and Fitted From Regime Switching Model In-Sample (1952 – 2017) and Out-of-Sample(2018 – 2020:2)
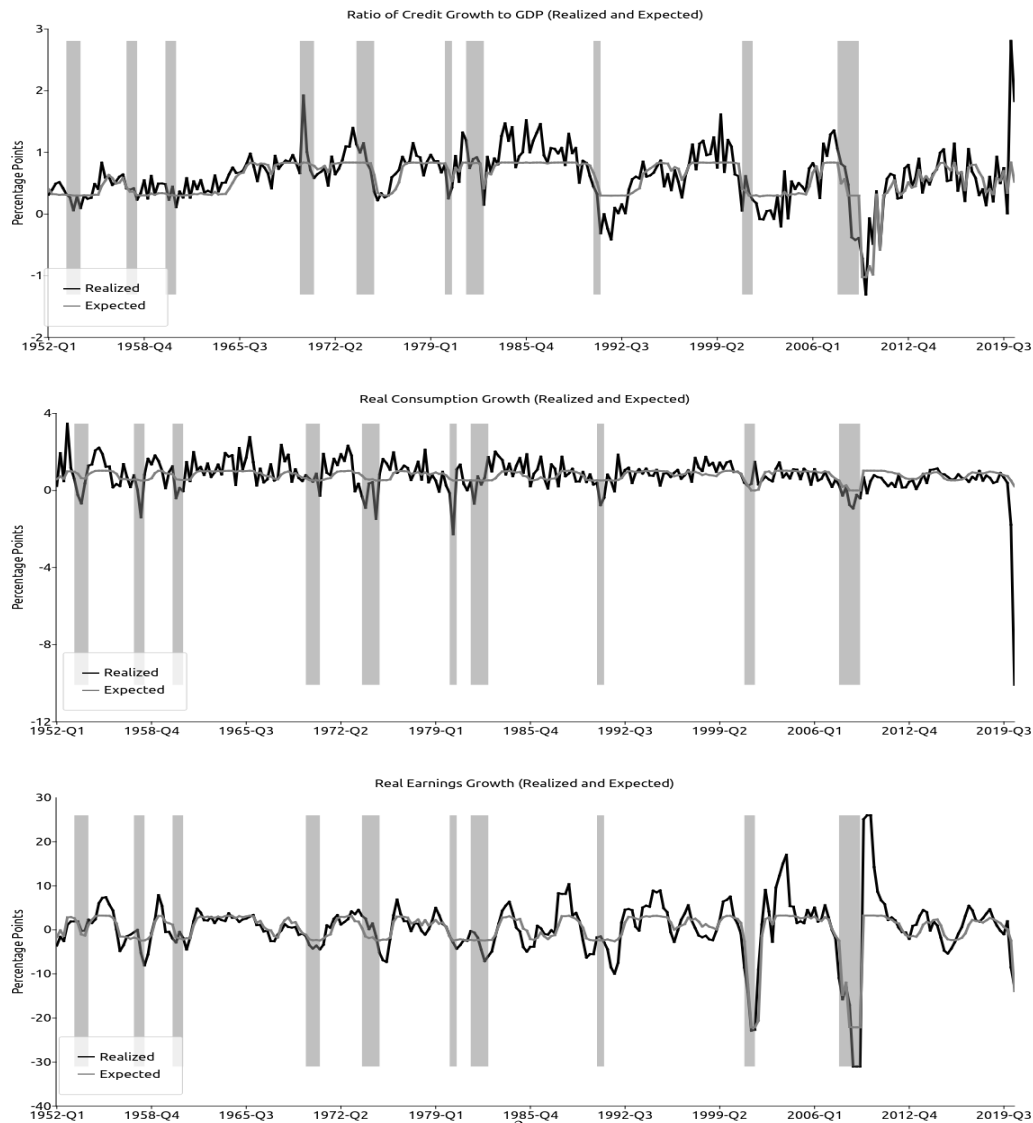
The expected growth explains realized growth with R^2s of 63.9%, 18.6%, and 40.1%, for credit growth, consumption growth, and aggregate earnings growth, respectively.

Figure 10: In-Sample (1952 – 2017) and Out-of-Sample(2018 – 2020:2) Probabilities of the States From Regime Switching Model (1952 – 2017)

State 1: High Credit - High Growth

State 2: High Credit - Low Growth

State 3: Low Credit - High Growth

State 4: Low Credit - Low Growth

State 5: Low Credit - Disaster Growth

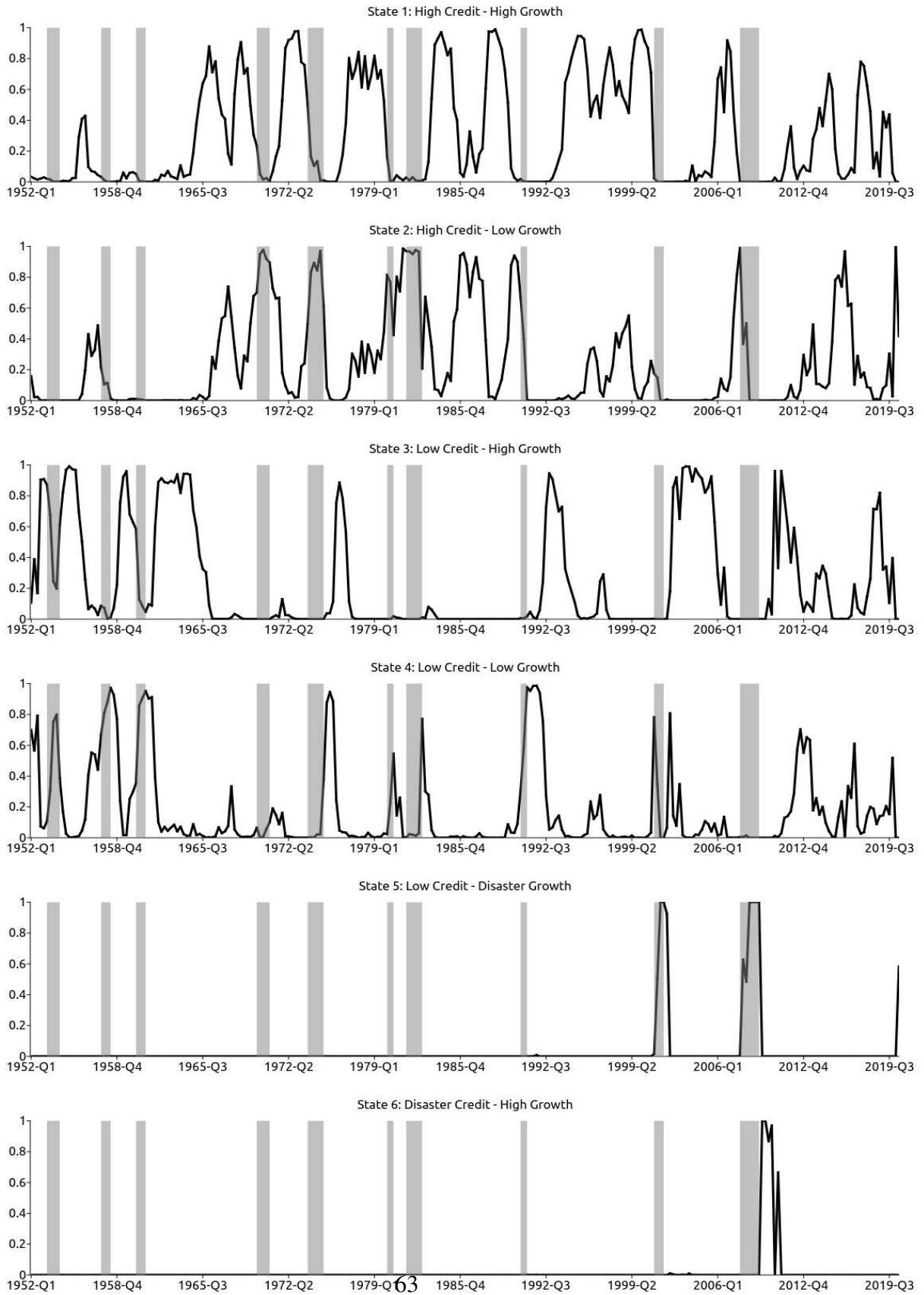State 6: Disaster Credit - High Growth

Figure 11: Data Approximated and Model (With CRA) Zero Upfront Spreads on CDO Tranches In-Sample (1952 – 2017) and Out-of-Sample(2018 – 2020:2)
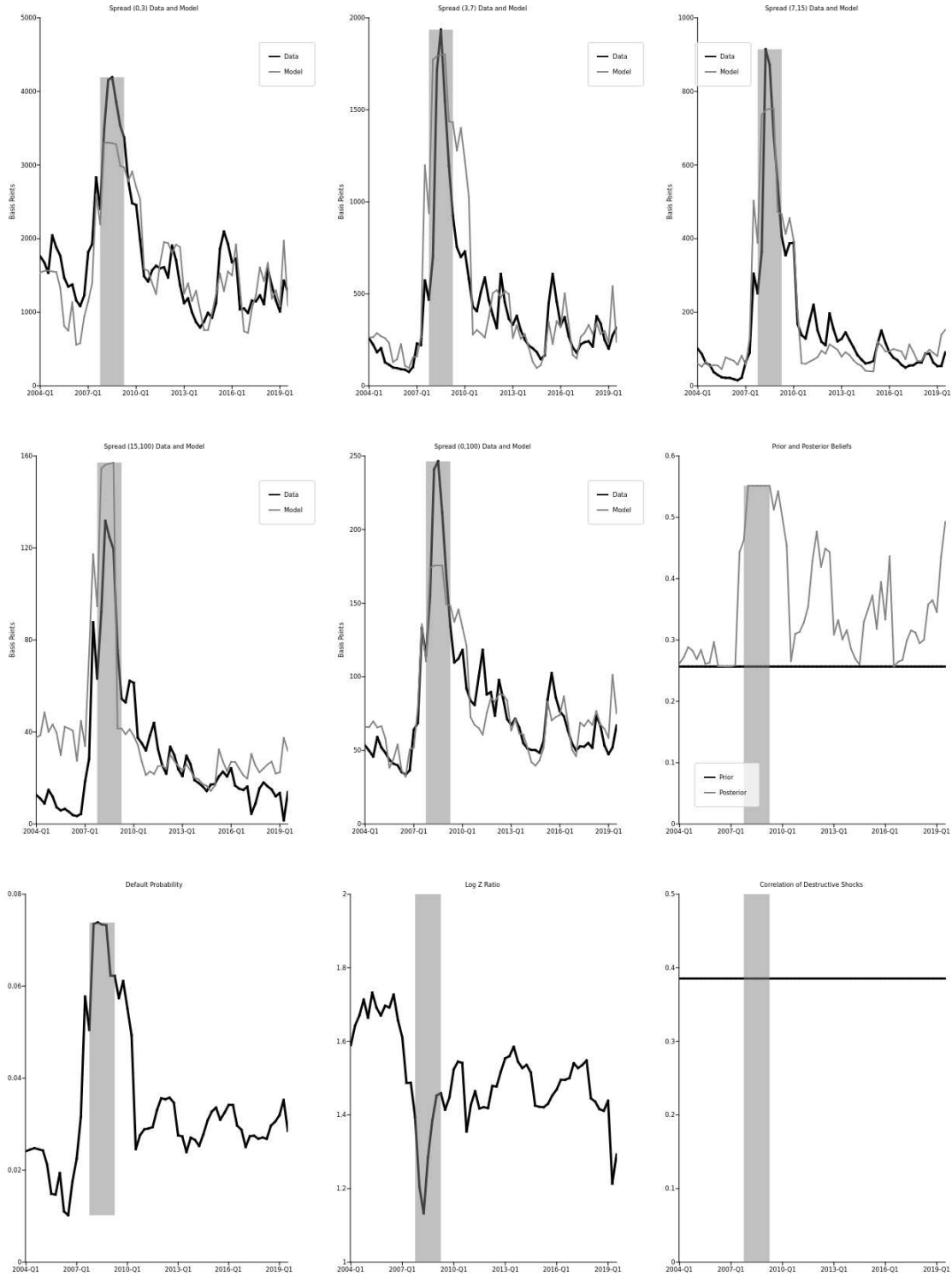
Figure 12: ZUV Spread (0,3) Approximated by our Model and by Coval, Jurek, and Stafford (2009a)

Zero Upfront Variable Spread For (0,3) Tranche

Approximated by our Model
Approximated by Coval et. al.