Bank Asset Problem and Financial Stability*

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Abstract

We propose a bank asset problem and analyze its impacts on the banking system. The bank loan business in general equilibrium is that some loans become non-performing held by some banks. However, removing non-performing loans becomes unconditional if a bank's regulatory capital is compromised. We model non-performing loans on the bank balance sheet with the following inputs: good or bad news on loan status, early or patient bank lenders, no panic-based or panic-based bank runs, orderly or distressed fire-sale discounts, heterogeneous bank size, conditional lender of last resort, and fair-value mark-to-market accounting guidance. Our model has identified an asset recovery switch. After liquidating non-performing loans, a bank can keep business as usual upon a rapid asset recovery. Conversely, a slow asset recovery causes panic bank runs. Banks with capital reserves as the acquirer offer deep discounts, and a small bank will be liquidated. The Silicon Valley bank's bankruptcy and acquisition is one of the examples. On the other end of the bank size spectrum, systemically important banks (SIBs) are collectively under capital constraints when the costs of non-performing loans exhaust capital buffers, even if the central bank can neutralize shocks of bank runs. More critically, financial stability is compromised because small banks do not have capital buffer. Our model also predicts two abnormal operations. Libor scandal and bank capital manipulation. Large banks report higher equity capital to the FED to cover up noncompliance with the Basel capital requirements in 2008 yet truthfully report to SEC to avoid litigation risk.

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*Comments are welcome

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1 Introduction

Banks perform maturity and risk transformation by borrowing from household deposits and lending to risk-taking entrepreneurs. The literature has advanced the frontier on bank functions and financial crises. For a comprehensive survey, refer to two papers [1]. Among others, the Nobel Committee cites two theoretical contributions in Diamond and Dybvig (1983) and Diamond (1984) and an empirical contribution in Bernanke (1983).

Banks are vulnerable to bank runs. Panic runs can cause even fundamentally healthy banks to fail. Therefore, deposit insurance and government liquidity are justified. Because bank failure by cutting loan supply hurts real economic activities, policy responses aim to maintain credit availability to all borrowers during the 2008 global financial crisis. In a nutshell, the root of the problem and the solutions are on the side of bank liabilities.

We propose to study a problem on the bank *asset* side. The extension of the literature can be summarized as follows. Diamond (1984) has implied a general equilibrium of having bad loans in the banking system, and the cost subsidizing helps to reduce borrowing costs. This study addresses the question of how many bad loans are too many. We show that the bank asset problem is independent of bank runs, either panic-based or no panic-based. More important, the bank asset problem can cause bank runs. Externalities range from leading to small bank liquidation to compromising financial stability.

To study bank asset problems, our study is motivated by two banking crises. We summarize each case with five stylized facts. The first case was the collapse of Silicon Valley Bank (SVB) in 2023Q1. The second refers to a process in which a group of systemically important U.S. banks (SIBs) had collectively developed capital constrained during the 2008 financial crisis. However, the outcomes of the two cases are different. We develop a model for the general asset problem and rationalize the different outcomes for small or large banks.

To explain the bankruptcy of SVB, the puzzle is why bank runs begin after the bank

[1] Financial Intermediation and the Economy, Advanced information Scientific Background. Anil Kashyap's article Explaining the Rationale for the 2022 Nobel Prize in Economics.

has liquidated all its assets reporting unrealized losses. The puzzle of the 22 large U.S. banks[2] points to the opposite direction. They kept adding non-performing assets to balance sheets from 2007Q3 to 2009Q1. Although forced to downsize and recapitalize in 2009Q2 and onward, they walked out of the 2008 crisis unscathed. The undeniable fact is that these SIBs compromised financial stability in 2009.

Our analysis stems from one of the central insights in Diamond (1984). Delegated monitoring can reduce borrowing costs. Because a bank must maintain a large and diversified loan portfolio, it is implied that the intermediary bank must allocate profits of performing loans to subside the losses of non-performing loans. We focus on the costs associated with non-performing loans and address three questions. First, how does the bank asset problem affect the core bank functions of maturity and risk transformation? Second, what is the role of the bank asset problem in small bank liquidation? Third, being plagued by an asset problem, what can the large bank do? How could large banks plagued by the problem compromise financial stability?

From the literature arsenal, we borrow the following tools. Panic-based runs, and early and patient households as bank lenders are in Diamond and Dybvig (1983). Banks originate many loans to address the monitoring-the-monitor problem in Diamond (1984). The cost of credit intermediation is from Bernanke (1983, 2023). The general equilibrium of asset sales is in Shleifer and Vishny (1992). No panic-based bank runs are from Allen and Gale (1998).

The main difference is that banks in this study practice *two-tier* future assets maximization. Each bank operation includes two considerations. Unconditionally, the transformations of risk and maturity aim to improve the efficiency of debt intermediaries, as in Diamond and Dybvig (1983) and Diamond (1984). When originating each bank loan, the bank maximizes the utility of the bank, instead of consumers and entrepreneurs.

The two-tier maximization is better understood through two financial contracts. Every bank intermediation involves two contracts. In the first lender-borrower contract, households

[2]The average assets in 2006 is $\$537bn$.

are the lender and the bank is the borrower. In the second contract, the bank is the lender and the borrowers are entrepreneurs. The nature of business operations of entrepreneurs is risk taking, which can fail. Therefore, the bank that is fully committed to its lenders in the first contract must be active to reject borrowing demand from business owners that are unlikely to meet their debt service obligations in the second contract.

Therefore, a bank's choices to maximize its utility are different from what is suggested by the mathematical formula. Because reducing borrowing costs is good for the economy, the time T should be a finite large number. Banks cannot cherry pick a loan's (μ) and (σ) unless the borrower's income generating capacity is of concern. Therefore, every bank maxes out its capital to make as many loans as possible with its risk-based capital in compliance with Basel requirements.

Acknowledging an equilibrium that part of the loan portfolio of some banks is non-performing, this study focuses on the non-performing loans. A bank can keep non-performing loans if the net income of performing loans can absorb the cost of its non-performing loan. However, a bank must sell its non-performing loans if the loans jeopardize its Basel capital ratio in the next period. How does liquidating non-performing loans cause a bank liquidation?

We identify a cause that we term *slow asset recovery.* To model slow asset recovery, we define loan news, bank runs, and fire-sale discounts following the literature. Loan news can be good or bad. Households can be early or patient bank lenders and can change from patient to early if they are hit by repeated bad news. Bank runs can be panic-based or no panic-based. Fire-sale discounts can be orderly or forced. We do not include deposit insurance, so we can demonstrate the impacts of panic-based bank runs. For simplicity, we do not model the tax either. Operating loan business, no news is good news. However, two loan situations are bad news.

After liquidating non-performing loans at t, a bank must use its risk-based capital to absorb liquidation costs, including fire-sale discounts, deposit principal and associated interest rates, cost of credit intermediation, and other operating costs. Fire-sale discounts have two unique characteristics compared to those in Shleifer and Vishny (1992). Fire sales are caused by regulatory capital requirements, not to meet debt payments. The opportunity cost of capital, rather than the re-deployable asset, drives the fire-sale discounts.

After a bank removes non-performing loans and residual standby capital cannot make a new loan, early bank lenders withdraw their deposits. Such withdrawals can escalate to no panic-based bank runs if the proceeds from liquidation cannot cover the liquidation costs. The result is bank liquidation. If a bank can survive household withdrawals, the bank has only performing loans on the balance sheet and satisfies regulatory capital requirements. The rest of the bank lenders will wait for the next update.

If the performing loans can generate enough income, together with the standby capital after liquidating the non-performing loans, the bank can recover its loan portfolio in the next period. Because the bank has demonstrated rapid asset recovery, patient lenders remain patient. They roll over their deposits, again.

We demonstrate slow asset recovery if a bank shows weak profit-generating capacity after suffering capital losses. Slow asset recovery shares two characteristics. In period t, after paying the liquidation costs of the bad loans, the residual standby capital is insufficient to replace a new loan in the same period t. In $t+1$, with the income from performing loans, the bank cannot recover its assets in t. Patient lenders lose confidence and withdraw rather than roll over. Such withdrawals are panic-based for two reasons. The bank loan portfolio is performing; the bank capital satisfies the Basel capital adequacy requirements.

In this study, we explain panic by rational fear hit by bad news twice. Slow asset recovery demonstrates the mechanism for bank runs on the Silicon Valley Bank. These are our stage 1 results. The results of stage 1 justify two bank run solutions ($BRSs$) and three fire sale solutions ($FSSs$).

$BRS1$: Liquidating non-performing loans is necessary and sufficient to prevent no panic-based bank runs, but only necessary for panic runs. $BRS2$: Central bank liquidity can neutralize the liquidity shocks from panic bank runs. However, due to moral hazard concerns,

$BRS2$ applies only to SIBs. However, liquidating non-performing loans causes fire-sale discounts. $FSSs$ are to address fire-sale discounts.

$FSS1$: bank loan borrower pledges overcollateralization equal to two missing coupon payments. $FSS2$: In the small-large banking system, the large bank maintains a capital buffer. $FSS3$: Fire-sale transactions can be orderly or forced. The difference is that discounts for the former are smaller. According to FASB, banks can mark the fair value of their balance sheet assets against orderly discounts when recent market transactions are forced.

Our analysis indicates the side effect of $FSS3$. A bank cannot liquidate its non-performing loans because its ask price is higher than recent market transaction prices. The misalignment effect facilitates the development of asset problems among large banks.

When liquidating non-performing loans, excessive fire-sale discounts are not a surprise. Our analysis identifies two disadvantages of the small bank. First, the lender of last resort does not support the small bank. Second, the small bank is plagued by slow asset recovery due to its small performing loans. Both are why a small bank is more likely to be liquidated.

Our analysis predicts five large bank operations. Operation 1: To maximize its utility, the large bank offers deep discounts in acquisition when the troubled small bank is on sale[3]. Operation 2: Write down bad collateral loans. Operation 3: adding non-performing loans with good collateral, expecting excessive discounts to go back to normal in the near future.

Operation 3 is a double-sided sword. $FSS3$ allows the bank to mark the assets upward in period $t + 1$. However, the large bank cannot sell the assets due to the misalignment effect. When holding, capital buffer must pay for the funding cost of non-performing loans because the latter do not generate income. Therefore, the capital buffer is stretched to cover two

[3]I refer to three examples. (a) J.P. Morgan acquiring Bear Stearn, the final offer price is \$10 per share as compared to \$32 per share close before acquisition. Kelly, Kate. "The Fall of Bear Stearns: Bear Stearns Neared Collapse Twice in Frenzied Last Days; Paulson Pushed Low-Ball Bid," Wall Street Journal, 29 May 2008. (b) Mr. Robert E. Diamond Jr. of Barclays paid \$250 million for Lehman's business with trading assets of \$72 billion and liabilities of \$68 billion. Randall Smith, Diya Gullapalli, and Jeffrey McCracken. "Lehman, Workers Score Reprieve; Barclays's President Agrees to Buy Bulk Of the Prime Assets, Less the Risky Ones" Wall Street Journal, September 17, 2008. (c) First Citizens bought around \$72 billion of Silicon Valley Bank's assets at a discount of \$16.5 billion. Peter Hoskins & Nick Edser. "Silicon Valley Bank: Collapsed US lender bought by rival" BBC, 27 March 2023

functions. Replenishing equity capital is the official role, but paying funding costs inevitably compromises the priority function.

Unfortunately, the large bank develops capital constrained once accumulative financing costs exhaust its capital buffer. As a result, the large bank must reduce its risky assets and issue new equity. Reducing the balance sheet of the large bank compromises financial stability because the small bank lacks the capital to fill the service vacuum.

Moreover, our analysis predicts two more abnormal operations of the large bank. Operation 4: Manipulate the market Libor to increase its income, known as the Libor scandal in the UK. Operation 5: Report a higher total equity capital to the Fed than to the SEC to comply with regulatory capital requirements while avoiding litigation risk.

Here is the contribution of this study. We have identified a problem on the bank asset side. The bank asset problem is independent of bank runs, where the problems are on the bank liability side. Furthermore, the bank asset problem can cause bank runs and a variety of negative impacts, from causing small bank liquidation to compromising financial stability.

Rooted in the general equilibrium in which the banking system holds non-performing loans in Diamond (1984), this study identifies the "switch", the profit from the performing loans and the non-performing assets in a bank's balance sheet. A bank can continue business as usual on the good flip. However, the switch can be turned to the bad side under two conditions. A bank must liquidate non-performing assets to comply with the Basel capital requirements. The next income from the performing loans is lower than the liquidation costs. Slow asset recovery rationalizes panic-based bank runs.

Capital and liquidity buffers are effective for addressing bank liability side problems. To address asset problems, we suggest the revenue buffer. In Chu and Ou (2022), we empirically explore the cross-borrower spillovers of the bank asset problem.

The remainder of the paper is organized as follows. We describe two cases of the asset problem in 2023 and 2008 and set up our model in Section 2. In Section 3, we model the impacts of a bank asset problem on the small bank, the large bank, and the banking system.

We discuss the policy recommendation in Section 4 and conclude in Section 5.

2 Bank Asset Problem, Model Framework, and Setup

We propose a bank asset problem that could cause widespread shocks. We develop a model to analyze the shock development mechanism. The bank asset problem and the model we will develop in this study are motivated by the stylized facts of two banking crises summarized in Section 2.1. We define the bank asset problem on three elements.

The first element is the general equilibrium of non-performing assets, or GEONPA for abbreviation. In this study, the bank assets are bank loans. We distill the term from the second central insight of Diamond (1984)[4]. GEONPA refers to the general equilibrium fact that, in the banking system at any time, some loans held by some banks become non-performing. I must mention that GEONPA does not imply or indicate the shirk of a bank. In fact, banks practice credit screening as part of their two-tier utility maximization, defined in Section 2.3. Credit screening with due diligence can reduce the odds of but cannot completely avoid non-performing loans. This is simply due to uncertain future operating income, which is the nature of risk-taking business operations. Having non-performing loans is socially optimal because borrowing costs can be reduced to an affordable level in the economy.

The second element is the GEONPA switch, and the switch has a good or bad flip. When the switch is on the good side, as assumed in Diamond (1984), banks can continue their functions of maturity and risk transformation with bad loans. However, we are interested in when the switch is flipped from the good side to the bad side. When bad loans reach a certain point, a bank must reduce its risk and maturity transformation functions, in general, and specifically the bad loans.

[4]There are two central insights in Diamond (1984). The first is the delegated bank monitoring. To efficiently channel household savings to support the risk taking of entrepreneurs, households must hire a specialist named banker. However, the intermediary bank leads to a follow-up question. How do households monitor banks, the delegated specialist? The second insight is quite novel. The bank should be large and originate many loans. In perfect diversification, the bank will never fail and the foreclosure of the borrowers will never happen

The third element addresses the question of why a bank must sell its bad loans. Bank runs, panic-based or no panic-based, cause banks to sell their assets but are not what is investigated in this study. We are interested in the bank asset problem. This is due to the regulatory capital requirements. Bans must address the bad side of the GEONPA switch to satisfy the Basel capital adequacy ratio. When Diamond (1984) was published, there were no regulatory capital requirements. The Basel I: the Basel Capital Accord was published in 1998 and enforced by law in the Group of Ten countries eight years later in 1992.

Let us clarify three different reasons that a bank must sell its assets. The first two reasons are panic runs by Diamond and Dybvig (1983) and no panic runs by Allen and Gale (1998). Either type of bank runs causes problems on the bank liability side. The third reason for a bank asset problem is a bank's concern that it won't satisfy the regulatory capital requirements due to the bad loans on its balance sheet. Bank asset problem is independent of bank liability problems. Specifically, we are interested in the liquidation decisions when neither bank run has started. Furthermore, as we will show, the bank asset problem will cause bank runs of panic- or no panic-based.

Here are five stylized facts for two cases of bank asset problems. The first case is the collapse and acquisition of Silicon Valley Bank. The second case refers to a group of systemically important banks and their asset changes during the 2008 financial crisis.

2.1 Motivation: two cases of bank asset problem

Case A: bank asset problem and liquidation of midsized banks

Case A is about the liquidation of a midsize bank. The five stylized facts we aim to model are the bankruptcy and the acquisition of Silicon Valley Bank (SVB). (a) From 2021 to 2022, SVB invested in long-term Treasury and mortgage bonds due to the influx of large deposits. (b) In 2022, SVB started to record unrealized losses of long-term bonds due to the rising Federal funds rate. However, SVB met Basel capital requirements. (c) On March 8, 2023, SVB removed long-term bonds that reported unrealized losses, announced a $1.8

billion loss, and planned to raise $2 billion equity. (d) On March 9, 2023, CEO Becker stressed that the bank is well capitalized and has a high quality balance sheet. (e) However, panic bank runs accelerated. Regulators took action on March 12, 2023, and SVB depositors were under FDIC protection. On March 26, 2023, First Citizens BancShares acquired SVB's clean assets at a huge discount[5].

Case B: bank asset problem and capital-constrained SIBs

Case B is about capital constrained SIBs. We turn to the balance sheet activities of 22 large U.S. banks from 2006Q4 to 2009Q4. The 22 large U.S. banks are part of 97 leading intermediaries identified in the follow-up empirical study Chu and Ou (2022), which tests cross-borrower spillovers, one of the predictions of this study. In Table I, we collect balance sheet data entries from FR Y-9C and 10-Q for the 22 large US banks[6].

The 22 banks share US domicile and report FR Y-9C to the Federal Reserves, in addition to 10-Q to the SEC. The 22 large US banks reported average assets (Compustat AT) of $537 billion at the end of 2006, higher than the bar of $250 billion for the SIBs. The 97 intermediaries account for more than 70% of the total assets of the finance sector (SICCD 6000-6999) with more than 1,800 financial institutions in 2006. Meanwhile, the 97 intermediaries account for more than 80% of the debt intermediary service on syndicate loans, credit lines, and corporate bonds from 2002 to 2006[7].

[5]Bloomberg: SVB's 44-hour collapse was rooted in treasury bets during pandemic, March 12, 2023. ABC News: A timeline of the Silicon Valley Bank collapse, March 15, 2023.Collapse of Silicon Valley Bank at wikipedia

[6]Data sources are FR Y-9C and Compustat. In column (1), we add BHCKA223 from 22 large banks and set the sum for 2006Q4 at 100. Therefore, the number 137 in 2009Q1 indicates that the 22 banks cumulatively added 37 percentage points of new risky assets to their balance sheets from 2006Q4 to 2009Q1. We repeat the exercise from column (1, BHCKA223) to column (6, BHCK3459). Column (5) is the calculated book equity (EC), a sum of 5 items: BHCK3283, 3230, 3240, B530, and A130 in FR Y-9C. Or it equals column (3) minus column (4). Columns (7) and (8) are equal-weighted ratios of the 22 large banks.

[7]We merge 97 leading intermediaries into FR Y-9C through bank names on the Federal Financial Institutions Examination Council (FFIEC). We collect quarterly balance sheet data to analyze bank operations in compliance with Basel capital requirements. There are 27 initial matches, and we drop further 5 institutions after checking the consistency of the data. We compare the common equity–total in Compustat (CEQQ) and the total equity capital in FR Y-9C (BHCK3210). The 5 institutions dropped are Barclays Group U.S. Inc., HSBC North America Holdings Inc., Deutsche Bank USA Corporation/Taunus Corporation, and ABN AMRO North America Holding Company because the BHCK3210/CEQQ<33% from 2002 to 2006 as well

Here are the five stylized facts. (a) Banks increase their maturity transformation, so the risk-weighted assets increase on their balance sheets; see column (1) from 2007Q1. (b) The added bank assets do not perform, so the retained earnings peak and then start to decrease; see column (4) since 2007Q4. (c) Banks satisfy the Basel capital ratio throughout the window. See column (7). Therefore, they must commit more equity capital. (d) However, the sources of bank capital are unclear. (e) Banks collectively faced capital constraints from 2009Q2 to 2009Q4. They reduced their assets and issued new common shares. See columns (1) and (6) from 2009Q2 to 2009Q4.

Here are two details of the stylized fact (d). First, from 2007Q4 to 2009Q1, common shares increased 19 percentage points, but reported book equity increased 153 percentage points. See column (5) and (6). The gap is too wide to be explained by the capital buffer.

Here is the second detail of stylized fact (d). There exists a reporting discrepancy between two government agencies. Refer to Table I, panel A, column (8). Total equity capital is reported as BHCK3210 in FR Y-9C and common equity total (CEQQ) in Compustat (SEC, 10-Q). Being the same accounting item filed with the Federal Reserve (Fed) and the SEC, only the item in FR Y-9C limits the bank's risk taking. We calculate *ratio of total equity capital* (BHCK3210 in FR Y-9C divided by CEQQ in 10-Q). The ratio is calculated at each bank's quarter level and equally weighted across 22 large banks. The ratio ($BHCK3210/CEQQ$) should be equal to one, and the number has been very stable at 98% from 2002 to 2007Q2.

The ratio increased to 101% in 2007Q3, and the discrepancy has two characteristics. (1) Whenever there is a discrepancy, the capital reported to the Fed is always higher than that reported to the SEC. (2) The discrepancy was not noticeable until 2007Q3, the quarter in which retained earnings peaked. The discrepancy has increased and peaked in 2008Q4 and 2009Q1. Equity capital reported to the Fed was more than 30 percentage points higher than reported to the SEC. The discrepancy is sizeable because one (1) percentage point equals

as UnionBanCal Corp/MUFG Americas Holdings Corporation. The total assets of the 22 FR Y-9C large banks account for approximately 20% of the total assets of all institutions in the financial sector (SICCD 6000-6999).

7.2 billion dollars for the equity capital of the 22 large banks in 2006Q4.

To rationalize the stylized facts for both cases, we model the bank utility. We borrow the following tools from the literature[8]. We apply tools for maturity and risk transformation, early and patient bank lenders, and panic-based bank runs in Diamond and Dybvig (1983). Financial intermediation can reduce the cost of monitoring loan borrowers Diamond (1984). The intermediary service comes at a cost, identified as the cost of credit intermediation (CCI) in Bernanke (1983, 2023). The general equilibrium of asset sales is from Shleifer and Vishny (1992). A no panic-based bank run is from Allen and Gale (1998).

CCI is defined as the costs of channeling funds from household savings to end borrowers, net of risk-free interest rates. We identify two cost components of CCI. Component 1: Practice active banking service. A bank is active if it screens loan applications and only grants access to qualified applicants. See Chu and Xiao (2023) for more details on why banks reject liquidity demands that most need external debt. Component 2: Pay households and monitor borrowers. When signing a bank loan contract, a bank must sign a household-bank contract in which the bank is the borrower. There are two costs in two contracts. Banks must pay households and devote resources to monitor borrowers.

2.2 Our model framework and building blocks

In this section, we demonstrate the intuitions of our model building blocks on how we understand the bank asset problem. In each building block, we have two choices. We apply a building block that has been proposed in Diamond (1984); Diamond and Dybvig (1983) if we agree with it. To be consistent with the narrative, we follow some terms in Diamond (2023) in the bullet points. In the second choice, we propose our independent thoughts. We will list both arguments if ours is different from that in the literature. We report a building block that we think is necessary, but is not found in the literature.

[8] We follow a chronological year order for citations. For three papers published in 1983 and 1984, we follow the order of Diamond and Dybvig (1983), Diamond (1984), and Bernanke (1983) in the scientific background paper titled "Financial Intermediation and the Economy" by the Committee for the Prize in Economic Sciences in Memory of Alfred Nobel in 2022.

2.2.1 CRRA utility function and an agent's utility

- In (Diamond, 2023, page 2608), risk-averse investors (households as the bank lender in this study) prefer liquid assets to illiquid assets.

We add the following extension. There are three economic agents in this study: entrepreneurs as bank borrowers, bankers, and households as bank lenders. All agents are risk-averse and recognize the same CRRA utility function. Entrepreneurs run businesses to produce products or offer services. Banks originate loans. Both business operations of enterprises and banks are risk-taking. I interpret that CRRA indicates the *risk attitude* of the economic agent toward the returns of a risk-taking asset or project. By human nature, all agents prefer more over less.

Households, like two other agents, prefer an asset with a higher certainty equivalent return. This extension allows us to model that households prefer term deposits to Treasury; the interest rate of the former is higher than the latter. Lending to banks (term deposits) is also riskier than purchasing Treasury because bank operations are risk-taking.

- Banks maximize the utility of consumers in Diamond and Dybvig (1983) and the utility of entrepreneurs in Diamond (1984).

In this study, utility is different from a utility function. The utility (or happiness, or best interest) of an agent is to maximize the agent's long-term assets. Each agent maximizes *its own* utility, not someone else. Because transformation of risk and maturity is good for the economy, banks practice two-tier utility maximization, as we define in Section 2.3.

2.2.2 Conflicts of interest in lending and delegated monitoring

- In Diamond (1984), the main conflict of interest between a borrower and a lender is how to get the borrower to pay back. The answer is *delegated monitoring.* In Diamond (2023), delegated monitoring and its benefits can be summarized in the following five characteristics. (A) Delegated monitoring ensures that borrowers pay back to banks.

> For example, a bank originates many loans. Exactly half of the borrowers pay 2 and the rest half pay 1. The expected payments to the bank will be 1.5. (B) A debt contract between banks and investors promises to pay the latter 1.05. (C) The 0.45 residual income incentivizes the bank to monitor so that the bank can collect payments higher than 1.05. (D) In perfect diversification, the bank will never fail. (E) Borrower foreclosure will only be a threat but will never be implemented.

We propose the building block of the delegated monitoring *with a bank asset problem*[9]. This building block has three pillars. First, the bank asset problem could affect (A). We refer borrowers who pay 2 as good borrowers and those who pay 1 as bad borrowers. The bank asset problem is that bad borrowers may pay much less than 1. If bad borrowers pay no more than 0.1, with (B) unchanged, the incentive for bank monitoring drops to zero in (C). Without monitoring, even good borrowers may pay less than 2. Without keeping its promise to investors, the bank will fail in (D) even starting with an ex ante perfectly diversified loan portfolio. Borrower foreclosure will occur in (E).

Why is it well grounded that bad borrowers could pay less than 1? This is due to the nature of risk-taking business operations that generate stochastic corporate revenues. Business owners will default on their loan payment obligations when their business operations fail. Therefore, banks must be active in rejecting debt demand applications with a poor track record of business operations. This is consistent with "a sound credit granting process" in BIS bcbsc125.

Even if starting as a good loan (paying debt obligations on time in full amount), the loan can change to a bad loan (missing coupon payments) before maturity. Again, the change is due to the same reason as stochastic corporate revenues. Furthermore, it is a general equilibrium that some loans on the balance sheet of some banks have changed from good at $t-1$ to bad at t.

Once a loan has changed from good to bad, we want to draw attention to the second

[9]We start with the term "delegated monitoring" to homage Diamond's contribution to this literature.

pillar of our building block. How long can a bank keep a bad loan on its balance sheet? In (A), the loan the borrowers pay 1 is a bad loan. This is because investors lend 1 to a bank and expect to receive 1.05 from the bank. When borrowers pay the bank 1 after they borrowed 1 for one period, this loan is a bad loan or non-performing loan. The reason why the bank can keep the bad loan on the balance sheet is that half of the borrowers pay 2.

What if the bank cannot receive the 2 from other borrowers? This question is not trivial because borrowers who have signed loan contracts with higher coupons have a higher probability of default. Because the transformation of risk and maturity is good for the economy, the life of a bank should be as long as possible. Therefore, the bank is certainly exposed to bad loans at times.

The third pillar, the regulatory capital requirements, addresses the question of monitoring the monitor. As we will analyze in Section 2.3, maxing out a bank's capital to originate as many qualified loans as possible becomes the most popular choice for most banks to maximize their long-term assets. If borrowers are not monitored, some loans become bad. The bank risks not complying with the Basel capital ratio. This is bad news that can cause bank runs.

2.2.3 Bank asset problem and bank runs

- Bank runs in Diamond and Dybvig (1983) are caused by self-fulfilling prophecy, which can cause more depositors to withdraw in one period than the bank liability structure of one period versus two periods has designed for. Bank runs can bring down a bank with all good loans.

The bank asset problem in our analytical framework has altered the foundation of all good loans in Diamond and Dybvig (1983). While bank runs driven by a self-fulfilling prophecy still work in our setting, we have to analyze the new situation. How does the bank asset problem change the decisions of early or patient bank lenders?

We observe that households as an economic agent are less wealthy than the two other agents of bankers and entrepreneurs, either good or bad, in the social pyramid. Entrepreneurs and bankers practice risk-taking operations to reach for higher returns. Here is our assumption. Households with limited wealth do not practice risk-taking operations. This assumption is consistent with the well-recognized stylized fact that households with assets below the 70^{th} percentile distribution do not own public equity Campbell (2006). Their main income comes from salary.

If households have savings, they can invest in the Treasury or in a term deposit. Therefore, the bank must offer a term deposit contract with an interest rate measured by certainty equivalent returns higher than that of the Treasury. In this case, risk-averse households prefer term deposits to the Treasury. In our analysis in Section 3.2, the interest rate for term deposits is 4% and the Treasury return is 2%.

Because households are lower in the social pyramid, they are in a disadvantageous position in the lender-borrower contract with banks. Therefore, they choose to invest 1 saving as a term deposit for one period but are happy to roll over the term deposit to the next period if they do not hear bad news on bank loans. If they hear bad news, they will withdraw their savings in the same period, and the bank must honor the early withdrawal requests.

Similarly to Diamond and Dybvig (1983), we have households as early and patient bank lenders. Furthermore, the duration of the bank loan portfolio is longer than the maturity of the term deposits. Here are the differences. Due to their disadvantageous position in the financial contract, every household is an early bank lender. They set the maturity of the term deposit equal to one period. If they hear bad news during this period, they withdraw their deposit as planned. They invest the withdrawal proceeds in the Treasury. However, they are happy to roll over the term deposit to the next period without bad loan news.

We make the above assumptions on the condition that the bank is fully transparent on its loan quality status, bad or good. In reality, our assumptions are more natural than the early and late investors in Diamond and Dybvig (1983); Diamond (2023) on two fronts. From the perspective of households, every household is opportunistic if the family can only choose between term deposits and the Treasury. This is due to human nature that more is better than less. From the perspective of banks, we rarely see that a bank claims that 25% of its time deposits are one period and the rest of the 75% deposits are multiperiods. In fact, that SVB assumes the maturity of its term deposits to be 7 years is one incorrect assumption that leads to its collapse and acquisition.

2.3 Model bank future assets

While we are deeply indebted to the insights of Diamond and Dybvig that banks improve the efficiency of debt intermediaries, here is our extended assumption. Banks in this study practice *two-tier* utility maximization. Every bank operation is the result of two considerations. Ex ante, the transformations of risk and maturity aim to improve the efficiency of debt intermediaries. When a bank executes each bank operation, the bank maximizes the utility of the bank, instead of consumers or entrepreneurs.

We summarize three characteristics of two-tier maximization. Characteristic 1: Banks do not cherry pick debt requests submitted by business owners. Unlike entrepreneurs who select high return projects, banks accept all liquidity demand requests and give them full consideration. This operation enables them to maintain a portfolio of diversified loans. Characteristic 2: Banks grant access to some, but reject other applications after reviewing all received applications with due diligence. Characteristic 2 specifies the bank's own interest. Characteristic 3: To maximize long-term assets, banks should max out bank capital to originate as many loans as possible while satisfying regulatory capital requirements. Characteristic 3 fulfills the first two characteristics together. However, we will reveal the challenge for the banking system if every bank is maxing out its capital to achieve growth.

It is easy to appreciate that rejections are the result of the bank utility maximization. However, rejections on a sound economic ground are necessary to maintain a bank's long-term commitment to improve the efficiency of debt intermediaries. Justified rejections can improve the quality of borrowers to whom banks originate loans. For more details on justified rejections, please refer to Chu and Xiao (2023).

Even if banks have practiced due diligence screening, bank loans can change their status from good loans to bad loans, not paying debt obligations. This is a general equilibrium of the bank loan business. This study is devoted to the negative effects of bad loans and how to address them.

In my humble opinion, that a bank, an entrepreneur, or a household keeps its independence and maximizes its utility sets the cornerstone for ex ante fair financial contracts. Let us take the point of view of a bank. Each debt intermediary business includes two financial contracts. In the contract with households as bank depositors, banks are borrowers. In the bank loan contracts with entrepreneurs, banks are lenders. Each contract must be fair to both parties; otherwise, the contract will not last, ex ante.

However, it is possible to break a contract even if the contracts have been fairly established ex ante. Bank runs, panic or no panic based, break the contract where banks are the borrowers. The literature has many studies on this category of banking crisis.

$$NI_t = AT_t \times (BL_{bwr} - \beta FL_{td,t} - CCI - OpEx).\ CCI = \theta;\ OpEx = \eta. \tag{1}$$

$$(RE + EC)_t = (RE + EC)_{t-1} + NI_t. \tag{2}$$

$$BaselR = \frac{\textit{Total risk-based capital}}{\textit{Total risk-weighted assets}} = \frac{RE_t + EC_t}{AT_t}. \tag{3}$$

$$ROA_t = \frac{NI_t}{AT_t};\ dROA_t = \mu dt + \sigma dW_t. \tag{4}$$

$$\begin{aligned} \max_{\mu,\,\sigma} \quad & E\left[AT_0 \times (1 + ROA_t)^T\right] \\ \text{s.t.} \quad & \text{insufficient } \beta,\ BL_{bwr} \le FL_t, \\ & \text{Basel capital requirements, at every } t. \end{aligned} \tag{5}$$

We will study contract breaking where the entrepreneur default events are the origin of the shock to the bank-entrepreneur (with the identity of lender-borrower) contract. Meanwhile, there is no bank run in the household-bank contract. We will demonstrate the mechanism through which entrepreneurs' defaults can shock the continuation of the household-bank contract. Because bank runs are caused by a breach of the bank-entrepreneur contract, we will analyze the potential solutions and possible outcomes.

Our model starts with the system of equations (1) to (5) that describe how a bank grows its future assets. The core of asset growth is to make profits, as in Equation (1). Operating profits will increase the total risk-based capital, Equation (2). We assume that the Basel capital ratio ($BaselR$) is constant. Therefore, the bank originates new loans due to a higher bank capital, in Equation (3), and grows its assets.

Net income (NI_t) is the profit generated by existing bank loans. To generate profits, a bank must borrow low from households and lend high to corporations through two separate contracts. In the household-bank contract, banks borrow from households and pay interest rates on term deposits $FL_{td,t}$. In the situation of bank runs where banks cannot borrow from households, banks must access alternative funding liquidities such as market Libor ($FL_{libor,t}$) or central bank liquidity ($FL_{cbl,t}$). We use the manipulated Libor ($FL_{mlibor,t}$) to model the Libor scandal. Households can access two financial products: Treasury and term deposits. Because the bank loan business is risk taking, $FL_{td,t}$ is higher than the Treasury risk-free rate $FL_{td,t} > r_{rf}$. Households also maximize their long-term future assets, growing their wealth from current savings. We will set the criteria for household choices of financial products in Section 2.5.

In the bank-entrepreneur contract, BL_{bwr} is the loan coupon rates that banks charge business owners. We assume that risk-taking projects undertaken by entrepreneurs generate returns and associated risks at different levels. Therefore, a bank charges varying BL^j_{bwr} on different projects. Because the duration of bank loans is longer than that of bank funding liquidity $FL_{td,t}$, BL^j_{bwr} does not have a subscript t.

To originate bank loans in period t, a bank must borrow β of AT_t from households, where $\beta = 1 - BaselR$. The $AT_t \times \beta FL_{td,t}$ is the cost component of CCI paid to households. We measure monitoring costs by (θ). Operating expenses ($OpEx$), measured by (η), include the costs of bank rejecting unqualified borrowing applications. Three types of $OpEx$ are related to salary compensation, office, and sales and marketing. Because it is a continuous exercise rejecting unqualified liquidity demands and monitoring existing borrowers, both monitoring and operating costs are constant (θ and η) proportional to the assets of a bank.

Following FR Y-9C, we decompose the total risk-based capital into two balance sheet entries: retained earnings (RE) and book equity capital (henceforth equity capital, EC). $(RE + EC)_{t-1}$ in the previous period will be updated by net income (NI_t), which could be positive or negative. Due to constant capital requirements ($BaselR$), the updated $RE_t + EC_t$ decides whether a bank can increase assets or must downsize or recapitalize in $t + 1$.

That NI_t can be negative because some entrepreneurs who are bank borrowers fail risk taking operations. It is true after banks have practiced due diligence by screening applications, and non-performing loans are a general equilibrium. It is implied in Diamond (1984) and the foundation of this study. A bank can keep the non-performing loans on its balance sheet as long as GEONPA is on the good flip. However, we shall not forget that these business owners cannot meet their debt obligations (BL_{bwr}) in bank-entrepreneur contracts.

As long as other entrepreneurs fulfill their loan service obligations and profits are higher than losses, the bank can sail its loan business smoothly to the future. However, there is no doubt that bank revenues become uncertain. Throughout the loan portfolio, the return on assets (ROA_t) of a bank becomes a stochastic process, as in Equation (4).

We model the process of bank business returns by an arithmetic Brownian motion, with the return mean as (μ) and volatility (σ). A negative net income can be absorbed by the profits of other performing loans. However, if negative net income escalates to the level of a bank's loan portfolio, the bank must update its capital downward. The stochastic process underlines the shock from the bad flip of GEONPA.

We start from a simple case where a bank originates one loan with maturity m_1. Under the mean-reverting return process, the bank can expect future assets $AT_0 \times (1+\mu)^{m_1}$. Completing a loan successfully indicates that the lending bank has received all loan coupon payments and principal repayment at maturity. Bank assets grow after successfully completing a loan. The bank can grant access to more borrowing requests from different entrepreneurs whose projects decide the loan maturities. Although perpetual loans exist, they are rare. Most loan maturities range from less than a year to a few years. Therefore, in this section, we extend our one-bank analysis to J loans ($J > 1$). We will extend our analysis to two banks ($I = 2$) with heterogeneous sizes in Section 3.4.

In reality, loan maturities are shorter than the duration of the banking business (T). The maturity of the loan is measured by (m_j) from the initial concept to the sales of the new products. The duration of banking business is a finite large number T. With $J > 1$ and $m_j < T$, bank i's ($i = 1$) future assets will be

$$\sum_{j=1}^{J} AT_{i,0} \times (1 + \mu_j)^{m_j}, \text{ where } \sum_{j=1}^{J} m_j = T. \tag{6}$$

Now, we are in a good position to discuss three variables (T, μ, and σ) and one constant ($BaselR$) to appreciate the choices of banks in terms of maximizing long-term future assets. In corporate identities, entrepreneurs or banks are free to maximize their long-term assets. From the point of view of entrepreneurs, selecting high μ projects would be their priority.

However, banks shall not maximize their long-term assets by selecting high μ loans for two reasons. First, banks should be the debt intermediaries for all qualified corporate borrowers in an economy. Banks are necessary for the economy, but only because banking intermediary services improve the efficiency of debt intermediaries. Lending to many borrowers can reduce borrowing costs. In other words, banks must maximize their best interests on the basis of being good for the economy. This is the two-tier future asset maximization we advocate.

The second reason is for the bank's own sake, and the concerns are on nonqualified borrowers. Because bank intermediaries are good for the economy, T should be a finite large number. μ and σ are two sides of the same coin. Higher risks (σ) are associated with higher returns (μ). Bank credit screening process shall reject loan requests from business owners whose operating income history suggests that these borrowers are unlikely to meet loan interest rate payments or loan principal repayment Chu and Xiao (2023).

Even if banks have rejected high-risk borrowing demand, some will turn to bad loans due to the nature of risk-taking business operations. Such non-performing loans are general equilibrium. This study will demonstrate a variety of shocks from the bad side switch of GEONPA driven by σ. To maximize long-term assets, banks should max out bank capital to originate loans such that their capital ratios satisfy regulatory requirements.

2.4 Setup: good or bad news on bank loan in two periods

One task of this study is to model on what condition a bank decides to remove the non-performing loans from its balance sheet. Once a borrower has drawn down the loan, monitoring loan performance is part of credit intermediation. To maintain performing, a borrower must meet all debt repayment obligations on time. However, a loan becomes non-performing if the borrower misses at least one coupon payment. In equilibrium, the loan portfolio of the banking industry includes some non-performing loans.

In the base analysis at stage 1, lending banks do not demand overcollateralization. Since stage 2, we include overcollateralization and model that a bank will remove non-performing loans if the borrower misses two consecutive coupon payments. The updated feature is consistent with the convention in the bank loan business[10].

Why do banks liquidate their non-performing loans? The answer is straightforward. A bank will not comply with regulatory capital requirements if the bank keeps non-performing loans on the balance sheet. Furthermore, keeping loans on the balance sheet that have missed multiple coupon payments could trigger bank runs. Spreading costs across all loans is the key to reducing monitoring costs. However, too many bad loans may impair a bank's capital adequacy ratio.

[10]It is also consistent with academic exercises to model default events by the first-passage-time process. Once a default is confirmed by two missing coupon payments, the borrower will not make coupon payments for this loan

We decompose risk-based capital ($RE + EC$) into two pieces $LC + SC$ with $RE + EC = LC + SC$. LC_κ refers to the bank capital committed with loans, where κ is an integer indicating the number of standard loan facilities (SLF). In this study, one (1) stand loan facility is equal to \$100. SC is the standby capital less than the amount to originate one new SLF, $LC_{\kappa=1} = SLF \times BaselR$. See Equation (7).

$$LC_\kappa \text{ and } SC = \begin{cases} LC_\kappa : & \text{loan commitments } LC_\kappa = \kappa \times SLF \times BaselR; \\ SC : & \text{standby capital } SC < LC_{\kappa=1}. \end{cases} \tag{7}$$

SC captures two pieces of bank capital. The first piece is the net income from the current performing loans. The second piece is the residual risk-based capital insufficient to make a new loan after a bank removes non-performing loans and pays the liquidation costs.

Being bank lenders, households classify different bank practices into good or bad operations. Assume information transparency. We set bank loan news in two dimensions. First, good or bad news comes and updates in time series. Second, households further classify each piece of bad news by the severity of its negative impacts. Households make different decisions depending on whether a bank operation is level 1 or 2 bad news. We will link the bad news to two types of bank runs in Section 2.5.

There is good and bad news in period t. Because banks originate loans after screening borrowing applications, no news is good news. All loans are performing. This is the first good news. The first bad news is that banks are losing capital. As in Equation (8), when a bank is forced to remove non-performing loans at $AT_{t,NP}$ to comply with the Basel capital ratio, bank capital must absorb the liquidation costs of non-performing loans.

$$\text{loan news at } t = \begin{cases} bwr^1 \; news_t^g : & AT_t, \text{ keep performing loans;} \\ bwr^2 \; news_t^b : & AT_{t,NP}, \text{remove non-performing loans.} \end{cases} \tag{8}$$

$$\text{loan news at } t+1 = \begin{cases} news^{g}_{t+1}: & AT_{t+1} \geq AT_t;\ bwr^1 + bwr^{new}. \\ news^{b}_{t+1}: & AT_{t+1} < AT_t;\ bwr^1. \end{cases} \tag{9}$$

The good and bad news in period $t+1$ is in Equation (9). A bank may not survive upon bad news, $news^b_t$, due to bank runs, as we will show in Section 2.5. However, if the bank survives $news^b_t$ after removing non-performing loans of bwr^2, there are two outcomes. It is good news if the profit-generating capacity of the remaining performing loans is strong, defined as a rapid asset recovery, where a bank can recover its performing loans at $t+1$ after removing non-performing loans at t. Otherwise, it is bad news if recovery takes more than one period, and we define it as slow asset recovery.

2.5 Setup: early or patient bank lender, panic- or no panic-based bank runs

The bad loan news ($news^b_t$) and ($news^b_{t+1}$) in two periods can be further differentiated by the severity of adverse shocks. With bad news, regardless of its level, households could not earn higher interest rates from term deposits in the next period because SC is not enough to make a new loan after absorbing liquidation costs. The difference is the deposit recovery of the current period. With the bad news of level 1, household depositors can completely withdraw their deposit principal and associated interest rates in the current period.

$$\text{Level 1 } new^b: \begin{cases} \text{retrieve deposits and interest rates at } t. \\ \text{no term deposits at } t+1. \end{cases} \tag{10}$$

$$\text{Level 2 } new^b: \begin{cases} \text{loss of deposits and interest rates at } t. \\ \text{no term deposits at } t+1. \end{cases} \tag{11}$$

However, households lose some of their savings and associated interest rates with level 2 bad news. Level 2 bad loan news can occur at t or $t+1$. Bad loan news in Equation (8) ($news_t^b$) can be level 1 or level 2 at t. The difference is due to the liquidation costs. Assume that one SLF becomes non-performing and needs to be removed. If $LC_{\kappa=1}$ can cover the liquidation costs, that is bad news at level 1. However, it is elevated to level 2 bad news if $LC_{\kappa=1}$ is insufficient to cover liquidation costs. Furthermore, at $t+1$, bad loan news in Equation (9) ($news_{t+1}^b$) is level 2 due to household concern about slow asset recovery, in Equation (11), that we will define.

The bad news of level 2 causes bank runs at t or $t+1$. The difference is panic-based or no panic-based. The $news_t^b$ in Equation (8) causes no panic-based bank runs. At t, if $LC_{\kappa=1}$ is insufficient to cover liquidation costs, the households will run as predicted in Allen and Gale (1998). Runs are *rational* because the expected recovery from liquidation is insufficient to cover the full rights of households in the household-bank contract.

$news_{t+1}^b$ in Equation (9) is also level 2 bad news but causes panic-based bank runs due to slow asset recovery. The final results of no panic-based or panic-based bank runs are the same: bank liquidation. What causes panic? In Diamond and Dybvig (1983), panic is fear. In this study, we propose that a slow asset recovery causes *rational fear*. We define slow asset recovery as a bank operation in which a bank cannot recover its asset in $t+1$ after liquidating its non-performing loans in t. Here is the intuition.

Households as bank lenders have a heterogeneous sensitivity to bad news about the bank where they have deposited. We can separate them as early or patient bank lenders as in Diamond and Dybvig (1983). Early bank lenders withdraw their deposits on the level 1 bad news. Patient bank lenders maintain patience on level 1 bad news, but run on level 2 bad news. Suppose that a bank wants to argue that having non-performing loans is just bad luck; it can happen to any bank. The concern about the bank's profit-generating capacity is rational if the bank cannot recover its assets in the next period.

We want to draw attention to two characteristics of panic-based bank runs. When panic runs occur, a bank has liquidated its bad loans and has more than sufficient bank capital

($SC > 0$) for the bank's loan portfolio. We will demonstrate the details of how slow asset recovery causes panic runs in Section 3.2. Therefore, panic in our study is a rational fear.

At the end of each period, all households return to the bank and make one of the three decisions. Without bad news, they all roll over. They are patient bank lenders. For those hit by level 1 bad news, they withdraw their savings and interest rate and park the withdrawals in the Treasury. They are early bank lenders, and their withdrawals are in full amount. The good news is that, although they withdraw, they do not run. Furthermore, their withdrawals do not lead others to run. The worst case is that some households are hit by level 2 bad news. Their runs either are no panic-based or lead to a self-fulfilling prophecy that others will follow. The result is bank liquidation without government intervention. We will demonstrate bank runs and their impacts in Section 3.

3 Non-performing Loans, Bank Runs, and Impacts at Three Stages

Every bank loan business involves two contracts, and the bank is the only player in both contracts, although playing different roles. In the base model, we model the behaviors of three contract players, known as two qualified entrepreneurs as the bank borrower, one bank as the debt intermediary, and multiple households as the bank lender. Our model covers three time periods ($t = 0, 1, 2$). As our analysis develops, we will include nonbank financial institutions, two banks with heterogeneous sizes, and the central bank as the lender of last resort in model extensions.

Critical reasoning flows from (a) and (b) to (c). (a) households can be early or patient bank lenders; (b) two bank borrowers release good or bad loan news. Being bad news, it can be level 1 or level 2; (c) when the bank liquidates non-performing loans on bad news, households may practice one of the four behaviors: being patient, withdrawing their savings without runs, running without panic, or running with panic. Intuitively, savings withdrawals or bank runs are natural responses to bad news about the liquidation of bad loans. It seems counterintuitive how households can remain patient. As we demonstrate in the analysis, being patient is not a surprise if the bank has a strong profit-generating capacity.

We assume that each household has a \$1 saving and can choose between Treasury and term deposits (TD) as the bank lender. The costs of funding liquidity that banks pay for TD are $FL_{td,t}$. Because the bank loan business is risk taking, $FL_{td,t}$ are higher than the Treasury risk-free rate, $FL_{td,t} > r_{rf}$. Households prefer term deposits without bad news but will withdraw term deposits and switch to Treasury upon hearing bad news. For self-protection, households set the maturity of term deposits equal to one period. If a household withdraws its term deposit at the end of the period t, it is an early bank lender. Otherwise, a household is a patient bank lender if it rolls over the term deposit.

A bank B has a loan portfolio (AT_t) with two borrowers, bwr^1 and bwr^2. In the base model, both entrepreneurs borrow the same amount at the same coupon rates. In model extensions, entrepreneurs can borrow different loan sizes, but at the same coupon rates. In period t, only bwr^1 is performing but bwr^2 misses the due coupon payments ($BL_{bwr} = 0$). Therefore, bwr^2's loan reports losses ($L_{t,bwr2}$); The loan (AT_t) of bwr^1 generates profits ($P_{t,bwr1}$). See Equation (12).

$$NI_t = \begin{cases} AT_t^1 \; P_{t,bwr1}: & AT_t^1 \times (BL_{bwr} - \beta FL_{td,t} - \theta - \eta); \\ AT_{t,NP}^2 \; L_{t,bwr2}: & -AT_t^2 \times (\beta FL_{td,t} + \theta + \eta). \end{cases} \tag{12}$$

Our analysis starts with the base case, followed by a numerical example. Then we extend the base-case analysis through five additional examples. To facilitate extension, we set two changeable variables. The first is the relative size of performing vs. non-performing loans. In the base case, the relative size is 1 to 1. The second changeable variable is the normal or forced fire-sale discounts.

Example 2 is about over-collateralization. Example 3 analyzes a large bank. Example 4 adds a capital buffer. Example 5 considers a large bank and distressed fire-sale discounts. Taking every measure that could help prevent bank runs in the first five examples, we analyze small bank liquidation in Example 6. The choice of numbers of all numerical examples is without loss of generality, but for better illustration.

Due to the larger loan portfolio that meets debt payment obligations and potential intervention of central banks, the large bank operations after being notified of bad loans are quite different from those of a small bank. We dedicate our analysis of the large bank to Section 3.5. Understanding large bank operations while holding and even actively purchasing non-performing loans, we utilize Section 3.6 to demonstrate the ultimate negative effects on financial stability.

We calculate each borrower separately with the Basel capital ratio. We do so only for an easy demonstration. B splits positive net income to increase equity capital by $\gamma \times NI_t$ and retained earnings by $(1-\gamma) \times NI_t$. γ is a positive constant for banks if $NI_t > 0$ or 0 if a bank reports negative net income, NNI_t, defined as $NI_t < 0$. No bank can increase its equity capital in period t due to NNI_t. Furthermore, banks must deduct NNI_t from cumulative retained earnings. See Equations (14) and (13).

$$\gamma = \text{constant} \in (0,1) \text{ if } NI_t > 0; \text{ or } \gamma = 0 \text{ if } NI_t \leq 0. \tag{13}$$

$$EC_t + RE_t = \begin{cases} bwr^1 \ NI_t^1 > 0: & EC_t^1 = EC_{t-1}^1 + \gamma NI_t^1; \ RE_t^1 = RE_{t-1}^1 + (1-\gamma) NI_t^1; \\ bwr^2 \ NI_t^2 < 0: & EC_t^2 = EC_{t-1}^2; \ RE_t^2 = RE_{t-1}^2 - |NNI_t^2|. \end{cases} \tag{14}$$

Assume that both borrowers satisfy Basel capital requirements at $t-1$. However, the situation changes at t. bwr^1 has capital more than satisfying the Basel requirement ($BaselR <$), but the capital of bwr^2 is lower than the Basel requirement ($BaselR >$). The difference is because of whether the loans are performing. See Equations (15) and (16). Following Diamond (1984), B subsidizes bwr^2's loan losses with loan profits from bwr^1 at the bank level. One of the two scenarios must be true. B complies with or does not comply with the regulatory capital ratio.

$$bwr^1: \; BaselR = \frac{RE^1_{t-1} + EC^1_{t-1}}{AT^1_{t-1}}; \; BaselR < \frac{RE^1_t + EC^1_t}{AT^1_t}. \tag{15}$$

$$bwr^2: \; BaselR = \frac{RE^2_{t-1} + EC^2_{t-1}}{AT^2_{t-1}}; \; BaselR > \frac{RE^2_t + EC^2_t}{AT^2_t}. \tag{16}$$

Complying with the regulatory capital ratio, B can continue to hold nonperforming loans. However, if not compliance with the Basel ratio, B has two options: raise new capital or reduce assets, or practice both. Raising new capital is rational only if current stocks are overvalued Myers and Majluf (1984). Obviously, this is not why B needs new equity. In reality, raising equity due to bad news will trigger panic-based bank runs. Bank runs on SVB are the recent case. Therefore, B liquidates bwr^2's coupon-missing loan ($AT_{down} = AT_{t,NP}$) at t, as in Equation (17).

$$BaselR > \frac{RE_t + EC_t}{AT_t}, \; AT_{t+1} = AT_t - AT_{down}; \; AT_{down} = AT_{t,NP}. \tag{17}$$

3.1 Liquidating non-performing loans and fire-sale discounts

However, B cannot recover the fair value when liquidating non-performing loans. The difference is known as fire sale discounts, and here are two reasons. First, liquidating $AT_{t,NP}$ must be completed in period t because missing loan coupon payments will compromise the bank's Basel capital ratio at t+1 if not liquidated; see Equation (17). Second, non-performing loans ($AT_{t,NP}$) will be sold to nonbank financial institutions because other banks max out their equity capital to hold loan assets to maximize their long-term assets. The fire-sale discounts of non-performing loans ($Dis^{fs}_{t,NP}$) are defined in Equation (18). A capital buffer is not included in the base case analysis but will be added in stage 2.

$$AT^{fs}_{t,NP} < AT_{t,NP}, \; Dis^{fs}_{t,NP} = AT_{t,NP} - AT^{fs}_{t,NP}. \tag{18}$$

We decompose $Dis_{t,NP}^{fs}$ into CCI and the opportunity cost of capital (R_t^{ococ}). The costs in CCI are associated with screening loan applicants and monitoring existing borrowers. The bank's CCI (CCI_B) is lower than that of nonbank (CCI_{NB}) financial institutions. This is because banks have cost advantages in the credit granting process and in maintaining a healthy loan portfolio for ongoing operations. Furthermore, a bank or nonbank institution as the transaction buyer will demand an opportunity cost of capital (R_t^{ococ}) higher than an average market rate of returns (μ_{mkt}) even through orderly transactions. This is because the seller is under time pressure to liquidate. Every potential buyer can take advantage of the selling bank to maximize the former's return, which is consistent with the utility maximization in Equation (5). We define fire-sale discounts through orderly transactions $Dis_{t,NP}^{fs,nor}$ in Equation (19). Discounts could be even more severe when market transactions are distressed (or forced) $Dis_{t,NP}^{fs,frc}$, as in Equation (20).

$$CCI_B < CCI_{NB};\ R_t^{ococ} > \mu_{mkt};\ Dis_{t,NP}^{fs,nor} = AT_{t,NP} \times (CCI_{NB} + R_t^{ococ}). \tag{19}$$

$$R_t^{ococ,frc} > R_t^{ococ};\ Dis_{t,NP}^{fs,frc} > Dis_{t,NP}^{fs,nor}. \tag{20}$$

Shleifer and Vishny (1992) has documented the rapid sale and the general equilibrium of asset sales as two reasons for fire-sale discounts. However, the fire sales of non-performing loans in our study have two unique characteristics. First, fire sales are caused by regulatory capital requirements, not by companies to meet debt payments. Moreover, satisfying such requirements is under time pressure. Second, it is the sum of the cost of credit intermediation and the opportunity cost of capital, rather than the redeployable asset, that matters. In reality, the opportunity cost of capital drives the discounts.

3.2 Stage 1: slow asset recovery and bank liquidation

We model multiple results where a bank holds non-performing loans. B and its loan portfolio follow Equation (12). Consider each t representing a quarter. At the beginning of

t, B has committed $LC_\kappa = RE + EC$ to loans of bwr^1 and bwr^2, respectively. However, only $bwr^1 \rightarrow bwr^P$ generates net income, but $bwr^2 \rightarrow bwr^{NP}$ misses the coupon payments. B liquidates loans missing coupon payments.

We analyze B's cash flows after liquidating bwr^2's non-performing loan. B must honor the withdrawal requirements, and the face value of all term deposits is ($TD = \beta \times AT_{t,NP}$). The change of equity capital after liquidation includes three inputs at t, summarized in Equation (21). (1) Add net income from bwr^1's performing loans, $P_{t,bwr1}$; (2) Record fire-sale losses, $-Dis^{fs}_{bwr2}$; (3) Record the interest rate of term deposits and costs of CCI and $OpEx$, $-|L_{t,bwr2}|$, as shown in Equation (12). The sum of (2) and (3) is the liquidation costs ($LiqCst_{bwr2}$) of bwr^2's non-performing loans, which is covered by $LC_{\kappa=1}$. The change in risk-based bank capital at the end of t, $\Delta(EC + RE)_t$, will be described as $LC_{\kappa-1} + SC_t$. $LC_{\kappa-1} + SC_t$ can be surplus or deficit as in Equation (23).

$$LC_{\kappa-1} + SC_t = P_{t,bwr1} - \underbrace{(Dis^{fs}_{bwr2} + |L_{t,bwr2}|)}_{LiqCst_{bwr2}}. \tag{21}$$

In the surplus scenario (scenario 1), as in Equation (22), B can orignate a new SLF to replace the non-performing loans of bwr^2 in the same period t. Please note that B is not subject to the Basel capital constraints in the surplus scenario. Since B has to pay fire-sale discounts Dis^{fs}_{bwr2} in liquidation, B has two choices.

$$\text{At } t, \text{ scenario 1: } P_{t,bwr1} > LiqCst_{bwr2}. \tag{22}$$

Choice one, B chooses to liquidate non-performing loans if the fire-sale discounts are normal. These are the lowest costs to remove bad loans. Furthermore, B can originate a new SLF that will generate income in $t+1$. Choice two, B chooses to hold non-performing loans on the balance sheet to save the liquidation costs. Choice two is rational if the current discounts $Dis^{fs,frc}_{bwr2}$ are excessive and B expects that excessive discounts will be back to normal in the near future. In fact, the surplus scenario offers the rationale for the insight of cross-loan subsidization of GEONPA in Diamond (1984). In surplus scenario, all households will remain patient and roll over their deposits in $t+1$.

The deficit scenario, in Equation (23), indicates the bad flip of GEONPA. B is subject to binding regulatory capital constraints. The deficit scenario has three outcomes: household withdrawals without bank runs, no panic-based bank runs, and panic-based bank runs.

$$\text{At } t\text{, scenario} \begin{cases} 2a: & P_{t,bwr1} < LiqCst_{bwr2} < LC_{\kappa=1}; \\ 2b: & P_{t,bwr1} + LC_{\kappa=1} < LiqCst_{bwr2}. \end{cases} \tag{23}$$

Scenario 2a: the profit from the good loans is lower than the liquidation costs, but not as low as in scenario 2b. After the new profits $P_{t,bwr1}$ are exhausted, B must use its capital ($LC_{\kappa=1}$) to absorb the additional liquidation cost. Because the residual capital SC is insufficient to make a new loan, as in Equation (24), B is forced to terminate the term deposit contracts with households that support bwr^2.

$$0 < SC = LC_{\kappa=1} - (LiqCst_{bwr2} - P_{t,bwr1}) < LC_{\kappa=1}. \tag{24}$$

B makes an announcement about the removal of non-performing loans before all households come to the bank. The latter will choose between withdrawal or rollover at the end of t. Bank tellers have two files ready for each household depositor. Every household that supports loans of bwr^2 will receive the file of a contract termination note. In the second file, it is a check with the full amount of the term deposit principal and the associated interest rate for period t. These households walk out of the bank without panic because they have fully withdrawn their savings with investment interest rates. They may be disappointed because they cannot earn a higher $FL_{td,t+1}$ next period. However, there is no reason for them to run.

Households supporting bwr^1's loans receive the same check for the term deposit interest rates and one invitation for the term deposit contract rollover. They have no reason to panic. They do not panic because they do not see panic among households with contract termination. They know the bad news of bank asset reduction. However, because the bad news is level 1 because the current bank loans are performing and they will earn higher interest rates by rolling over the deposit contract, they accept the rollover invitation.

In scenario 2a, households withdraw their term deposits and complete their role as early bank lenders after B liquidates bwr^2's non-performing loans. Households supporting bwr^1 are patient bank lenders. They roll over term deposit contracts. There are no bank runs.

In scenario 2b in Equation (23), the situation is worrisome. The sum of the profit from the performing loans (bwr^1) and bank capital for the non-performing loans ($LC_{\kappa=1}$) is insufficient to cover the liquidation costs. This is a level 2 bad news, as in Equation (11), and no panic-based bank runs will start. Some households will not be paid when B has exhausted the cash from liquidating $AT^2_{t,NP}$.

We pay attention to the current situation. B has no bad loans. Because unpaid households do not know the income of B's good loans in $t+1$, they panic. The result of the panic is these unpaid households will run. Therefore, B must liquidate the performing loans of (bwr^1) to pay for the unpaid households. In this case, no panic bank runs lead to panic bank runs. The final result is that B will be liquidated. In Example 1 below, we show details of the households' decision-making for scenario 2b in Equation (23).

Since scenario 1 is business as usual and scenario 2b is game over, we revisit scenario 2a for B at the next period $t+1$. Now, we introduce the *asset recovery switch* in Equation (25). In Scenario 2a, B adds the income from the performing loans ($P_{t+1,bwr1}$) at $t+1$.

$$\text{At } t+1, \text{ asset recovery} \begin{cases} \text{rapid}: & P_{t+1,bwr1} + SC \geq LC_{\kappa=1}; \\ \text{slow}: & P_{t+1,bwr1} + SC < LC_{\kappa=1}. \end{cases} \tag{25}$$

There are two outcomes. In the scenario of rapid asset recovery, B can make a new loan because income from the performing loans and the standby capital (SC) are sufficient to make one unit of new loans, $LC_{\kappa=1}$. Current and new households will be patient. Otherwise, B suffers from the bad switch of slow asset recovery, defined as $P_{t+1,bwr1} + SC$ is not enough to make one unit of new loans. To households rolling over their deposits supporting loans of bwr^1, this is the second bad news. Because a slow asset recovery cause panic, households supporting bwr^1 decide to withdraw, rather than roll over their contracts. Bank runs force B to be liquidated if the central bank does not intervene.

Equations (21) to (25) are the basic mechanism and conditions (at stage 1) through which a bank holding non-performing loans can lead to different results. The worsr case is bank liquidation. We will calibrate different results through a numerical example in which we observe an individual bank with other banks and nonbank financial institutions in the background. Basel capital requirements are in place. However, no capital buffer is required. The size of the bank is homogeneous. Every bank maxes out risk-based capital to maximize asset holding. Bank loans do not require overcollateralization.

3.2.1 Example 1: base case of non-performing loans and impacts

We observe three quarters ($t = 0, 1, 2$). At the beginning of $t = 0$, B has a loan portfolio of $AT_0 = \$200$ with two borrowers (bwr^1) and (bwr^2). Each has borrowed one unit of SLF, equal to \$100 face value. The Basel capital requirement is 13%. B maintains its balance sheet in line with Basel capital requirements with $(RE + EC)_0 = \$26$. Term deposits are $TD_0 = \$174$ for \$200 loans from 174 households ($H_1, ..., \; and \; H_{174}$). In each quarter t, $BL_{bwr} = 2.5\%$, $FL_{td,t} = 1\%$, $\theta + \eta = 0.5\%$, $r_{rf} = 0.5\%$[11].

In the second half of $t = 0$ quarter, bwr^2 informs B about missing \$2.5 coupon, and B issues a notice of default to bwr^2. B's due cash outflows are \$1.37, of which \$0.87 are interest rates for term deposits and \$0.5 for CCI and $OpEx$. With the income of \$1.13 from bwr^1's performing loans, B is exposed to \$0.24 bank capital shortage. Due to non-compliance with

[11] We try numbers close in reality. For example, because t is by quarter, measured by APR, the loan coupon rate, the household deposit rate, and the risk-free Treasury rate are 10%, 4%, and 2% respectively.

regulatory capital, B must downsize at $t = 0$.

We now make two local assumptions in Example 1. (1) B has extra \$0.24 retained earnings from $t = -1$ quarter to fill the cash flow gap. (2) B assumes that missing the first coupon is an accident and bwr^2 will pay the next coupon on time. At the end of Example 1, B will learn that both assumptions are mistakes.

All 174 households come to B on the last day of quarter $t = 0$. The bank tellers give two envelopes to all 174 households, the first includes the interest rates of the deposit in quarter $t = 0$. The second envelope has a rollover contract for quarter $t = 1$. All 174 households will not withdraw and roll over their deposit contracts.

$t = 1$: liquidation of non-performing loans

At $t = 1$, bwr^2 misses the coupon the second time ($BL_{bwr} = 0$). B records $NNI_{1,NP} = -\$1.37$, again. Without the additional \$0.24 capital endowments, B's $(RE + EC)_1 < \$26$. If keeping bwr^2's loan, B won't comply with the regulatory capital ratio because the \$1.13 income can not cover the holding cost \$1.37. To bank lenders, non-compliance with regulatory capital could trigger panic-based bank runs. Therefore, B must liquidate $bwr2$'s loan and associated collateral at $t = 1$.

Assume $Dis^{fs}_{1,NP} = \$2$. B receives \$98 cash after liquidating $bwr2$'s loan and collateral, so $AT_{1,NP} = 0$. On the last day of $t = 1$, all 174 households return to B for their decisions at $t = 2$. Within the bank, the information records show that households (H_1 *to* H_{87}) offer their term deposits for bwr^1 and households (H_{88} *to* H_{174}) offer their term deposits for bwr^2.

B informs all 174 households that here are your checks for the deposit interest rates at $t = 1$. For the 87 households (H_{88} *to* H_{174}), here are your checks of the deposit principal. Because we (B) have removed the bad loans, we safely pay back your savings. For the remaining 87 households (H_1 *to* H_{87}), here are the invitations to roll over your deposit contracts. 87 households (H_{88} *to* H_{174}) walk out of the bank without panic because each of them receives a check of \$1.01, the full amount of the saving principal plus the interest rates at $t = 1$. None of the 174 households is in panic.

The next question for the remaining 87 households (H_1 *to* H_{87}): should they roll over as patient bank lenders or withdraw in $t = 1$? We assume that every economic agent, including banks, entrepreneurs, and households, maximizes its long-term assets, as in Equation (5). If they choose to withdraw and invest in the Treasury, their income in $t = 2$ will definitely be lower. Although the bank loan portfolio is lower at $t = 1$ than at $t = 0$ ($AT_1 = \$100$), the loan is perofrming. Furthermore, the total bank capital ($LC + SC$) of \$23.76[12] is more than enough for the current one unit of SLF. The rational choice would be to roll over at $t = 1$ and be vigilant at $t = 2$.

$t = 2$: slow asset recovery

At $t = 2$, $bwr1$ confirms its coupon payments. B expects $NI_2 = \$1.13$. Updated risk-based capital is 24.89. Furthermore, B does not announce new capital issuance. Therefore, 24.89 capital is insufficient to maintain \$200 loans. With the profit generating capacity of the \$100 performing loans, B expects that cumulative risk-based capital will be \$26.02 at $t = 3$ such that it can commit the second \$100 loan.

This is the slow asset recovery, the level 2 bad news. Households from H_{88} to H_{174} become panic and decide to withdraw at $t = 2$. Please note the major difference between the no panic-based bank runs and panic-based bank runs. When panic bank runs occur, B has all loans performing and satisfies regulatory capital requirements with a comfortable margin of \$11.89. Without the intervention of the central bank, panic-based runs force B to be liquidated at $t = 2$.

There are solutions to prevent bank liquidation in Example 1. Panic-based bank runs can be prevented if a bank has more performing loans, for example, $AT_1 = \$200$. Or, B has additional financial resources to reduce fire-sale discounts. That is what we will discuss in the solution package.

[12]The total capital of B equal to \$23.76, a sum of three pieces. (1) $LC_\kappa = \$13$ for performing loan $AT_1 = \$100$. (2) $NI_1 = \$1.13 = 2.5 - 1.37$ profits from performing loans. (3) $SC = \$9.63 = 100 - 2 - 87 - 1.37$ from liquidated non-performing loans.

3.3 Solution package for bank runs and fire-sale discounts

One of the two inputs that can change the end results in the stage 1 analysis is the fire-sale discounts. Up to now, we assume orderly discounts and focus on one-bank operations. In stage 2, we elevate our analysis to the banking system, and distressed fire sales are possible. We include all available solutions to maintain financial stability. These solutions can be categorized into three branches: bank practices, government intervention, and accounting guidance, all rooted in the trilogy of papers that won the Nobel Memorial Prize in Economic Sciences 2022 to understand the nature of banking business and banking crises.

We start with bank-run solutions. To prevent no panic-based bank runs, a bank must completely remove non-performing loans. However, panic bank runs can occue after removing non-performing loans. In other words, the removal of non-performing loans is a necessary condition to prevent panic-based bank runs. The good news is the lender of last resort. The central bank can neutralize the liquidity shocks of panic-based bank runs by guaranteeing liquidity access to funding (β) and a profitable interest rate margin in Equation (12). However, there is a caveat: moral hazard concern. The lender of last resort is justified only for systemically important banks. Equation (26) lists the necessary condition to avoid bank runs ($BRS1$) and the central bank liquidity as the sufficient condition ($BRS2$) for the large bank.

$$\text{bank run solutions} = \begin{cases} BRS1: & \text{removal non-performing loans} \\ BRS2: & \text{central bank liquidity guarantee.} \end{cases} \tag{26}$$

The side effect of liquidating non-performing loans through $BRS1$ is fire-sale discounts. We summarize three solutions to address the fire-sale discounts in Equation (27). The first solution is the overcollateralization. The second solution is an asymmetric banking system design with a large bank maintaining a capital buffer. The third solution is the fair-value accounting guidance on mark-to-market practice.

$$\text{fire-sale solutions} = \begin{cases} FSS1: & \text{loan overcollateralization} \\ FSS2: & \text{small and large banks and capital buffer.} \\ FSS3: & \text{fair-value mark-to-market accounting.} \end{cases} \tag{27}$$

In $FSS1$, the banking industry demands an appropriate overcollateralization (OC), which is the sum of two consecutive missed coupon payments, as in Equation (28). Pledging overcollateralization adds a layer of protection for banks. However, we must keep fair in contract design and maintain the balance between lenders and borrowers. Following industry practice, we use two missed coupon payments to confirm the non-performing status. The status triggers the liquidation of non-performing assets.

$$FSS1: OC = AT^{mc} \times 2 \times BL_{bwr} \tag{28}$$

Example 2: overcollateralization. Ceteris paribus, assume that bwr^2 pledges $\$5(= 2 \times 2.5)$ overcollateralization and $Dis^{fs}_{1,NP} = \$2$. After $bwr2$ miss two \$2.5 coupon payments, B liquidates the loan and its collateral. Liquidation is the only option, the same as in Example 1. If keeping bwr^2's loan, B^S won't comply with the required capital ratio because the \$1.13 income can not cover the holding cost \$1.37. After liquidation, B^S has \$15.52 risk-based capital[13], which is sufficient to add another \$100 SLF at $t = 1$.

Here is the difference between two examples. Due to overcollateralization, B can recover assets at $t = 1$ as $AT_1 = \$200$ in Example 2. This is a quick asset recovery compared to $AT_1 = \$100$ in Example 1. Early bank lenders from H_{88} *to* H_{174} will become patient bank

[13]The overcollateralization offers time and resource buffers since $t = 0$ to book bank income and losses. When missing the first coupon income of bwr^2 at $t = 0$, B books $NI = \$1.13 = 2.5 - 1.37$ for AT^{mc}_0 and issues the notice of default to bwr^2. B will repeat the same accounting operation booking $NI = \$1.13$ for $AT_{1,NP}$. Doing so, B has accumulated $\$4.52 = 4 \times 1.13$ from \$200 loans at $t = 0, 1$. At $t = 1$, the interest rates on the deposits of all 174 households are paid. Meanwhile, B liquidates the collateral of the non-performing loans ($AT_{1,NP}$), writes off $AT_{1,NP}$, honors deposit withdrawal requests from 87 early bank lenders, and has $\$11(= 105 - 2 - 5 - 87)$ capital in cash. With \$4.52 income, the total SC is \$15.52.

lenders and roll over their deposits as patient bank lenders from H_1 *to* H_{87} to support bwr^1.

Example 3: large bank size. In $FSS2$, the large bank and its capital buffer could help prevent bank runs. In Example 3, we show that larger banks (with more performing loans) can recover from bad loan shocks faster. Like in Example 1, there is no overcollateralization. Due to the larger size, extra capital \$0.24 from $t = -1$ is not needed.

Consider B^L a large bank with two sizes $AT_0 = \$300$ or $AT_0 = \$400$. For the size $AT_0 = \$300$, B^L will demonstrate quick asset recovery at $t = 2$ with $SC_2 = \$15.04$, which is sufficient to originate a new SLF[14]. Liquidate the same \$100 non-performing loans at $t = 1$. If the bank size is larger, say $AT_0 = \$400$, with more performing loans, B^L will accumulate $SC_1 = \$15.04$ risk-based capital[15]. Asset recovery is faster at $t = 1$. The 348 households will be patient at $t = 1$. This is the intuition why larger banks are more resilient to bad loan shocks.

Example 4: capital buffer. For B in Example 1, consider a larger bank with capital buffers as the potential buyer. In Shleifer and Vishny (1992), capital buffers help avoid fire-sale discounts when non-performing loans are sold to the large bank. If B^L has \$2 in capital reserves and does not demand discounts of $Dis^{fs}_{1,NP} = \$2$, B can liquidate non-performing loans without fire-sale discounts[16]. B wil have sufficient capital for a new SLF at $t = 2$ as the accumulated $SC_2 = \$13.89$[17]. Thanks to rapid asset recovery, B can avoid panic bank runs. However, we will show that the large bank B^L will not tender its own capital reserves to help B in a difficult time in Stage 2 analysis.

Next, let us focus on large bank that holds the non-performing loans. Dis^{fs}_{NP} is expected

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[14] At $t = 0$, there are $AT^P_0 = \$200$ and $AT^{mc}_0 = \$100$. The $SC_0 = NI_0 = 2.26 - 1.37 = \$0.89$. Because missing coupon again at $t = 1$, AT^{mc}_1 and its collateral will be liquidated. SC of non-performing loans after liquidation is $SC^{NP}_1 = 9.63 = 100 - 2 - 87 - 1.37$. The NI_1 from \$200 performing loans are \$2.26. The total $SC_1 = \$12.78 = 0.89 + 9.63 + 2.26$. $SC_2 = \$15.04 = 12.78 + 2.26$

[15] At $t = 0$, there are $AT^P_0 = \$300$ and $AT^{mc}_0 = \$100$. The $SC_0 = NI_0 = 3.39 - 1.37 = \$2.02$. Because missing coupon again at $t = 1$, AT^{mc}_1 and its collateral will be liquidated. SC of non-performing loans after liquidation is $SC^{NP}_1 = 9.63 = 100 - 2 - 87 - 1.37$. The NI_1 from \$300 performing loans are \$3.39. The total $SC_1 = \$15.04 = 2.02 + 9.63 + 3.39$.

[16] $SC^{NP}_1 = \$11.63 = 100 - 87 - 1.37$ from liquidated non-performing loans. B has \$2 more after liquidation because B^L has the capital reserves and does not demand \$2 fire-sale discount.

[17] $SC^P_1 = \$1.13$. $SC^P_2 = \$1.13$. The total $SC_2 = \$13.89 = 11.63 + 1.13 + 1.13$.

if B^L needs to liquidate AT_{NP}. When the large bank needs to liquidate non-performing loans, the fire-sale discounts could be larger due to the size of the loans. Because larger capital is needed, a higher opportunity cost of capital should be expected $R_t^{ococ,frc} > R_t^{ococ}$ as in Equation (20).

Example 5: large bank and distressed fire-sale discounts. We demonstrate that distressed fire-sale discounts could force large bank liquidation. The overcollateralization is equal to \$5, the same as in Example 2. The assets of B^L are the same as in Example 3, $AT_0 = \$300$.

Fire-sale discounts vary depending on the capital available in the market from other banks. From time to time, the discount could be very high. For example, when First Citizens BancShares acquired the commercial banking business of SVB on March 26, 2023, the former bought around \$119 billion in deposits and \$72 billion of SVB's loans discounted by \$16.5 billion, while around \$90 billion of SVB's securities remained in receivership. SVB shareholders absorbed the loss of \$16.5 billion. In the case of acquiring Lehman Brothers, Barclays paid only \$250 million for Lehman's business with trading assets of \$72 billion and liabilities of \$68 billion, or roughly 6.25% of Lehman's shareholder equity. To model such deep discounts, we assume $Dis^{fs,frc} = \$10$ when large banks hold non-performing loans and are forced to liquidate. At $t = 2$, B^L has standby capital $SC_2 = \$12.04$[18], which is insufficient for a new SLF. In other words, B^L shows slow asset recovery.

As a result, all 174 patient lenders for \$200 performing loans will change to early lenders and demand withdrawals. The change leads to panic bank runs. Without the intervention of the central bank, B^L will be liquidated at $t = 2$.

$FSS3$ intends to address the panic bank runs in Example 5. $FSS3$ is a fair-value,

[18]The overcollateralization offers time and resource buffers since $t = 0$ to book bank income and losses. When missing the first coupon income of bwr^2 at $t = 0$, B^L books $NI = \$1.13 = 2.5 - 1.37$ for AT_0^{mc} and issues the notice of default to bwr^2. B^L will repeat the same accounting operation booking $NI = \$1.13$ for $AT_{1,NP}$. Doing so, B^L has accumulated $\$6.78 = 6 \times 1.13$ from \$300 loans at $t = 0, 1$. At $t = 1$, the interest rates on the deposits of all 261 households are paid. Meanwhile, B^L liquidates the collateral of the non-performing loans ($AT_{1,NP}$), writes off $AT_{1,NP}$, honors deposit withdrawal requests from 87 early bank lenders, and has $\$3 (= 105 - 10 - 5 - 87)$ capital in cash. The total SC_1 is $\$9.78 = 3 + 6.78$ and $SC_2 = \$12.04 = 9.78 + 2.26$.

mark-to-market accounting guidance when market transactions impose excessive fire-sale discounts. According to FSP FAS 157-4[19], banks can mark to market non-performing assets on the balance sheet against "orderly" fire-sale discounts ($Dis^{fs,nor}$) if the market discounts are "distressed" ($Dis^{fs,frc}$).

However, by marking non-performing loans by orderly discounts when market discounts are distressed, banks cannot liquidate these non-performing loans because they ask for a price higher than those in the recent market transactions. In other words, if a bank practices $FSS3$, the operation violates $BRS1$, and the latter is the necessary condition to prevent bank runs. We will demonstrate its side effects.

3.4 Stage 2: Non-performing loans and small bank liquidation

After introducing all safety nets in reality in 2023 and demonstrating their effects, we analyze the non-performing loans in the banking system in stage 2. There are two scenarios in which one of the small and large banks holds non-performing loans. We demonstrate strategic decisions to hold or trade non-performing loans and include non-bank financial institutions as the trading counterparty. In Section 3.4, we analyze the first scenario. How do non-performing loans lead to small bank liquidation?

The banking system includes a small bank (B^S) and a large bank (B^L) with equity capital of EC^S and EC^L[20]. Following regulatory requirements, B^L allocates ($EC^{L,R}$) from (EC^L) to reserves. To maximize growth, B^S and B^L commit the available capital to the balance sheet. Due to the higher equity capital ($EC^{L,B} = \alpha \times EC^{S,B}$, where $\alpha > 1$), B^L can hold more loans, $\alpha \times AT_t^S$. Equations (29) to (30) will be the setup of the banking system before one bank announces the removal of non-performing loans.

$$EC^S = EC^{S,B};\ BaselR = \frac{RE_t^S + EC_t^S}{AT_t^S}. \tag{29}$$

[19]FASB issues final staff positions to improve guidance and disclosures on fair value measurements and impairments, April 9, 2009

[20]Superscripts B, S, L and R are for the balance sheet, small, large, and reserves.

$$EC^L = EC^{L,B} + EC^{L,R} \tag{30}$$

$$BaselR = \frac{RE_t^L + EC_t^{L,B}}{AT_t^L} = \frac{\alpha \times (RE_t^S + EC_t^S)}{\alpha \times AT_t^S},\ \alpha > 1; \tag{31}$$

B^S, AT^S_{NP}, OC are the same as in Example 2. $EC^S_t = EC^S_{t,P} + EC^S_{t,NP}$. bwr^2 missed two coupon payments at $t-1$ and t. Equation (32) shows the balance sheet at the end of t after adjusting with profits from performing loans and holding costs of non-performing loans.

$$\frac{RE^S_{t,P} + EC^S_{t,P}}{AT^S_{t,P}} > BaselR > \frac{RE^S_{t,NP} + EC^S_{t,NP}}{AT^S_{t,NP}} \tag{32}$$

Example 6: non-performing loans and small bank liquidation. Our starting point is Example 2, B^S demands overcollateralization of \$105 for every \$100 face value loans. There are two borrowers bwr^1 and bwr^2. bwr^1 always makes coupon payments on time. However, bwr^2 has missed two coupons worth of \$5. B^S must liquidate $AT^S_{t,NP}$ with a face value of \$100 at t. Otherwise, the profits of the performing loans $AT^S_{t,P}$ are insufficient to cover the costs of non-performing loans such that the loan portfolio of B^S satisfies the Basel capital ratio. In previous examples, the reference adjustments for one SLF are $P_{t,bwr1} = NI_t = \$1.13$ and $L_{t,bwr2} = NNI_t = -\$1.37$.

Several recent market transactions show that fire-sale discounts are around $Dis^{fs,nor} =$ \$3, above the historical orderly discounts at \$2 per \$100 face value loans. Now, with the large bank available as a potential trading counterparty and its capital reserves, B^S' hope to get a better deal. We assume that $AT^S_{t,NP}$ has good collateral, as we will differentiate AT_{NP} with good or bad collateral when analyzing B^L.

We analyze three transactions, named A, B, and C, to demonstrate the different interests of small and large banks. Recall that all banks maximize their future assets as in Equation (5). Specifically, B^S's interests decrease from transaction A to transaction C. However, transaction A (C) adds the least (most) to B^L's interests.

Here is transaction A. B^S contacts B^L due to the latter's capital resources from a large portfolio of performing loans and capital reserves. B^S wishes to sell the collateral of the non-performing loans with fair value at \$105. If the transaction price is completed as B^S wishes, B^S has $SC_1 = \$13(= 105 - 5 - 87)$, which is enough to originate new \$100 SLF at $t = 1$ due to the saving of \$2 of fire-sale discounts. Transaction A maximizes B^S's interests.

However, purchasing non-performing loans with good collateral at fair value is the least of B^L's interests. B^L has paid the fair value \$105 for a good collateral but the loan will not make coupon payments because B^S confirmed the non-performing status. Expected NI for B^L is \$0. Therefore, B^L passes on transaction A.

Because the small bank must liquidate $AT^S_{t,NP}$ at t, B^S moves to transaction B by contacting NB. NB's offer is with excessive discounts $Dis^{fs,frc} = \$6$. The \$6 discounts are the best offer because NB has a higher CCI, demands a higher R^{ococ}, may need additional time to find a buyer, and must make a profit when closing the position on $AT^S_{t,NP}$.

B^L can beat NB's offer if B^L wishes due to the cost advantage over NB ($CCI_B < CCI_{NB}$), as in Equation (19). Assume that the tick size of the tender offers is \$1. B^L can acquire $AT^S_{t,NP}$ with expected $NI = \$3(= 5 - 2)$, where \$5 is B^L's tender offer and \$2 is orderly fire-sale discounts.

After $AT^S_{t,NP}$ are sold to NB, the SC of B^S will be \$11.52 at t[21]. The 87 early bank lenders (H_{88} to H_{174}) withdraw their deposits supporting bwr^2 at t. Meanwhile, the remaining 87 households (H_1 to H_{87}) are patient because B^S's balance sheet is clean, the loans are performing and B^S has additional equity capital, \$11.52.

At $t+1$, B^S predicts that it could not make a new loan due to insufficient capital \$12.65 but can recover the pre-liquidation loan portfolio at $t+2$, where it has accumulated standby capital $SC^S_{t+2} = \$13.78$. However, this is a slow asset recovery. The remaining 87 households who are patient at t change to early bank lenders at $t + 1$ and demand withdrawals. This change triggers panic-based bank runs.

$$BaselR < \frac{RE^S_{t+1} + EC^{S,B}_{t+1}}{AT^S_{t+1}}. \tag{33}$$

[21] After fire sales, B^S receive $\$7 = 105 - 5 - 6 - 87$ standby capital and cumulated income $\$4.52 = 4 \times 1.13$ for $t - 1, t$. B^S's total standby capital (SC^S_t) is \$11.52.

Let us freeze the moment when panic-based bank runs have not started at $t+1$. The assets are performing. Furthermore, the capital ratio of the B^S's balance sheet is higher than the Basel capital requirements, with a standby capital \$12.65; see Equation (33).

However, due to panic-based runs, B^S cannot attract enough (β) household deposits to support the performing loans of bwr^1. The central bank cannot offer a liquidity guarantee due to moral hazard concerns. Switching to Libor does not help because Libor could be more expensive than the loan coupon rates ($FL_{libor,t+1} > BL_{bwr}$) when other banks know B^S is suffering bank runs.

Being subject to liquidity constraints, for sale is the only option for B^S. This is transaction C. B^L will win the transaction if NB intends to compete for two reasons. First, the cost of credit intermediation of banks is lower than that of nonbank financial institutions ($CCI_B < CCI_{NB}$). More important are the regulatory capital requirements for banks, which are important to protect households as bank lenders. Nonbank financial institutions are not subject to capital requirements. Second, the size of performing loans can facilitate the takeover without issuing new equity. If B^L's performing loans are no less than \$1200, one period $NI_{t+1} = \$13.56 = 1.13 \times 12$ is enough to acquire B^S's \$100 performing loans. This is not unrealistic. In Table I, the 22 large banks have large performing assets.

$$RE^L_{t+1} = (RE^S_{t+1} + EC^{S,B}_{t+1}) - \min[\,(Dis^{fs,frc}_P - \epsilon), (RE^S_{t+1} + EC^{S,B}_{t+1})] \tag{34}$$

$$AT^L_{t+2} = AT^S_{t+1} - \min[\,(Dis^{fs,frc}_P - \epsilon), (RE^S_{t+1} + EC^{S,B}_{t+1})] \tag{35}$$

B^L's offer is the bank equity of RE^L_{t+1} to acquire the entire loan portfolio of B^S. RE^L_{t+1} is part of the retained earnings from B^L's performing loan portfolio, as in the left hand side of the Equation (34). If NB can offer $Dis^{fs,frc}_P$, which is smaller than $RE^S_{t+1} + EC^{S,B}_{t+1}$, B^L's offer a marginally smaller discount (ϵ) due to cost advantage of $CCI_B < CCI_{NB}$. If the discount of NB's offer is very steep ($Dis^{fs,frc}_P > RE^S_{t+1} + EC^{S,B}_{t+1}$), B^L offers the discount of $RE^S_{t+1} + EC^{S,B}_{t+1}$ and wins the transaction. Through this transaction, B^L has acquired the performing loans of B^S in Equation (35) on the discounts of B^S's risk-based capital.

Of all three transactions, transaction C maximizes the interests of B^L the most for two reasons. First, unlike transactions A or B, the acquired are performing loans that will generate income in the future. Second, the transaction price is only a fraction of the fair value of the acquired loan portfolio.

Here is the prediction of the model that could explain multiple bank mergers and acquisitions since 2008. In Equation (34), a small fraction of B^S's capital is sufficient to acquire B^S when it is on sale due to bank runs. Such jawdropping bank $M\&A$s occur because the fire-sale discounts are excessive ($Dis_P^{fs,frc}$); this is the immediate reason.

The ultimate reason is due to the bank asset problem, which can cause bank runs. A bank with extra capital can demand very high discounts to acquire another bank that is on sale because bank runs have started, and the bank hit by bank runs is without central bank liquidity guarantee. Equations (34) and (35) demonstrates the basic mechanism of the deep discounts when a small bank is eventually on sale after liquidating bad assets. First Citizens acquiring the Silicon Valley Bank is one example. When SVB was acquired, the bank did not carry unrealized losses. Furthermore, its balance sheet was fully in compliance with the regulatory capital ratio. Two earlier cases are that J P Morgan acquired Bear Stearns and Barclays acquired Lehman Brothers in 2008.

3.5 Stage 2: when large bank holds non-performing loans

We move on to the second scenario. The balance sheet of B^S is healthy, being the same as Equation (29). However, B^L has the bank asset problem. What will B^L do after being informed that part of its loan portfolio becomes non-performing?

We pay attention to the difference in two dimensions. Unlike B^S, B^L maintains capital reserves ($EC^{L,R}$) as in Equation (36). The second difference is the scale of assets, especially performing loans. Although B^L may hold non-performing loans in higher dollar amount than B^S, the performing loans of the large are even larger than those of the small. The larger performing assets give B^L more resources when holding non-performing loans.

$$EC^L = EC^{L,B}_{t,P} + EC^{L,B}_{t,NP} + EC^{L,R};\ EC^{L,B}_{t,P} = \alpha \times EC^S_t. \tag{36}$$

$$\frac{\alpha \times (RE^S_t + EC^S_t)}{\alpha \times AT^S_t} > BaselR > \frac{RE^L_{t,NP} + EC^{L,B}_{t,NP}}{AT^L_{t,NP}}; \tag{37}$$

$$NI^L_{t,P} > NNI^L_{t,NP}. \tag{38}$$

To facilitate analysis, we rewrite EC^L from Equations (30) and (31) to Equations (36) and (37). Here are the similarities and differences between B^S in Equation (32) and B^L in Equation (37). Both equations indicate non-performing loans on the balance sheet. However, the major difference is that B^S will be capital constrained but B^L will not. For B^S, Equation (32) tells us that bank runs will start if B^S does not liquidate $AT^S_{t,NP}$ at t because households will spread the news that B^S does not have enough bank capital. However, B^L is not subject to the same regulatory capital constraints as B^S because B^L's income from performing loans is higher than the cost of the non-performing loans, as in Equation (38)

In other words, B^L's overall loan portfolio satisfies regulatory capital requirements, although one or more loans are nonperofrming. B^L can keep $AT^L_{t,NP}$ on its balance sheet at $t+1$. Without the news of liquidating non-performing loans, households keep rolling over their deposits as patient bank lenders.

Even if bank runs occur, the central bank playing the lender of last resort is justified because the large bank is systemically important. After early bank lenders withdraw their deposits, and Libor is expensive, central bank liquidity could neutralize funding liquidity constraints. B^L can borrow enough β at a profitable interest rate ($BL_{bwr} > FL_{cbl,t}$), where $FL_{cbl,t}$ is the interest rates offered by the central bank. For the remaining loans that have not been hit by bank runs, household deposits are the funding liquidity ($FL_{td,t}$).

Without being constrained by funding liquidity or equity capital, B^L practices due diligence to review alternative choices for non-performing loans ($AT^L_{t,NP}$). Doing so can maximize its capital efficiency. Again, we need utility maximization to motivate this practice.

We follow the standard first-passage-time default assumption that borrowers will not resume coupon payments after default confirmation, missing two consecutive coupon payments. Lenders can only recover the loan principal through the loan collateral.

Now, we introduce collateral deterioration. By nature, collateral depreciates over time. However, care can make a difference. We define non-performing loans with good collateral (AT_{NP}^{gc}) where the underlying collateral has been maintained well when the loan has been identified as non-performing. Non-performing loans with bad collateral (AT_{NP}^{bc}) indicate that the underlying collateral only carries residual value when the loan has been identified as non-performing.

Next, we combine orderly or forced transactions with collateral variations. Good collateral sales prices in orderly transactions ($AT_{NP}^{gc} - Dis_{GC}^{fs,nor}$) are higher than in distressed transactions ($AT_{NP}^{gc} - Dis_{GC}^{fs,frc}$). However, the fetching price of bad collateral in an orderly transaction ($AT_{NP}^{bc} - Dis_{BC}^{fs,nor}$) will be similar to that of a distressed transaction ($AT_{NP}^{bc} - Dis_{BC}^{fs,frc}$)[22].

$$AT_{NP} = \begin{cases} AT_{NP}^{gc}: & AT_{NP}^{gc} - Dis_{GC}^{fs,nor} > AT_{NP}^{gc} - Dis_{GC}^{fs,frc}; \\ AT_{NP}^{bc}: & AT_{NP}^{bc} - Dis_{BC}^{fs,nor} = AT_{NP}^{bc} - Dis_{BC}^{fs,frc}. \end{cases} \tag{39}$$

With the collateral heterogeneity in Equation (39), we propose the criteria of rational decisions to liquidate or keep non-performing loans. It is rational to hold non-performing loans on the balance sheet if the holding costs are lower than expected valuae appreciation. Say, a bank has confidence in the collateral, but the markets dislocate the collateral away from its fair value. Holding non-performing loans is rational if the expected value appreciation ($[\,AT_{NP}^{gc} - Dis_{GC}^{fs,nor}] - [\,AT_{NP}^{gc} - Dis_{GC}^{fs,frc}]$) is higher than the funding cost of holding non-performing loans, as shown in Equation (40). Because the bank has confirmed the non-performing status, we assume, just for simplicity, that CCI and $OpEx$ drop to zero. Equation

[22]The descriptions for the subscripts are *nor* (orderly transactions), *frc* (forced or distressed transactions), GC (good collateral), and BC (bad collateral).

(40) predicts three large bank operations.

$$\text{Criterion to hold } AT_{NP}: \ AT_{NP}^{gc} \times FL_{cbl,t} < Dis_{GC}^{fs,frc} - Dis_{GC}^{fs,nor}. \tag{40}$$

Operation 1: liquidate non-performing loans with bad collateral. Although lower than market interest rates, the central bank interest rate is yet higher than zero. From Equation (39), B^L knows $Dis_{BC}^{fs,nor} = Dis_{BC}^{fs,frc}$ for loans with bad collateral. When replacing good collateral with bad collateral in Equation (40), the conclusion has changed because the costs of holding non-performing loans are higher. Therefore, writing off non-performing loans with bad collateral is rational.

This prediction is consistent with the MBS writedown during the 2008 financial crisis. The collapse of the subprime mortgage markets began in July 2007 (Longstaff, 2010). By September 2008, large banks reported 79.8% of cumulative MBS writedown losses from the finance sector[23].

Operation 2: Hold the existing non-performing loans. Following $FSS3$ to mark to market non-performing loans with good collateral against orderly transactions ($AT_{NP}^{gc} - Dis_{GC}^{fs,nor}$) when the market transaction prices are distressed ($AT_{NP}^{gc} - Dis_{GC}^{fs,frc}$), B^L cannot liquidate existing non-performing loans. Therefore, the large bank must pay funding costs until the loan is liquidated. Holding non-performing loans is rational only if the holding cost $AT_{NP}^{gc} \times FL_{cbl,t}$ is lower than the difference between two fire-sale discounts as in Equation (40). B^L must reevaluate Equation (40) in each period.

Operation 3: Equation (40) also predicts continuous risk taking, purchasing more non-performing loans at excessive discounts ($AT_{NP}^{gc} - Dis_{GC}^{fs,frc}$) as long as the large bank is not subject to capital constraints. This is because the right hand side of the Equation (40) becomes expected profits higher than the holding costs. This is consistent with increasing risk-weighted assets after 2007Q3 for the 22 leading U.S. banks, column 1, panel A Table I.

[23]Source: Yalman Onaran and Dave Pierson, Banks' Subprime-Related Losses Surge to $591 Billion, Bloomberg, September 29 2008.

The model in Shleifer and Vishny (2010) also predicts the purchase of distressed securities during the 2008 crisis. However, their paper has a different mechanism and predicts different results. In Shleifer and Vishny (2010), fire sales have wiped out bank equity, and banks cannot borrow more. Purchasing distressed securities generated higher returns than normal loan projects could match. In summation, purchasing distressed securities has a crowd-out effect on bank's traditional functions on risk and maturity transformation.

3.6 Stage 3: non-performing loans and banking system downsize

A natural extension of the small bank failure is to analyze how the non-performing loans of the large bank could compromise financial stability. We demonstrate the process through which non-performing loans on the balance sheet of the large bank progress to constrain the capital of the banking system. The negative net income of B^L's loan portfolio, $NNI_{t,NP}$, could increase for two reasons.

First, purchasing non-performing assets at deep discounts at t and practicing $FSS3$, B^L can book asset appreciation in the next period $t+1$. However, there are two side effects. Side effect (1), B^L cannot close the position without reporting losses if fire-sale discounts remain distressed. Side effect (2), B^L must pay the net founding cost as long as the non-performing loans stay on the balance sheet. Once side effect (2) outweighs the discount difference, the dislocated non-performing assets purchased as a bargain will hurt B^L balance sheet by reporting $NNI_{t,NP}$. This is the first reason why $NNI_{t,NP}$ will increase.

Second, it is natural to assume that households will not deposit their savings to support purchasing non-performing loans. Therefore, B^L borrows from the market libor ($FL_{libor,t}$). During crisis periods, market libor ($FL_{libor,t}$) is more expensive than household deposits ($FL_{libor,t} > FL_{td,t}$).

$NNI_{t,NP}$ impacts B^L's loan portfolio as follows. See Eqaution (41). Thanks to the profits from the performing loans ($NI_{t,P}$), equity capital (EC) increases ($\gamma NI_{t,P}$). However, the lower retained earnings are due to the holding costs of non-performing loans, $NNI_{t,NP}$.

$$EC + RE = \begin{cases} EC_{t,P} = EC_{t-1,P} + \gamma NI_{t,P};\ EC_{t,NP} = EC_{t-1,NP}. \\ RE_{t,P} = RE_{t-1,P} + (1-\gamma)NI_{t,P};\ RE_{t,NP} = RE_{t-1,NP} - |NNI_{t,NP}|. \end{cases} \tag{41}$$

$$RE\ warning:\ EC_t > EC_{t-1} \text{ but } RE_t < RE_{t-1} \because (1-\gamma)NI_{t,P} < |NNI_{t,NP}| \tag{42}$$

Here is our definition of *RE warning.* The costs of holding nonperforming loans are higher than the net income of performing loans allocated to retained earnings; see Equation (42). Upon a RE warning, the bank reports divergence of increased EC but decreased RE in period t compared to $t-1$; see Equation (41).

The RE decrease is what banks should be concerned about. After the RE warning in period t, the large bank should liquidate the non-performing loans under the utility maximization assumption that B^L has maxed out its equity capital as any small banks. However, the large can continue to practice its maturity and risk transformations for k periods ($k \geq 1$) as long as net income from performing loans is higher than the holding costs of non-performing loans ($\gamma NI_{t,P} + (1-\gamma)NI_{t,P} > |NNI_{t,NP}|$).

The warning becomes a red flag if the net income of the performing loans becomes insufficient to cover the funding costs of the nonperforming loans, as in Equation (43). At $t+k$, $RE_{t+k} + EC_{t+k} < RE_{t+k-1} + EC_{t+k-1}$. The large bank must deploy capital reserves, or it is capital constrained. Assume that the capital reserves satisfy $EC^R + NI_{t+k,P} - |NNI_{t+k,NP}| > 0$. The large bank complies with the Basel capital ratio in period $t+k$, as in Equation (44). The large bank can purchase non-performing loans with good collateral through distressed transactions from the market, justified in Equation (40).

$$\text{Red flag}:\ NI_{t+k,P} < |NNI_{t+k,NP}|. \tag{43}$$

$$\text{Because } BaselR < \frac{RE_{t+k} + EC_{t+k} + EC^R}{AT_{t+k}},\ AT_{t+k+1} = AT_{t+k} + AT_{up}. \tag{44}$$

However, here is the caveat. Funding costs of nonperforming loans reduce the capital reserves by $|NNI_{t,NP}|$ in each period. If the markets remain distressed, the large can not liquidate the non-performing loans. Funding costs for the bad loans will exhaust the capital reserves from period $(t+k)$ to period N. Once capital reserves drop to zero. The revenue of performing loans is less than the holding costs of non-performing loans $(NI_{N,P} < |NNI_{N,NP}|)$, and the large bank becomes capital constrained, as in Equation (45).

$$EC^R < \sum_{t+k}^{N} NNI_{t,NP};\ BaselR > \frac{RE_N + EC_N}{AT_N}, \tag{45}$$

Once the large bank is subject to capital constraints, two textbook operations are issuing new equity and downsizing assets. However, being large, here are two abnormal operations. **Abnormal operation 1**: Manipulate borrowing costs. Typically, Libor has been elevated when market transactions are distressed. B^L draws part of its funding liquidity from the Libor market. With the power to set Libor, B^L is incentivized to manipulate the market Libor $(FL_{mlibor,t})$ so that it can increase its revenue $(FL_{mlibor,t} < FL_{libor,t})$, Equation (46). An example of abnormal operation 1 is the Libor scandal, a criminal offense in the UK.

$$NI_t = AT_t \times (BL - \beta FL_{mlibor,t}) > AT_t \times (BL - \beta FL_{libor,t}). \tag{46}$$

Abnormal operation 2: Report an artificially higher equity capital $(RE^L + EC^L)$. With a higher capital, the large bank complies with the Basel ratio without the consequences of bank runs. To avoid litigation risk, the large bank reports a higher capital to the Federal Reserve Board only, while reporting the noninflated equity capital to the SEC. This is consistent with what we have observed in Table I column (8).

Issuing new equity and downsizing become essential if the large bank is still below the Basel capital requirements after practicing the aforementioned normal and abnormal operations. In table I Panel A, this is in 2009Q2. Large banks added 19 percentage points of common shares outstanding from 117 in 2009Q1. They cumulatively issued 121 percentage points of new equity from 2009Q1 to 2009Q4.

Furthermore, large banks reduced risk-weighted assets by 6 percentage points from 2009Q1 to 209Q4. Because B^S does not have capital capacity, the downsize of B^L indicates that the entire banking system has practiced less maturity and risk transformation since 200Q2. Simply put, financial stability was compromised in three quarters in 2009, and this is a systemic crisis. Our model has delivered five stylized facts for large banks in the 2008 crisis.

$$BaselR > \widehat{BR}_N^L = \frac{\alpha \times (RE^S_{N,P} + EC^{S,B}_{N,P}) + RE^L_{N,NP} + EC^{L,B}_{N,NP}}{\alpha \times AT^S_{N,P} + AT^L_{N,NP}}, \text{ and } EC^{L,R} = 0. \quad (47)$$

The mechanism of the bank asset problem also indicates a unique spillover effect. Large banks will not liquidate non-performing loans ($AT^L_{N,NP}$) due to distressed fire-sale discounts, as shown in Equation (47). Therefore, they sell performing loans because fire-sale discounts ($\alpha \times AT^S_{N+1,P} - AT_{down}$) are smaller. For bank borrowers, the spillover is unique. When banks must downsize due to non-performing borrowers, spillovers are actually on the performing borrowers. We term this unique spillover as the cross-borrower spillover and will explore its empirical spillover effects in Chu and Ou (2022).

3.7 What can cause bank asset problem

In all of the aforementioned analyses, households are bank lenders only. In this subsection, we extend the assumption that some households are mortgage loan borrowers. After inducting household mortgage loans, we add two more extensions. Extension one, banks that originate mortgage loans can securitize these loans to MBS and sell the MBS to other banks. In extension two, households can default on their mortgage loans.

When we analyze the impacts of bad loans, a defaulted corporate loan, due to its size, will exert a larger shock on the banking system than any individual household mortgage loans. Fortunately, corporate default events have been idiosyncratic in history. This is because the credit risk of entrepreneurs is unique.

Conventional wisdom has long applauded the benefits of loan securitization for economies and financial markets. However, limited attention has been paid to its side effects. Securitization practice of pooling and transforming millions of individual mortgage loans into standard MBS has achieved two changes. First, the process exposes mortgage loan borrowers to a *homogeneous* risk of falling housing prices or rising interest rates. Second, the mass of MBS has become much larger than any individual corporate bond issuance.

We summarize the two changes as risk aggregation. Although not good news, it could be less of a concern if bank exposure is limited. However, the concern becomes alarming when large and small banks purchase non-trivial MBS as their balance sheet assets as we know from the 2008 financial crisis.

The risk premium of corporate debt coupons includes two main elements: credit risk and interest rate risk. Although the former is idiosyncratic, the interest rate risk is homogeneous to all debt borrowers. Therefore, a period of interest rate increases could be the environment for the bank asset problem to brew. The collapse of Silicon Valley Bank is one example.

4 Policy Recommendation and Discussions

Our policy recommendation is about the solution to slow asset recovery. Because banks must make many loans and because part of borrowers defaulting on their loans is natural, bank runs can be avoided if we can prevent the slow asset recovery. Our policy recommendation is in Equation (48). We recommend that banks must mark to market the profits of good loans and liquidation costs of the bad loans and ensure that the former is higher than the latter $LiqCst_{bwr2} = Dis^{fs}_{bwr2} + |L_{t,bwr2}|$. Similar to the Basel capital buffer, we term our recommendation the *revenue buffer*.

$$P_{t,bwr1} > LiqCst_{bwr2}; \tag{48}$$

Liquidation costs include three major elements. The two elements on deposit's interest rate and fire-sale discounts have been covered in previous sections. We quickly mention $OpEx$, which includes three costs: salary compensation, office, and sales and marketing. What is important but usually overlooked is that bank employees are fired when banks are liquidated. Let us be clear. By salary, we never mean the money paid or to be paid to a bad CEO or a rogue trader that just screwed up a financial institution that has served the economy for more than 100 years. We mean the employees whose salary is in the range of mean or below that of the financial industry.

5 Conclusion

In the banking industry, a bank must make many loans to reduce the cost of monitoring the monitor Diamond (1984). The rationale is that revenues from successful loans can subsidize unsuccessful loans. Therefore, monitoring costs in an economy could be reduced. Lower borrowing costs facilitate the efficiency improvement of debt intermediaries by channeling household savings to risk-taking operations by entrepreneurs.

We ask the question what if the revenues from successful loans struggle to subside the losses from unsuccessful loans? We ask this question to rationalize bank operations in reality. Two pillars support the proposed rationale. The first is the change in regulatory requirements. Four years after Diamond's publication, Basel I was published in 1988 and enforced in 1992. Second, banks practice two-tier future asset maximization. In an effort to improve intermediary efficiency, banks also maximize their long-term assets, rather than the utilities of households or entrepreneurs.

Here is a summary of the mechanism when some of a bank's loans become non-performing. Cross-subsidizing works but with a limit. The limit is the Basel capital ratio. When a bank must liquidate its non-performing loans, fire-sale discounts can not be avoided and will be deducted from bank capital. Furthermore, fire-sale discounts vary and could be steep in distressed markets. After liquidating non-performing loans, a bank could end up with one of the two results. If the profit-generating capacity is strong, a bank will quickly recover its pre-liquidation asset. The current patient bank lenders will keep their patience. This bank can maintain its resilience after the shock and maintains its growth. Otherwise, patient bank lenders will become early lenders due to slow asset recovery. Their withdrawal requests will lead to panic-based bank runs.

For small banks, the asset problem could cause more damages for two reasons. They have less performing loans due to their small size. The central bank's liquidity guarantee as the lender of last resort could not apply to small banks due to moral hazard concerns. Once plagued by a bank asset problem, the odds are high that liquidation of bad loans will lead to the small bank being liquidated. Acquiring banks only pays a small fraction of the bank capital of the acquired bank. This is an example of the SVB collapse and acquisition.

Large banks have two advantages that small banks lack. They have capital buffers. Furthermore, the lender of last resort can neutralize liquidity shocks when they experience panic-based bank runs. With the advantages, large banks can keep non-performing loans with good collateral on the balance sheet. They can also purchase new non-performing assets with good collateral at deep discounts. Both are what happened in the 2008 crisis.

However, holding non-performing assets requires funding costs. Due to limited performing loans, the capital buffer has to pay the holding costs of non-performing loans. If the capital buffer is exhausted before the recovery of an orderly market, lending banks are under capital constraints. During the 2008 crisis, large banks were affected. The intermediary sector was compromised because small banks did not have the capacity to fill the service vacuum. As in stylized fact (e), the banking sector downsized and recapitalized in most of 2009. This is evidence of financial instability.

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Table I
Large Banks' Basel Capital and Aggregate Borrowing

Panel A: Large banks' risk transformation, equity capital, and source of capital

This panel reports balance sheet items reported in FR Y-9C by 22 large banks overlapping with 97 leading intermediaries. Columns (1) to (4) and (6) to (8) are equal-weighted ratios or indexes. For an FR Y-9C index item (code starting with BHCK), the value of each individual bank is set equal to 100 in 2006Q4. The formula ratio is given in columns (4) and (8). The ratio of column (5) is calculated as (BHCK3210-BHCK3247)/BHCK3210.

Time	BHCKA223 total risk-weighted assets	BHCK2170 total assets	BHCK3210 total equity capital	BHCK3247 retained earnings	Calculated book equity sum of 5 items	BHCK3459 common shares outstanding	BHCK7205 total risk-based capital ratio	Calculated Ratio of total equity capital FR Y-9C/10-Q
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
2006Q4	100	100	100	100	100	100	13%	98%
2007Q1	102	104	101	103	99	100	12%	98%
2007Q2	107	109	102	106	94	100	12%	98%
2007Q3	111	113	104	108	96	99	12%	101%
2007Q4	113	114	103	104	101	98	12%	101%
2008Q1	115	116	107	103	117	100	12%	102%
2008Q2	117	116	112	102	132	102	13%	104%
2008Q3	121	126	111	100	132	103	12%	105%
2008Q4	131	134	132	90	219	110	15%	131%
2009Q1	137	136	144	91	254	117	15%	132%
2009Q2	133	135	148	92	265	136	15%	120%
2009Q3	131	136	151	92	275	212	15%	115%
2009Q4	131	135	148	90	269	238	15%	113%