Information Spillovers, Funding Liquidity, and Financial Stability*

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Abstract

We develop a theory of beliefs and financial stability in which beliefs about fundamentals are shaped by the availability of liquidity in funding markets. Agents exchange debt contracts to invest in a risky asset. They receive private signals about the fundamental of the asset and are subject to idiosyncratic liquidity shocks. Agents can partially, but never fully, infer their counterparties' private information from asset prices or the cost funding. As a result, liquidity shocks in funding markets lead to belief-driven booms or busts. Abundant funding liquidity leads to exuberance, while tight liquidity leads to pessimism. Information spillovers, whereby asset prices affect agents' beliefs, may amplify or dampen the likelihood and severity of fire sales depending on the underlying shock. Central bank lending facilities may stabilize markets in part by reducing the informativeness of asset prices, thereby reducing the volatility of beliefs.

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1 Introduction

Financial markets are inherently unstable and prone to episodes of panic. An important function of financial markets is aggregating private information about financial assets and facilitating price discovery. Yet in times of financial stress, when perhaps information matters the most, this function seems to break down, with sharp and temporary increases in uncertainty and excessive pessimism about fundamentals.[1] These episodes of financial panic are characterized by sudden and sharp declines in asset prices and a rise in risk premia, leading to credit and liquidity freezes which may come at great social costs. Moreover, a large literature has argued that asset price booms, in which spreads are compressed and valuations are stretched, are in part driven by investor exuberance, in which beliefs or expectations about asset returns become divorced from underlying fundamentals.[2] Therefore, an understanding of how beliefs are shaped over the financial cycle is important to better understand the nature of financial panics and provide appropriate policy recommendations.

The literature has made substantial progress in understanding the role of beliefs in financial markets. There is a vibrant literature on the intersection of dispersed information and financial stability, including Gorton and Ordonez (2014), Asriyan (2021), Asriyan et al. (2022), Bostanci and Ordoñez (2024), Garcia-Macia and Villacorta (2023), and Dávila and Walther (2023). There is also a literature studying in more general terms asset markets with information frictions and learning, including Simsek (2013), Kurlat (2016), Asriyan et al. (2017), Dávila and Parlatore (2021), and Asriyan et al. (2021). Finally, a large literature studies asset prices and fire sales in a context of financial frictions, including Lorenzoni (2008), Brunnermeier and Pedersen (2009), and Malherbe (2014).

In a similar spirit, this paper presents a theory of belief-driven asset price fluctuations . We make three contributions. First, we develop a model which shows how beliefs about fundamentals are shaped by the availability of liquidity in funding and asset markets. In the model, asset prices only imperfectly aggregate private information about fundamentals. That is, agents can partially, but never fully, infer their counterparties' private information from asset prices, including the price of debt. Asset prices shape agent's beliefs due to *information spillovers* in which the cost of financing affects agents' beliefs about fundamentals. As a result, idiosyncratic shocks to the availability of funding liquidity cause market participants to have overly optimistic or pessimistic beliefs about asset fundamentals.

Our second contribution is to spell out the financial stability implications of this mechanism.

[1]For example, the severity of the liquidity freezes in debt and money markets in 2008 and 2019 suggest that these episodes were driven at least in part by temporary changes in the perceived riskiness of the underlying assets (Covitz et al., 2013; Bernanke, 2023).

[2]This literature goes back at least to Kindleberger (1978), and more recently includes Shiller (2005) and Barsky and Sims (2012).

We show that idiosyncratic liquidity shocks can lead to *belief-driven booms or busts* in asset markets, resulting in excessive investment or panic-driven fire sales. Abundant funding liquidity leads to exuberance and over-investment, while tight liquidity leads to pessimism and panic-driven fire sales. We also identify the conditions under which information spillovers have a stabilizing or destabilizing effect on financial markets on average, as measured by the likelihood and severity of fire sales.

Our third contribution is to explore the policy implications of the theory. We show that, during episodes of stress marked by funding illiquidity, government interventions to stabilize financial markets, such as liquidity facilities and asset purchases, may operate in part by *reducing* the informativeness of asset prices, which can prevent market participants' beliefs about fundamentals from becoming excessively pessimistic.

Our theory is motivated in part by the nature of various financial markets, such as repo markets, markets for asset-backed paper, and over-the-counter markets for securities and derivatives, in which short-term, collateralized debt is used to finance investment in a longer-maturity asset. These markets have grown in importance in recent years and have featured occasional episodes of severe market disruption featuring a drying up of liquidity and panic-driven asset sales. We discuss in detail the real-world relevance of our model in section 1.1.

We build a three-period model of belief disagreement featuring learning and fire sales. In the initial period (date 0), a borrower issues short-term debt to a lender in order to finance investment in a long-term risky asset, which matures at the final period (date 2). Agents have a common prior belief about the date 2 return of the risky asset. The debt matures at the interim period (date 1) and is collateralized by the borrower's holdings of the risky asset. At date 1, the borrower must refinance this debt with the lender, potentially under new terms, or liquidate some of its holdings of the risky asset to cover any shortfall in its funding.

At date 1, each agent receives a private, noisy signal about the asset's future (date 2) return. In addition, the agent is simultaneously subject to an idiosyncratic shock to the opportunity cost of its funds at date 1 (a 'cost shock'), which is orthogonal to the asset's future return. Both the cost shock and the signal received by the agent are private information. For simplicity, we assume that only one agent receives both a cost shock and a private signal, while the other agent receives neither.[3] While much of our exposition here focuses on the case in which the lender receives the signal and shock, we show in section 7 the same insights apply for the other case even if the mechanics differ slightly.

[3]This assumption makes makes it more tractable to solve for agent's equilibrium beliefs. If, in contrast, both agents received private signals, then the equilibrium would depend not only on how one agent, e.g. the borrower, updates its beliefs in response to the actions of the other, but also on an infinite feedback loop in which also the borrower's actions affect the lender's beliefs, etc. This would make an analytical characterization of equilibrium belief formation extremely difficult.

In response to its private signal, an agent updates its prior belief about the risky asset's date 2 return (the fundamental) according to Bayes' Rule, and can reallocate its portfolio at date 1 given its posterior beliefs. In response to either an adverse cost shock or negative news about the risky asset's return, the lender is less willing to refinance the borrower's debt at date 1, since the debt is collateralized by the risky asset.[4] This results in a contraction in the supply of credit at date 1. Therefore, the equilibrium price of debt conveys information to the borrower about the lender's private information.

However, asset prices only *partially* convey private information in this environment. The borrower cannot perfectly disentangle the lender's private news about the fundamental and its cost shock by looking at the price of debt.[5] We refer to this as the 'identification problem' faced by the borrower. Hence, financial markets are *informationally inefficient* in that prices do not perfectly reveal agents' private information about the fundamental.[6] [7]

This environment leads to the key mechanism of the model: Beliefs about fundamentals are endogenously shaped by the availability of liquidity in funding markets due to an *information spillover* whereby asset prices affect agents' beliefs about the fundamental. To illustrate this, suppose the lender's opportunity cost of funds rises at date 1, causing it to reduce the supply of new credit to the borrower. (In other words, the shock makes it more expensive at date 1 for the borrower to roll over its initial debt). Because the borrower cannot perfectly infer both the cost shock and the lender's private signal from the price of its new debt, it does not know whether the lender reduced its supply of credit because it faces a higher cost of funds or because it received bad news about the future return of the risky asset, which ultimately serves as collateral for the loan. Therefore, the borrower puts some weight on this latter possibility, causing it to revise downward its belief about the risky asset's future return. In turn, this greater pessimism on the part of the borrower reduces its willingness to hold the risky asset. This happens even though the cost shock is orthogonal to the fundamental. Symmetrically, if the lender instead receives a positive cost shock such that the price of debt at date 1 is high, the borrower revises upward its belief about the risky

[4] Similarly, for the borrower, an adverse cost shock or negative news reduces it's willingness to hold the risky asset as opposed to cash at date 1.

[5] Therefore, each agent faces two layers of uncertainty when forming its posterior beliefs about the asset's fundamental: It takes into account its own noisy signal about the fundamental, and it takes into account asset prices, which serves as a noisy signal about its counterparty's private signal. In turn, to form a belief about the private signal of its counterparty, it computes likelihood of all possible realizations of signals and cost shocks consistent with observed asset prices.

[6] While there are various ways to define informational efficiency, we define it in this context as how well agents can infer the private information of their counterparties relative to the common information benchmark, for a given realization of shocks and signals.

[7] This informational inefficiency derives from the the incompeleteness of financial markets, which implies that the price of each asset reflects two unknowns. The underlying assumption is that agents can neither observe one another's private information, nor trade securities contingent on private information. This assumption is a stand-in for the various frictions which imply that financial markets may not be informationally efficient and risk markets are incomplete.

asset's fundamental, becoming more optimistic as a result.[8]

In standard models, such information spillovers are part of the workings of the price-signal mechanism whereby asset prices, in revealing agents' private information about asset fundamentals, facilitate the convergence of beliefs to fundamentals. In our model, by contrast, information spillovers perversely cause beliefs to diverge from fundamentals. Why, in this environment, does the price-signal mechanism break down?

To shed light on this question, we compare the workings of our baseline model to a counterfactual benchmark economy in which all information is *publicly* observed by all agents (the 'common information benchmark'). Note that information is still incomplete here: Agents still receive noisy signals about the asset's date 2 return. The difference is that here, signals are publicly observable rather than being private. Agents therefore have a common information set. A comparison of our baseline model to this benchmark reveals how the configuration of incomplete markets and asymmetric information in our model markedly changes the nature of the price-signal mechanism and the role of asset prices in disseminating private information.

In our baseline model environment, the informational efficiency of financial markets depends on the availability of funding liquidity. Put differently, particularly tight or loose funding liquidity may impair the ability of prices to reveal agents' private information about fundamentals. The reason is that asset prices play a dual role in our model: They allocate funds between assets, and they convey agent's private information about the fundamental. The absence of complete markets implies that the price of an asset has to perform both functions.[9] Therefore, the more that an asset price has to adjust in response to cost shocks in order to reallocate funds at date 1, the less it reveals information about agents' private signals. Put differently, the price of debt serves as a noisy signal to the borrower about the lender's belief, where cost shocks play the role of noise. Hence, shocks to the availability of funding affect the informativeness of the price of debt about the fundamental.

The result of this informational efficiency is that agents' beliefs may become divorced from fundamentals in response to funding illiquidity at date 1. An adverse cost shock to the lender which causes the date 1 price of debt to fall results in the borrower becoming pessimistic relative to the common information benchmark, despite the fact that the shock is orthogonal to fundamentals. In contrast, a positive cost shock to the lender which causes funding liquidity to rise results in excessive optimism on the part of the borrower. Moreover, the larger (smaller) is the cost shock

[8]In section 7, we show that similar insights hold for the opposite case in which borrower receives cost shock and signal. In particular, market illiquidity, defined as a low risky asset price at date 1, may cause the lender to become over-optimistic.

[9]Indeed, an old literature has shown that prices may reveal information only imperfectly in the absence of complete risk markets. For example, Stiglitz (1981) showed that in the absence of complete risk market, prices must not only clear markets and aggregate information, but also allocate risk. As a result, asset prices may relay information only imperfectly. In our paper, we show how the informational inefficiency of financial markets may vary according to liquidity conditions, and its implications for asset price booms and busts.

to the lender, the more pessimistic (optimistic) the borrower is about the risky asset relative to the benchmark.

Turning to financial stability considerations, we show that this optimism and pessimism gives rise to the possibility of belief-driven booms or busts in financial markets. To capture some notion of financial market distress, we introduce the possibility of fire sales of the risky asset at date 1, along the lines of Lorenzoni (2008) . These fire sales occur either when the borrower lacks the liquidity necessary to refinance its holdings of the risky asset at date 1, or when it has become sufficiently pessimistic about the asset's future return that it prefers to hold more cash at date 1.

Our paper shows that tighter funding liquidity can give rise to *belief-driven fire sales* in which the risky asset is liquidated because agents are excessively pessimistic about fundamentals. For instance, when liquidity conditions in funding markets are tight, resulting in a low bond price, the borrower may become overly pessimistic about the asset fundamental at date 1, believing the lender to have received negative news about it. As a result, the borrower may liquidate its holdings of the risky asset depressing the price of the asset below the value justified by its fundamentals, relative to the common information benchmark. The nature of such fire sales is different from much of the literature in that they are belief-driven: In response to tight funding liquidity conditions, the borrower liquidates the asset not simply because of its need for liquid funds, but rather because the borrower endogenously becomes more pessimistic about the fundamental value of the asset.

On the other hand, abundant funding liquidity can give rise to *investment booms* characterized by overly exuberant beliefs about fundamentals. In particular, a high price of date 1 debt leads the borrower to become overly optimistic about the fundamental value of the asset relative to the common information benchmark. This leads to over-investment in the asset at date 1 and to the borrower bearing excess losses in bad states of the world at date 2, relative to the benchmark.

We next analyze whether, in this environment, information spillovers have a stabilizing or destabilizing effect on financial markets on average. To that end, we study how information spillovers in our model affect the likelihood and severity of fire sale episodes relative to the common information benchmark.[10]

In response to adverse cost shocks which reduce funding liquidity, the information spillovers generated in our model *amplify* both the likelihood and severity of fire sales. This is because information spillovers leave agents more pessimistic about fundamentals in response to adverse cost shocks. As a result, information spillovers enlarges the region of the state space in which the fire sale regime applies, making fire sales more likely in expectation compared to the common information benchmark.[11] Moreover, conditional on a fire sale occurring at date 1, this greater

[10]The severity of a fire sale is defined by how much of the risky asset is liquidated at date 1.

[11]More precisely, the likelihood of a fire sale conditional on a particular cost shock, and evaluated over the possible news shocks, is higher under the model with information spillovers than in the common information benchmark.

pessimism results in more of the risky asset being liquidated and at a lower price.

By contrast, when tight funding liquidity is driven by bad news, the information spillovers *dampen* the likelihood and severity of fire sales, as agents are relatively more optimistic than they would be under common information. This optimism implies they are less likely to fire sell the risky asset, and if they do, they sell less and at a higher price. Thus, financial markets overreact to negative liquidity shocks but are less responsive to bad news about fundamentals, because of how these shocks differentially affect the borrower's beliefs relative to the common information benchmark.

The dynamics of the model derive from the interaction of two externalities—a standard pecuniary externality through the price of the risky asset, and an 'information externality' in which agents fail to internalize how their decisions affect the information set and beliefs of other agents. In particular, agents do not internalize how asset prices affect other agents' beliefs through the information externality. In addition, agents' beliefs affect asset prices, which reinforces fire sales through the pecuniary externality.

Turning to the model's policy implications, we show that central bank lending facilities which provide loans against risky assets, such as the various facilities employed during the Covid-19 pandemic, may stabilize markets in part by reducing the informativeness of asset prices, and reducing volatility of beliefs as a consequence. By stabilizing prices at government-administered rates, such interventions render those prices uninformative about agents' private information about fundamentals. In that sense, the policy intervention *destroys* information, further deteriorating the informational efficiency of the market.

Nevertheless, such interventions may be socially desirable. Reducing the informativeness of prices can prevent agents' beliefs about fundamentals from becoming excessively pessimistic and causing them to fire sell assets due to the information spillover. Intuitively, if agents know that the availability of funding is pinned down by the government rather than private information about fundamentals, they are less likely to become pessimistic about fundamentals in response to a tightening of private funding. Therefore, these interventions stabilize markets not only directly by providing liquidity, but also indirectly by preventing wild swings in the beliefs about fundamentals. Indeed, policymakers have frequently cited the effect of their interventions on a vague notion of investor confidence as a stabilizing force. Our model thus provides insight about these forces.

Finally, we show that the insights of our model obtain for the alternative case in which information spillovers propagate upstream, from the borrower to the lender, through the availability of market liquidity. Our analysis thus far focused on the case of downstream information spillovers in which the lender receives a cost shock and a private signal, and how funding liquidity affects the informativeness of the price of debt. In section 7, we show that similar insights obtain for the alternative case in which it is the borrower who receives a cost shock and a private signal. In that case, the risky asset is priced by the borrower. Therefore, the price of the risky asset serves as a noisy signal to the lender about the borrower's beliefs. Therefore, cost shocks to the borrower shape the lender's beliefs through the availability of market liquidity.

1.1 Discussion of Real-World Applications

Our model is designed to capture, in a parsimonious way, salient features of various financial markets. With respect to the structure of lending, for example, repo markets, markets for asset-backed commercial paper, and over-the-counter markets for securities and derivatives typically feature collateral to protect lenders against counterparty risk. Moreover, at their core, the debt in these markets is short-term debt and is used to finance investment in longer maturity asset. As such, this debt either explicitly needs to be rolled over, is renegotiable, or is subject to margin requirements which effectively render it short-term. The collateralized, short-term debt in our model is intended to capture such characteristics.

Another salient feature of these markets is that they typically feature private information. In certain cases, lenders may be better-informed about the assets backing the debt. For example, consider a repo contract between a broker-dealer and a hedge fund in which the hedge fund invests in a risky asset and borrows from a dealer using the asset as under collateral. Dealers typically have additional information about the return of the asset. Even if the collateral asset is a Treasury security, the value of the asset at the maturity of the repo contract may vary depending on changes in the yield curve affected by various macro factors, which dealers, by virtue of their expertise, may have more information about. Similarly, in markets for asset-backed commercial paper, the issuers of the securities may have better information on the value of the underlying collateral securities but would not hold the securities themselves and instead lend funds to leveraged buyers.

Moreover, lenders in this situation may be unable to credibly reveal their private information to their counterparties for two reasons. On the one hand, they may have an incentive to claim that the asset is worth less than in order to be able to lend against the asset at a higher price. On the other hand, they may have incentive to inflate the collateral value of the asset in order to prevent borrowers in the market from liquidating assets at fire sale prices.

In other cases, however, the borrower may be better-informed about the fundamental of the risky asset. For example, consider the secondary market for corporate bonds in which an asset manager finances its purchase of a corporate bond by borrowing from a dealer in an over-the-counter market. The asset manager may be able to monitor the issuer of the corporate bond more closely than the dealer, who is one step further removed from the corporate issuer, and therefore may obtain private information about the credit risk of the bond that the dealer does not get. Moreover, the borrowers in this situation may be unable to credibly reveal their private information, as they have an incentive to inflate the collateral value of their asset holdings. As we show in section 7, the core insights of our model still obtain in this case, although the mechanism differs slightly.

1.2 Related Literature

Our paper relates to several strands of the literature. The paper is similar in spirit to the theoretical literature studying liquidity freezes in financial markets, including Brunnermeier and Pedersen (2009), who study how market liquidity and funding liquidity interact to give rise to illiquidity spirals, and Gale and Yorulmazer (2013).[12] In a similar vein, Garcia-Macia and Villacorta (2023) show how information frictions between banks can cause liquidity hoarding and freezes in the interbank market. Plantin (2009) build a model of learning in which adverse selection leads to a self-fulfilling dry-up of liquidity. Relatedly, Malherbe (2014) presents a model in which the availability of liquidity affects the severity of adverse selection. Relative to these papers, our focus is on how liquidity conditions affect the formation of beliefs about fundamentals, leading to belief disagreement. Our model emphasizes how the interaction between counterparty risk and fire sale dynamics may contribute to the fragility of such markets.

In the context of bank runs, Chari and Jagannathan (1988) show that panic can arise due to information, which is orthogonal to the health of a bank. Building on Diamond and Dybvig (1983), Goldstein and Pauzner (2005) and Allen et al. (2006) study the role of beliefs and financial stability in the context of global games, while Infante and Vardoulakis (2021) study runs on collateral. In contrast to this literature, our focus is on the role played by asset prices, particularly the price of debt, in disseminating private information and the formation of beliefs. Moreover, we study the implications of this information structure in for financial stability in the context of fire sales rather than coordination failures.

Our model is also related to a more recent literature on information production in credit markets. A paper closely related to ours is Bostanci and Ordoñez (2024), which studies how the volume of trade in equity markets affects the informativeness of prices. The authors study the aggregate implications of the mechanism in a quantitative macroeconomic model. In contrast, we focus on the informativeness of prices in markets for collateralized debt, and on isolating the theoretical mechanisms by which prices affect beliefs and financial stability. Gorton and Ordonez (2014) and Dang et al. (2020) show how collateralized debt can alleviate problems of asymmetric information by making debt informationally-insensitive. In these papers, financial crises are effectively a switch from a regime in which agents have no incentive to produce costly information to a regime in which agents produce information due to adverse selection concerns. Asriyan et al. (2022) also develop a model with costly information production and collateral in which boom-bust

[12] These mechanisms are supported by the empirical facts documented in Gorton and Metrick (2012) and Copeland et al. (2014).

cycles emerge due to the effect of asset prices on the incentive to produce information. In this literature, debt contracts resolve a coordination failure problem in the production of information, which arises due to adverse selection. By contrast, we abstract from the decision to produce information and instead focus on how prices convey private information due to market forces, albeit imperfectly.

Several papers study information aggregation in general equilibrium. Simsek (2013) provides a general characterization of the effect of belief disagreement on asset prices in the presence of collateral constraints. Asriyan et al. (2017) show how, when asset returns are correlated, the information content of prices is determined endogenously by the volume of trade and may give rise to multiple equilibria. Asriyan et al. (2021) show that decentralized markets do not, in general, perfectly aggregate private information. Asriyan (2021) study how dispersed information and imperfect competition in secondary markets for contingent claims distort aggregate investment and output. Dávila and Walther (2023) study optimal leverage regulation when investors and creditors have distorted beliefs. We study similar positive implications of distorted beliefs but in a specific context of collateralized debt, and fire sales, and we abstract form normative implications. Dávila and Parlatore (2021) characterize the relationship between the volatility of prices and their informativeness about fundamentals. Babus and Kondor (2018) study the effects of decentralized trade on the diffusion of information in a network setting. Our focus is on the implications of for asset price fluctuations in a model of fire sales. The broad insights from these papers largely apply for our paper as well. Indeed, many of these papers build on the insights about the informational inefficiency of financial markets from Grossman and Stiglitz (1980). Our paper can be interpreted as an application of these general frameworks to the specific context of collateralized debt markets and fire sales.

Methodologically, our paper borrows from the literature on fire sales to capture a notion of financial crises, especially Lorenzoni (2008) and Kurlat (2016). Such fire sales imply the presence of pecuniary externalities, discussed in Dávila and Korinek (2018). In our model, the pecuniary externality interacts with an additional information externality.

Finally, our paper is related to the literature on leverage cycles developed by Geanakoplos (1997), Geanakoplos (2003), Geanakoplos (2010), and Fostel and Geanakoplos (2015). Similarly to these papers, we build a general equilibrium model with heterogeneous agents and collateralized debt contracts, but our focus is on the role of asymmetric information in amplifying leverage cycles.

2 Model Setup

There are three dates (0, 1, and 2), a single good, and three representative agents (a borrower, lender, and traditional sector) indexed by $i \in \{B, L, T\}$, respectively. The borrower and lender are risk-neutral and consume only at date 2 according to $U_i(C_2^i) = C_2^i$. Each agent is endowed with e_0^i of the consumption good at the beginning of date 0. At date 0, a representative borrower issues debt to a representative lender to finance investment in invest in risky asset. The risky asset is in positive net supply A and yields a gross rate of return of $R \in \{\underline{R}, \overline{R}\}$ at date 2, where $\underline{R} < 1$ and $\overline{R} > 1$. At date 0, agents have a common prior belief about R such that $R = \underline{R}$ with probability π_0. We further assume that $\pi_0 \underline{R} + (1 - \pi_0)\overline{R} = 1$.

The risky asset is illiquid at date 1: It cannot be converted into the consumption good at date 1, nor can new investment in the risky asset be initiated after date 0. In each period, there is a spot market for the risky asset where p_t denotes the price of the risky asset at time t. There is also a traditional sector along the lines of Lorenzoni (2008), who we include to introduce possibility of fire sales at date 1. The traditional sector participates in the spot market for the asset at date 1 (along with the borrower), and values the asset at a discount.

The borrower's date 0 debt matures after one period. Therefore, to maintain its holdings of the risky asset at date 1, the borrower must issue new debt at date 1, potentially under different terms. By financing long-term risky assets with short-term debt, the borrower will be exposed to liquidity risk at date 1.[13] Debt is issued at date $t = 0, 1$ in a competitive market at unit price q_t. The debt promises a gross interest rate of 1 at date $t + 1$, and is collateralized by the borrower's holdings of the risky asset. As we show later, the borrower chooses to default at date $t + 1$ if the return on the risky asset R is less than the promised gross interest rate 1. We therefore define the effective gross interest rate on the loan as $R_2^d \equiv \min\{R, 1\}$. Assumption 1 we introduce later ensures that there is no default at date 1.[14] Appendix B microfounds this contracting environment in terms a competitive market for repo contracts. Because the debt is backed by the risky asset, the lender's willingness to lend at t will depend on its beliefs about the fundamental, R.

Between periods, each agent can store the consumption good in a storage technology we call 'cash'. The gross return to holding cash is τ_t^i where we normalize the rate of return between dates 0 and 1 at $\tau_0^i = 1$ for $i \in B, L$. At date 1, the return to cash τ_1^i will control each agents' opportunity cost of funds.

Shocks At date 1, agents are simultaneously subject to two shocks. This is illustrated in figure 1. First, they receive a shock to the opportunity cost of their funds τ_1^i, which we refer to as a 'cost shock'. The cost shock is independent of R, and is drawn from a continuous and smooth distribution with probability density function $\lambda_T(\tau)$ with mean 1 and support $[\underline{\tau}^L, \bar{\tau}^L]$. We further

[13] One may wonder why the lender lends at date 0 at all instead of holding cash and lending at date 1. The reason is that investment in the risky asset can only be initiated at date 0. Therefore, any equilibrium with positive real investment must involve lending at date 0.

[14] If the borrower defaults on its debt, the lender seizes the collateral and liquidates it to the traditional sector. Both of these actions are payoff-equivalent, so we ignore the confiscation of collateral by the lender and instead assume that the borrower liquidates to the traditional sector and uses the proceeds to pay its debt.

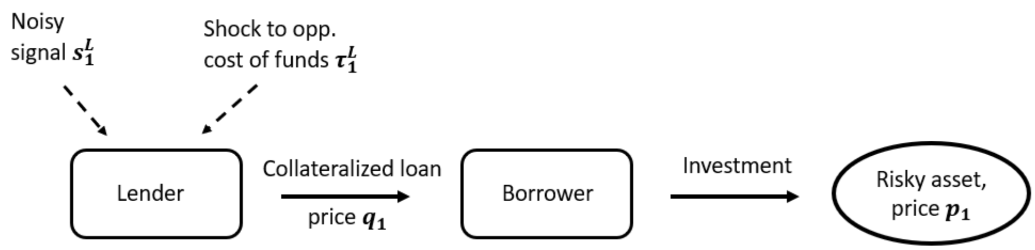

Figure 1: Model Setup

Note: This figures illustrates the setup of the model and the shocks at date 1.

assume that $\lambda_T(\tau)$ is decreasing in $|\tau - 1|$.

In addition to the cost shock, each agent receives private news about the fundamental R at date 1. In particular, at date 1, agent i gets a private noisy signal s_1^i about R, where $s_1^i = R + \varepsilon^i$ and where ε^i is drawn from a continuous and smooth distribution with cumulative density function $\Lambda_\varepsilon(\varepsilon)$, which has mean zero and variance σ_ε^2, and probability density function $\lambda_\varepsilon(\varepsilon)$ with support $(-\infty, \infty)$. We assume that $\lambda_\varepsilon(\varepsilon)$ is decreasing in $|\varepsilon|$, i.e., $\lambda_\varepsilon(\varepsilon)$ is single-peaked.

For simplicity, we assume that only one agent (the lender) receives both a cost shock and a private signal, while the other agent (the borrower) receives neither. Consistent with this case, we assume that τ_1^B is a fixed parameter such that $\tau_1^B = \tau_0^B = 1$, while τ_1^L is a random variable such that $E[\tau_1^L] = \tau_0^L = 1$. In addition, only the lender receives a private signal. These assumptions ensure the model is tractable and that we can solve for agent's equilibrium beliefs.[15] While much of our exposition focuses on the case in which the lender receives the signal and shock, as illustrated in figure 1, we show in section 7 the same insights apply for the case in which the borrower receives the signal and shock, although the mechanics differ slightly.

Information sets and beliefs Agents have the same information set and beliefs at date 0. At date 1, however, neither the lender's cost shock τ_1^L nor its signal about s_1^L are observable or verifiable to the borrower. Therefore, while the borrower's information set at date 1, I_1^B, consists only of publicly observable variables, including asset prices p_1, q_1, the information set of the lender, I_1^L, additionally includes the private shocks τ_1^L, s_1^L. Formally, the lender's information set at date 1 is $I_1^L = \{\Theta_1, \tau_1^L, s_1^L\}$, while the borrower's information set is $I_1^L = \{\Theta_1\}$, where Θ_1 denotes the set of all variables observable to all agents, including prices p_1, q_1 and the terms of the contract. At date 1, each agent updates their beliefs π_1^L, π_1^B that $R = \underline{R}$ in response to new information at

[15]If, in contrast, both agents received private signals, then the equilibrium would depend not only on how one agent, e.g. the borrower, updates its beliefs in response to the actions of the other, but also on an infinite feedback loop in which also the borrower's actions affect the lender's beliefs, etc. This would make an analytical characterization of equilibrium belief formation extremely difficult.

date 1 according to Bayes' rule. We denote agent i's expectation of variable x, conditional on its information set at time date t, as $E_t^i[x] \equiv E_t\left[x \mid I_t^i\right]$.

At date 1, after the realization of the cost shocks and signals, agents simultaneously settle the date 0 contracts and enter into another one-period contract at date 1. Uncertainty is fully resolved for all agents at date 2.

We make the following assumption about the endowments of each agent.

Assumption 1. *$e_0^B \tau_1^B < A\left(E_0^B[R]-1\right)$, $e_0^L > A\overline{R}/\underline{\tau}^L - e_0^B$, and $e_0^T > \max_{a\in[0,A]} aF'(a)$.*

The first part of the assumption ensures that borrowers cannot fund their asset purchases without debt, while the second part ensures that the lender's endowment is sufficiently large to satisfy the borrower's demand. Finally, the third part of the assumption ensures that the traditional sector always has enough cash to purchase any of the risky asset amount sold to them.[16]

We also make the following assumptions on the traditional sector.

Assumption 2. *$F'(A) \geq E_0^B[R] - e_0^B/A$ and $E_0^B[R]/\tau_1^B > F'(0)$.*

The first part of the assumption ensures that, at date 1, the liquidation value of the borrower's holdings of the risky asset is sufficient to repay all of its date 0 debt d_0 so that the borrower never defaults in equilibrium at date 1. (The borrower's liquidation value of its holdings A at price $p_1 = F'(A)$ is $AF'(A)$, while the maximum date 0 debt the borrower can have in equilibrium is $AE_0^B[R] - e_0^B$.) The second part of the assumption ensures that, ex ante, the borrower always values the asset more than the traditional sector does.

3 Equilibrium

The equilibrium conditions can be partitioned into two blocks: a real block which determines the allocation conditional on agents' beliefs, and a set of conditions characterizing the formation of beliefs given the real block. We first characterize each block and then turn to general equilibrium, which is pinned down by the joint determination of both blocks.

3.1 Real block conditional on beliefs

3.1.1 Lender's Problem

Each period, the lender solves a portfolio choice problem to maximize its date 2 expected consumption. In particular, at $t = 0, 1$, it chooses between cash κ_t^L and the borrower's debt d_t, taking

[16] This assumption is almost exactly the same as the assumptions in Simsek (2013) and Gottardi et al. (2019), serving the same purposes.

the unit price of debt q_t as given, and conditional on its opportunity cost of funds (the return to cash τ_t^L) and its beliefs π_t^L. The lender's budget constraints at dates $t = 0, 1$ are

$$\underbrace{\kappa_0^L}_{date\ 0\ cash\ holdings} + \underbrace{q_0 d_0^L}_{value\ of\ date\ 0\ loan} \leq \underbrace{e_0^L}_{date\ 0\ cash\ endowment}$$

$$\underbrace{\kappa_1^L}_{date\ 1\ cash\ holdings} + \underbrace{q_1 d_1^L}_{value\ of\ date\ 1\ loan} \leq \underbrace{\kappa_0^L}_{date\ 0\ cash\ holdings} + \underbrace{d_0^L}_{value\ of\ date\ 0\ loan},$$

respectively. At date 2, the lender consumes its resources C_2^L, which consists of the proceeds of its date 1 holdings of debt and the return on its cash holdings.

$$\underbrace{C_2^L}_{date\ 2\ consumption} = \underbrace{\tau_1^L \kappa_1^L}_{proceeds\ from\ cash\ holdings} + \underbrace{R_2^d d_1}_{proceeds\ from\ date\ 1\ loan}.$$

We state and solve the lender's full problem in appendix A.1. Assumptions 1 and 2 ensure that lending takes place in equilibrium so that $d_1 > 0$ and that the lender holds a strictly positive quantity of cash in equilibrium at date 1, $\kappa_1^L > 0$. Therefore, given these assumptions, the lender's first-order condition (FOC) for d_1 is

$$q_t = \frac{E_t^L\left[R_t^d\right]}{\tau_t^L}. \tag{1}$$

This condition states that the expected marginal return from lending at date 1 equals the expected marginal return on its opportunity cost from holding cash at date 1. The lender's willingness to hold debt is lower the higher is its opportunity cost of funds τ_1^L and the more pessimistic its beliefs about the fundamental R, since the loan is backed by the risky asset. Because the lender is the marginal buyer of debt, the competitive spot price of the contract at date 1, q_1, will ensure that this condition holds in equilibrium.

3.1.2 Borrower's Problem

At dates 0 and 1, the borrower decides how much to borrow and how to allocate its portfolio between the risky asset and cash in order to maximize its expected date 2 consumption $E^B\left[C_2^B\right]$. At date 0, the borrower has a cash endowment and receives a cash loan from the lender, which it can allocate between cash holdings κ_0^B and investment in the risky asset a_0^B. Therefore, its date 0 budget constraint is

$$\underbrace{q_0 d_0}_{date\, 0\, loan} + \underbrace{e_0^B}_{date\, 0\, cash\, endowment} \geq \underbrace{p_0 a_0^B}_{value\, of\, date\, 0\, asset\, holdings} + \underbrace{\kappa_0^B}_{date\, 0\, cash\, holdings}.$$

At date 1, the borrower must repay its date 0 debt. (Recall that assumption 2 ensures that it is never optimal to default at date 1, so we neglect this case here for simplicity.) However, because the risky asset only pays out at date 2, the borrower has no cash flow from the risky asset at date 1 to repay its debt. Therefore, to repay its debt, it must either issue new debt (which is equivalent to rolling over some of its date 0 debt), use any cash holdings it may have carried over from date 0, or raise cash by selling some of its holdings of risky asset to the traditional sector in a competitive spot market at price p_1. Accordingly, the borrower's date 1 budget constraint is

$$\underbrace{q_1 d_1 - d_0}_{net\, increase\, in\, loan} \geq \underbrace{p_1 \left(a_1^B - a_0^B\right)}_{net\, increase\, in\, asset\, holdings} + \underbrace{\kappa_1^B - \kappa_0^B}_{net\, increase\, in\, cash\, holdings}.$$

At date 2, the borrower chooses to default if the realized return on the risky asset R is less than 1, the promised gross interest on its debt. It then consumes its remaining resources.

$$\underbrace{C_2^B}_{date\, 2\, consumption} \leq \underbrace{\tau_1^B \kappa_1^B}_{proceeds\, from\, cash\, holdings} + \underbrace{a_1^B R}_{proceeds\, from\, asset\, holdings} - \underbrace{d_1 R_2^d}_{repayment\, of\, debt},$$

We state and solve the borrower's full problem in appendix A.2. Combining the borrower's optimality conditions for borrowing and investing in the risky asset yields a relation between the borrower's expected net return from buying the risky asset, $E_1^B[R] - \tau_1^B p_1$, and the cost of financing its holdings of the risky asset, $E_1^B\left[R_2^d\right] - q_1 \tau_1^B$, which in turn balances the benefit from being able to borrower more with the cost of a larger expected repayment.

$$\underbrace{E_1^B\left[R_2^d\right] - q_1 \tau_1^B}_{cost\, of\, financing\, asset\, holdings} + \underbrace{\xi_{\kappa_1}^B (p_1 - q_1)}_{shadow\, value\, of\, cash} = \underbrace{E_1^B[R] - \tau_1^B p_1}_{expected\, net\, return\, of\, risky\, asset} \tag{2}$$

This equation captures how the borrower's beliefs and the price of its debt q_1 affect its portfolio choice at date 1: The borrower chooses to hold less of the risky asset compared to cash if it is more pessimistic about the fundamental R (a higher π_1^B), or if the availability of funding is lower (lower price of its date 1 debt q_1). Thus, the borrower chooses to liquidate part of its holdings of the risky asset if it becomes sufficiently pessimistic about R, or if it is sufficiently liquidity constrained at date 1 such that its cash holdings κ_0^B are insufficient to repay its debt. Moreover, rearranging the optimality condition yields an expression for the borrower's marginal valuation for the risky asset, $p_1 = \frac{1}{\tau_1^B + \xi_{\kappa_1}^B}\left(E_1^B[R] - E_1^B\left[R_2^d\right]\right) + q_1$. For the purpose of exposition, we define $p_1^B \equiv \frac{1}{\tau_1^B}\left(E_1^B[R] - E_1^B\left[R_2^d\right]\right) + q_1$, which corresponds to the borrower's marginal valuation of the asset when the shadow value of cash is 0. By construction, $p_1 = p_1^B$ if and only if the borrower has positive cash holdings in equilibrium at date 1, $\xi_{\kappa_1}^B = 0$.

3.1.3 Traditional Sector's Problem

We incorporate a traditional sector which values the risky asset at a discount in order to introduce a notion of fire sales along the lines of Lorenzoni (2008). The traditional sector maximizes its date 2 consumption C_2^T. At date 0 it receives an endowment e_0^T with which it can buy existing risky assets in the spot market at date t, or store as cash between dates t and $t+1$, for $t = 0, 1$. Existing assets need to be managed subject to some increasing costs. The date-2 return of holding a_1^T of the risky asset at date one, net of these management costs, is given by $F(a_1^T)$ units of the consumption good, where $F' > 0$ and $F'' < 0$. In appendix A.3, we show that the traditional sector's optimality condition at date 1 is

$$p_1 = F'(a_1^T) + \xi_{a_1}^T. \tag{3}$$

This describes the traditional sector's marginal valuation of the risky asset. We assume that the expected return of the asset satisfies $E_0^B[R]/\tau_1^B > F'(0)$ so that the traditional sector always values the risky asset less than other agents do at date 0. This implies that the traditional sector is relevant for the equilibrium only at date 1. For simplicity, and without loss of generality, we assume the traditional sector's endowment of cash is sufficiently large that, at date 1, the traditional sector could feasibly buy all existing assets in equilibrium.

3.1.4 Market Clearing at Date 1

Recall that no new risky assets can be created after date 0. Therefore, the market clearing condition for the date 1 spot market for the risky asset implies that the total quantity of the asset held by the borrower and the traditional sector at date 1 is equal to the total amount of the asset created at date 0.[17]

$$a_1^B + a_1^T = a_0^B \tag{4}$$

The spot market for the risky asset is competitive, so the price p_1 will be pinned down in equilibrium at the marginal valuation of the marginal buyer. Since the lender cannot directly participate in the market for the risky asset, either the borrower or the traditional sector is the marginal

[17] For ease of exposition, we have already imposed a market clearing conditions for the market for debt at each date. These conditions are made explicit in appendix B.

buyer in equilibrium. The marginal buyer is the agent who has the maximum willingness to pay in equilibrium. Therefore, the risky asset price is given by

$$p_1 = \max\left\{ \frac{1}{\tau_1^B + \xi_{\kappa_1}^B} \left(E_1^B[R] - E_1^B\left[R_2^d\right]\right) + q_1,\ p_1^T \right\} \tag{5}$$

where the first term and p_1^T denote the marginal valuations of the borrower and traditional sector, respectively. We showed in the previous section that the traditional sector's marginal valuation is given by its optimality condition, $p_1^T \equiv F'(a_1^T) + \xi_{a_1}^T$.

3.2 Beliefs at Date 1

We now turn to the evolution of agents beliefs about the fundamental of the risky asset. Because agents have different information sets at date 1, their beliefs evolve differently.

Lender's beliefs at date 1 The lender's information set at date 1 consists of all commonly observed variables $\Theta_1 = \{d_1, q_1, p_1\}$ and its private cost shock and private signal τ_1^L, s_1^L. The lender's posterior beliefs about the fundamental of the risky asset R are determined by Bayes' Rule, where the likelihood function depends on its private signal s_1^L.

$$\pi_1^L = \frac{\pi_0 \lambda_\varepsilon\left(\varepsilon_1^L = s_1^L - \underline{R}\right)}{\pi_0 \lambda_\varepsilon\left(\varepsilon_1^L = s_1^L - \underline{R}\right) + (1-\pi_0)\lambda_\varepsilon\left(\varepsilon_1^L = s_1^L - \bar{R}\right)}. \tag{6}$$

This is derived formally in appendix C.1. The lender's pessimism about the fundamental, measured by π_1^L, is decreasing in s_1^L by the assumption on the distribution from which ε is drawn is single-peaked. We denote the reduced form cumulative distribution function of π_1^L as $G_\pi(\cdot)$ and its probability density function as $g_\pi(\cdot)$.

Borrower's beliefs at date 1 The borrower observes all prices and contract terms, but does not observe the lender's liquidity shock or private signal. Therefore the borrower's date 1 information set consists only of commonly observed variables $I_1^B = \{\Theta_1\}$. After observing equilibrium prices p_1 and q_1, the borrower updates its beliefs about the risky asset's return, where its posterior belief is denoted as π_1^B. In equilibrium, only the price of debt q_1 conveys additional information about the lender's private information. As discussed in section 3.3, p_1 is determined by the valuation of the marginal buyer of the risky asset. But since the lender cannot participate in the market for the risky asset, the price p_1 does not directly convey information about the lender's beliefs about R.

Nevertheless, the borrower can partially infer the lender's private signal about R based on the equilibrium price of debt q_1, which reflects the lender's beliefs about R through equation (6). However, the observed q_1 is not sufficient for the borrower to separately identify the lender's private news s_1^L versus its cost shock τ_1^L. This *identification problem* arises because there is only one observable q_1 to infer two unobservables s_1^L, τ_1^L. Hence, financial markets are *informationally inefficient* in that prices do not perfectly reveal agents' private information about the fundamental. We will more precisely define informational efficiency in this context in section 4.2.

This identification problem arises because the borrower faces two layers of uncertainty about the asset's return R. The price of debt q_1 is a effectively a noisy signal about the lender's private signal s_1^L, where the noise is introduced by the lender's idiosyncratic cost shock τ_1^L. The lender's private signal s_1^L itself is a noisy signal about R. The presence of two layers of uncertainty will give rise to endogenous belief disagreement, and will imply that the borrower's beliefs are shaped by liquidity conditions which affect the noisiness of q_1. Fundamentally, this problem arises due to the absence of risk markets, as discussed in Stiglitz (1981).

We now characterize the formation of the borrower's beliefs about R in light of the identification problem it faces. Recall from (1) that the price of debt q_1 contains information on the private information of lender τ_1^L, s_1^L through the equilibrium relationship

$$q_1 = \frac{1-\pi_1^L + \pi_1^L \underline{R}}{\tau_1^L}.$$

The borrower is rational and therefore understands how the price of debt q_1 reflects the lender's beliefs π_1^L through the mapping above. Moreover, the borrower also understands how the lender's beliefs π^L reflect its private news s_1^L about the fundamental through the mapping (6).

The identification problem faced by the borrower is illustrated in figure 2. The curve in the figure plots the combinations of the lender's cost shock and beliefs, $\left(\tau_1^L, \pi_1^L\right)$, which are consistent with the observed q_1 according to (6). Therefore, the observed q_1 is not sufficient to infer the true realizations $\left(\tau_1^{L*}, \pi_1^{L*}\right)$ separately.

In light of this identification problem, the borrower's posterior beliefs evolve according to

$$E_1^B\left[\tilde{\pi}_1^L\right] = \int_{\underline{\tau}}^{\bar{\tau}} \int_0^1 \pi \mathbb{1}\left\{\pi = \frac{1-\tau q_1}{1-\underline{R}}\right\} dG_\pi(\pi) d\Lambda_T(\tau), \tag{7}$$

where $\mathbb{1}\{\cdot\}$ is an indicator function, and $\Lambda_T(\cdot)$ is the distribution function of $\lambda_T(\cdot)$. By change of variables, the above equation can be rearranged as follows

$$\pi_1^B = \int_0^1 \pi \lambda_T\left(\frac{1-(1-\underline{R})\pi}{q_1}\right) \mathbb{1}\left\{\underline{\tau} < \frac{1-(1-\underline{R})\pi}{q_1} < \bar{\tau}\right\} dG_\pi(\pi). \tag{8}$$

Intuitively, the borrower computes the likelihood of all possible realizations of (τ_1^L, s_1^L) consistent with the observed value of q_1, based on its knowledge of the joint distribution of τ_1^L and s_1^L,

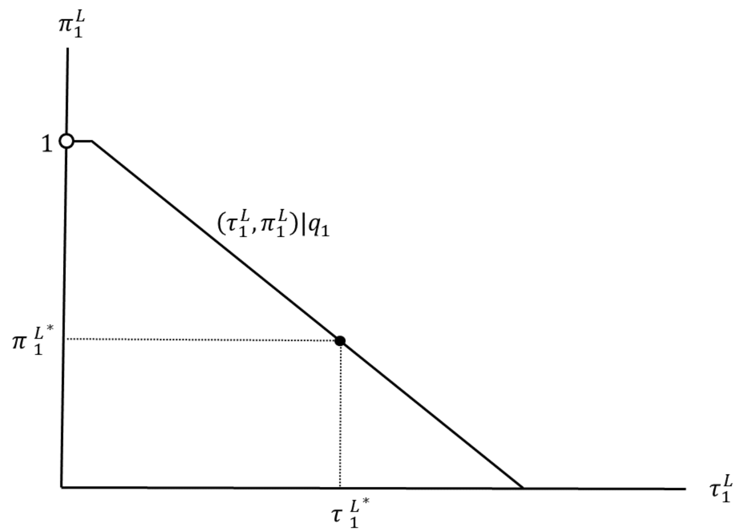

Figure 2: Identification Problem

Note: This figures illustrates the identification problem faced by borrowers at date 1. $\tau_1^{L^*}$ denotes the realized cost shock and $\pi_1^{L^*}$ denotes the lender's equilibrium belief given the realized signal it receives. q_1 is the equilibrium price of debt consistent with $(\tau_1^{L^*}, \pi_1^{L^*})$. The line plots the set of all possible realizations of (τ_1^L, π_1^L) consistent with the equilibrium q_1.

and given the mapping (6) between s_1^L and π_1^L. Based on the likelihood of these realizations, the borrower forms an expectation over the lender's private signal s_1^L and updates its own beliefs π_1^B accordingly.

This environment leads to the key mechanism of the model: Beliefs about fundamentals are endogenously shaped by the availability of liquidity in funding and asset markets. This arises due to an *information spillover* whereby the price of debt affects agents' beliefs about the fundamental, illustrated by the blue arrow in figure 3.

The proposition below summarizes how the borrower's beliefs are shaped by the price of debt due to the information spillover.

Proposition 1. *Borrower's posterior belief π_1^B has the following properties in equilibrium:*

1. *Borrower's belief is the same as the expected posterior of lender, i.e. $\pi_1^B = E_1^B[\pi_1^L]$.*

2. *π_1^B is decreasing in q_1.*

Proof. See appendix C.2. ■

Part 1 of the proposition shows that the borrower's posterior belief about the fundamental value of the asset is given by the borrower's expectation about the lender's belief. This is a direct result of Bayesian updating and simply reflects that the borrower knows the lender to be better informed due to its private signal.

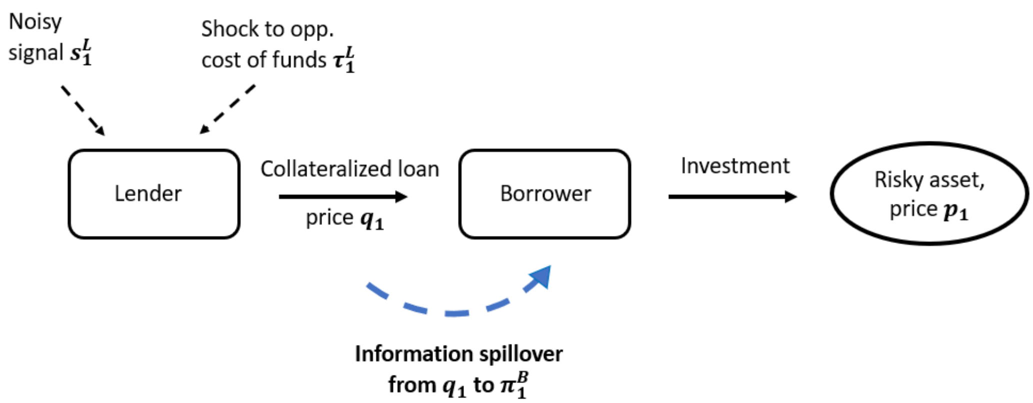

Figure 3: Downstream Information Spillover

Note: This figures illustrates the downstream information spillover in which the price of debt affects the borrower's belief about the fundamental of the asset.

Part 2 of the proposition shows that pessimism and optimism about the fundamental value of the asset arise endogenously in our model as a result of funding liquidity. In particular, if the borrower faces a tighter funding liquidity from the lender at date 1, then the borrower becomes more pessimistic (a higher π_1^B) about the fundamental value of the asset. Importantly, this is true even if the low price of debt q_1 is driven entirely by a an adverse cost shock τ_1^L which is orthogonal to R. By contrast, loose funding liquidity conditions (a high contract price q_1) leads to optimism (a low π_1^B). Hence, fluctuations in funding conditions cause B's beliefs to become divorced from fundamentals.

Information spillovers in standard models are part of the usual price-signal mechanism whereby asset prices aggregate and convey agents' private information. Indeed, this is usually the mechanism through which agents' beliefs converge with true underlying fundamentals. In our setting, by contrast, information spillovers perversely cause beliefs to diverge from fundamentals. Rather, beliefs are shaped by spurious shocks to the availability of liquidity. In section 4, we explore in more detail why information spillovers have such perverse effects.

3.3 Characterization of Equilibrium at Date 1

Definition of date 1 equilibrium An equilibrium at date 1 is a set of prices p_1, q_1, quantities $d_1, \kappa_1^L, a_1^B, \kappa_1^B, a_1^T$, C_2^L, C_2^B, C_2^T, and posterior beliefs π_1^L, π_1^B, satisfying the agents' optimality conditions and constraints, market clearing conditions, and the equations characterizing belief formation, taking as given variables determined at date 0 and the date 1 shocks τ_1^L, s_1^L. Note that a unique equilibrium always exists because of the linearity of the agents' objective functions and the constraints of the borrower's optimization problem, and the smoothness of the inverse demand function of the traditional sector, which determines the market clearing condition.

The date 1 equilibrium features two regimes, which we call 'normal times' and 'fire sales'. The regimes are determined by whether the marginal buyer of the risky asset is the borrower or the traditional sector at date 1. The lemma below describes the two regimes.

Lemma 1 (Two regimes in date-1 equilibrium)**.** *In equilibrium, the following holds:*

1. *When $p_1 \geq F'(0)$, the economy is in the 'normal regime' in which $p_1 = p_1^B$, $a_1^B = c_1 = a_0^B$, and $a_1^T = 0$.*
2. *When $p_1 < F'(0)$, the economy is in the 'fire sale regime' in which $p_1 = F'(a_1^T)$, $a_1^B = c_1^B < a_0^B$, and $a_1^T > 0$.*

Proof. See appendix C.5. ■

Which regime prevails at date 1 is determined by who the marginal buyer of the risky asset is in equilibrium. When the borrower's marginal valuation of the risky asset p_1^B, given by its optimality condition (2) exceeds the marginal valuation of the traditional sector $F'(0)$, then the borrower is the marginal buyer and the the normal regime obtains. By contrast, when the borrower's marginal valuation falls sufficiently such that $p_1^B < F'(0)$, then the fire sale regime obtains.

Normal Regime The normal regime obtains when cost shocks are not too large and the signal is not too bad. In this regime, all of the risky asset is held by the borrower, who has relatively optimistic beliefs. The borrower is at a corner solution in its portfolio choice and wants to hold only the risky asset at date 1. (Because no new investment in the risky asset can be initiated after date 0, the borrower's holdings of the risky asset remain unchanged from date 0, $a_1^B = a_0^B$.) The asset price p_1 is relatively high and is pinned down by the borrower's valuation.

In the normal regime, the borrower may still face a tightening in funding liquidity, given by a relatively low equilibrium price of debt q_1, due to adverse cost shocks or bad news. Moreover, this tight liquidity can cause the borrower to become more pessimistic about the asset fundamental through the effect of information spillovers on its beliefs. However, in the normal regime, these effects are not strong enough to lead to fire sales. The borrower finances any funding shortfall $d_0 - q_1 d_0$ out of its cash holdings stored from date 0 rather than by liquidating the risky asset to the traditional sector.

Fire Sale Regime When the cost shock or news are sufficiently severe, i.e., when q_1 is sufficiently low, the economy enters the fire sale regime. In the fire sale regime, the traditional sector holds a strictly positive amount of the risky asset at date 1, while the remainder is held by the borrower. In particular, when the cost shock to the lender is sufficiently large (τ_1^L is high) or its signal is sufficiently bad (s_1^L is low), the tightening of funding liquidity results in fire sales in which the borrower liquidates some of its holdings of the risky asset to the traditional sector. The proposition characterizes this regime.

Proposition 2. *In equilibrium, the following holds:*

1. ***Fire sales are more severe for tighter funding liquidity**: In the fire sale regime, a lower realization of s_1^L or a higher realization τ_1^L causes a_1^T to be higher and p_1 to be lower at the margin.*

2. ***Fire sales may be driven by liquidity needs or pessimism**: In a 'liquidity driven fire sale' in which the borrower is optimistic but is forced to liquidate assets to repay debt, we have $p_1^B > F'(0) > p_1$. In a 'belief driven fire sale' in which the borrower liquidates due to its pessimism, we have $p_1^B < F'(0)$.*[18]

Proof. See appendix C.6. ■

Part 1 of the proposition states that lower funding liquidity (lower q_1) causes more severe fire sales—that is, larger traditional sector holdings of the risky asset a_1^T and a lower price p_1 of the risky asset. For a given asset price p_1, tighter funding liquidity (lower q_1) worsens the tradeoff to investing in the risky asset by making the borrower's beliefs more pessimistic. To see this, recall the borrower's date 1 optimality condition (2)

$$\underbrace{E_1^B\left[R_2^d\right] - q_1\tau_1^B}_{cost\ of\ financing\ risky\ asset\ holdings} = \underbrace{E_1^B\left[R\right] - \tau_1^B p_1}_{expected\ net\ return\ of\ risky\ asset},$$

where we have imposed the result that $\xi_{\kappa_1}^B = 0$, so that the borrower holds a positive amount of cash. The proposition shows that tighter funding liquidity (lower q_1) affects the optimality condition directly through a liquidity effect—the direct effect of lower q_1—and through the effect of q_1 on the borrower's beliefs. Through both effects, tighter funding liquidity increases the cost of financing the risky asset, given by the left-hand side of the optimality condition, and reduces the expected return to holding the risky asset. This tilts the borrower's portfolio choice in favor of hold cash rather than the risky asset at date 1.

In the fire sale regime, equilibrium in the market for the risky asset is reached once the asset price p_1 falls sufficiently to restore equality in the borrower's optimality condition. The strict concavity of the traditional sector's technology $F(\cdot)$ implies that this equilibrium price $p_1 = F'(a_1^T)$ occurs at an interior optimum in which the borrower holds both cash and the risky asset.

Part 2 of the proposition distinguishes between liquidity-driven fire sales and *belief-driven fire sales*. In a liquidity driven fire sale, the borrower has insufficient funding to repay the date 0 debt,

[18]The price p_1 can be less than p_1^B in this case, if the borrower is extremely pessimistic and $p_1^B < F'(a_0^B)$. See appendix E.1 for the full characterization of this case.

and is forced to liquidate some of its risky asset holdings in order to repay its debt, despite being relatively optimistic about the fundamental R. In much of the literature, fire sales take this form.

Our paper shows, on the other hand, that fire sales may be driven by excessive pessimism about fundamentals. In a belief driven fire sale, the borrower liquidates some of its risky asset holdings due to its endogenous pessimism about the fundamental, despite having sufficient cash holdings to repay its debt without liquidating the asset. Such fire sales have the flavor of asset market turmoil driven by investor panic. Importantly, even when fire sales are liquidity-driven, the endogenous pessimism that arises from the information spillover here *exacerbates* the severity of the fire sale, causing the borrower to liquidate more of the asset and at a lower price.

We can partition the state space into the normal and fire sale regimes by defining the set of states $\left(\tau_1^L, \pi_1^L\right)$ consistent with $\hat{p}_1 \equiv F'(0)$, the threshold such that the economy enters the fire sale regime when the asset price is below the threshold $\hat{p}_1$. In our baseline economy, there is a one-to-one mapping from q_1 to p_1, and hence, the threshold asset price $\hat{p}_1$ corresponds to a threshold value $\hat{q}_1$. The lender's optimality condition defines the set of $\left(\tau_1^L, \pi_1^L\right)$ consistent with $\hat{q}_1$, given by

$$\pi_1^L = \frac{1-\tau_1^L \hat{q}_1}{1-\underline{R}}. \tag{9}$$

This equation (9) defines the frontier partitioning the state space into the normal and fire sale regimes, and is illustrated in figure 4. (Note that the line plotted in this figure is conceptually different from that plotted in figure 2. Figure 2 plotted the set of all (τ_1^L, π_1^L) consistent with the equilibrium q_1. By contrast, figure 4 plots such combinations consistent with the threshold price $\hat{q}_1$ which demarcates the two regimes. The equilibrium price q_1 is not, in general, equal to the threshold price $\hat{q}_1$.)

3.4 Equilibrium at Date 0

Given the equilibrium at date 1, we can solve recursively for the date 0 equilibrium. For tractability, we assume here that $F'(a) = \alpha$ for all $a \geq 0$.[19] The date 0 equilibrium is derived in appendix E.2, and the key properties are summarized in the following proposition. For our purposes, the key features of the date 0 equilibrium are that initial lending and investment in the risky asset are strictly positive.

Proposition 3. *Suppose that assumptions 1 and 2 hold. At date 0, the borrower is at a corner solution in its portfolio choice, maximizing its holdings of the risky asset at holding zero cash. That is,* $(c_0, a_0, \kappa_0^B) = (A, A, 0)$*— and the prices of the asset and contracts are* $(p_0, q_0) = (1 + e_0^B/A, 1)$.

[19]Effectively, this assumption implies that, in a fire sale at date 1, the borrower liquidates all of its holdings of the risky asset.

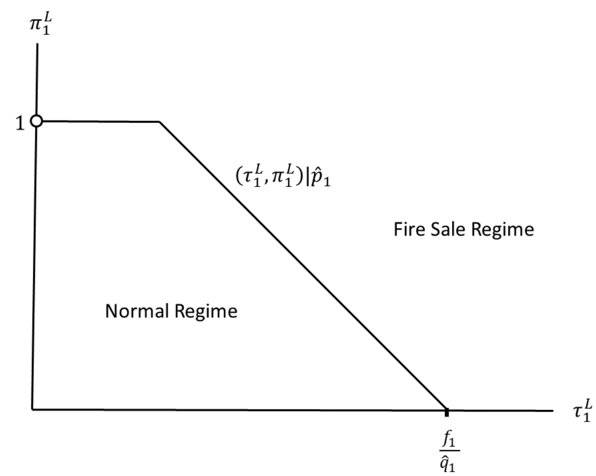

Figure 4: Two Regimes at Date 1

Note: This figure illustrates the bisection of the state space into two regimes at date 1. The equilibrium is in the Fire Sale Regime if and only if the equilibrium value of p_1 is below a threshold value $\hat{p}_1$ (or, equivalently, the equilibrium value of q_1 is below a threshold value $\hat{q}_1$). The curve in the figure plots the combinations of the states $\left(\tau_1^L, \pi_1^L\right)$ consistent the threshold $\hat{p}_1$ and marks the frontier between the two regimes.

Proof. See appendix E.2. ■

The date 0 loan from the lender to the borrower is strictly positive, the borrower invests only in the risky asset, and uses all of its holdings of the risky asset as collateral for the loan.

4 Counterfactual Benchmark Economies

In this section, we introduce counterfactual benchmark economies to shed light on the workings of the model. These benchmarks are useful to elucidate why the informational efficiency of financial markets depends on the availability of liquidity, and why, in contrast to standard models, information spillovers cause beliefs to deviate from, rather than converge to, fundamentals.

In our model, the interaction between beliefs and liquidity arises from two related but distinct features of the information structure:

1. Learning: In response to new information, agents update their prior beliefs about the fundamental value of the risky asset.

2. Asymmetric information: Because information is private, agents' beliefs evolve differently to new information.

To understand how each feature of beliefs interacts with market and funding liquidity, we conduct two separate exercises. In appendix E.3, we isolate the role of learning by comparing the date 1 equilibrium to that in an alternative benchmark economy in which agents do not update their beliefs. The appendix shows that the endogenous response of beliefs to changes in funding liquidity exacerbates fire sales by worsening market liquidity. In the next section, we isolate the role of asymmetric information by comparing the date 1 equilibrium to a separate benchmark economy in which all information is publicly observed, so that beliefs are homogeneous.

4.1 Benchmark economy with common information

To dig deeper into the role of information asymmetry per se, we consider a benchmark version of the model in which agents have common information at date 1: In this benchmark, all information is *publicly* observed by all agents. We use this 'common information benchmark' to assess how the configuration of incomplete markets and asymmetric information in our model markedly changes the nature of the price-signal mechanism.

In the benchmark economy, we assume that the borrowers can directly observe both the lender's signal s_1^L and the lender's shock to the opportunity cost of funds τ_1^L. In this sense, neither agent has private information. Note that information is still incomplete: Agents receive noisy signals about the asset's date 2 return, but signals are publicly observable rather than being private. Agents therefore have a common information set. A corollary of proposition 1 is that, under this common information benchmark, the borrower and lender have identical posterior beliefs, $\pi_1^B = \pi_1^L$ in every state. The form of the equilibrium is otherwise the same as that in the baseline economy with private information.[20]

To further elucidate the role of the adverse information spillovers in generating belief-driven fluctuations, we can divide the state space (π_1^L, τ_1^L) at date 1 into two regions, illustrated in figure 5. In this partition, we define the optimism of the borrower relative to the information set of the lender. This partition therefore shows how the borrower's beliefs about the fundamental deviate from its beliefs in the common information benchmark, for the same combination of shocks. As such, it captures fluctuations in beliefs due to adverse effect of information spillovers.

In the 'optimistic region' of the state space, the borrower is overly optimistic relative to the better-informed lender, i.e. $\pi_1^B < \pi_1^L$. The economy enters this optimistic region when the lender's cost shock is sufficiently good (low τ_1^L), for any signal. In the 'pessimistic region', the borrower is overly pessimistic relative to the better-informed lender, i.e. $\pi_1^B \geq \pi_1^L$. The economy enters this pessimistic region when the lender's cost shock is sufficiently bad (high τ_1^L) for any signal. The partition between these two regions, marked by the dotted line of figure 5, is defined by tracing out the borrower's posterior beliefs $\left(E_1^B\left[\tau_1^L\right], \pi_1^B\right)|q_1$ at different realizations of the equilibrium value

[20]In general, differences in the equilibrium allocations at date 0 across the baseline and benchmark may also manifest as differences in the date 1 allocation. We can show that the date 0 equilibrium allocations of the two cases would be the same under assumptions 1 and 2.

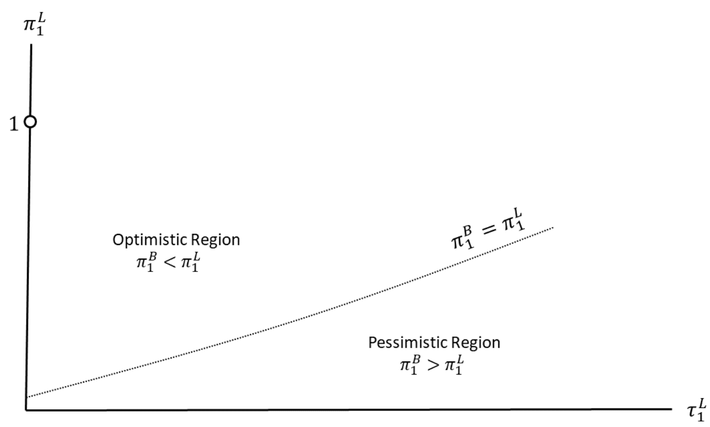

Figure 5: Optimism and Pessimism with Respect to the Common Information Benchmark

Note: This figure illustrates the bisection of the state space into regions based on the borrower's beliefs about the asset in the baseline economy relative to its beliefs in the common information benchmark (in which the beliefs of the borrower are identical to those of the lender). The dotted curve demarcates the region of the state space in which $\pi_1^B < \pi_1^L$ in equilibrium to the northwest of the curve, and the region in which $\pi_1^B > \pi_1^L$ to the southeast. The dotted curve is constructed by finding, for a given q_1, the borrower's posterior beliefs $\left(E_1^B\left[\tau_1^L\right], \pi_1^B\right)|q_1$ such that $\pi_1^B = \pi_1^L$, and then tracing this out at all possible realizations of q_1. In the northwest region, the borrower is optimistic about the risky asset relative to the lender (and relative to what its beliefs would be in the common information benchmark), while in the southeast region, the borrower is relatively pessimistic.

q_1. Note that the relative frequency with which the economy will end up in one region or the other depends on the distribution of shocks τ_1^L and news.

4.2 Informational efficiency varies with liquidity conditions

In this section we show how the informational efficiency of the economy varies with the availability of liquidity in funding markets. Conceptually, we think of informational efficiency as how well asset prices aggregate and reveal agents' private information about the fundamental. In our baseline model where the lender is the agent that receives private news, informational efficiency is reflected by well the borrower can infer the lender's private news s_1^L about R from the price of debt q_1.

We can define the informational efficiency of our baseline economy more precisely with respect to the common information benchmark, in which all signals about the fundamental are publicly observed. By construction, asset prices in the common information benchmark perfectly reveal private information. As a result, there is no dispersion of beliefs across agents, i.e. $|\pi_1^B - \pi_1^L| = 0$ in all states of the world. Therefore, a shorthand measure of the degree to which price reveal in baseline economy relative to common information benchmark is the extent of belief disagreement that emerges endogenously in equilibrium, averaged over the realization of shocks. Henceforth, expected belief dispersion $E\left[|\pi_1^B - \pi_1^L|\right]$ will be our measure of the informational efficiency of our baseline economy.

The following result shows that the availability of funding liquidity affects the informational efficiency of our baseline economy.

Proposition 4. *The expected difference between the borrower's posterior belief π_1^B and the lender's true posterior belief π_1^L, $E\left[|\pi_1^B - \pi_1^L|\right]$, is increasing in the absolute value of cost shock $|\tau_1^L - \tau_0|$.*

Proof. See appendix C.7. ■

This result implies that larger shocks to the availability of funding liquidity cause the borrower's beliefs to become more divorced from fundamental. More precisely, when lender's cost shock τ_1^L is especially large (small), borrower becomes more pessimistic (optimistic) than lender in expected sense.

Intuitively, particularly tight or loose funding and market liquidity impairs the ability of prices to reveal agents' private information about fundamentals. The reason for this is that the price of debt q_1 plays a dual role of allocating funds between the borrower and lender, and conveying the lender's private information about the fundamental R. The absence of complete markets implies that this price has to simultaneously perform both functions. Therefore, the more that the price of debt has to adjust in response to the lender's cost shock in order to reallocate funds at date 1, the less it reveals information about the lender's beliefs about the fundamental.

5 Financial Stability Implications of Information Spillovers

In this section, we make use of the common information benchmark to study the financial stability implications of information spillovers and show under what conditions they have a stabilizing versus destabilizing effect on financial markets.

First, we characterize how information spillovers affect the likelihood of fire sales. To that, we characterize the affects the partition of the state space between the normal and fire sale regimes in the common information benchmark and compare it with that in the baseline model. Recall that $\hat{q}_1$ is the threshold q_1 separating the two regimes in the baseline economy, defined implicitly by $\left(1-\pi_1^B(\hat{q}_1)\right)\left(\overline{R}-1\right)+\tau_1^B\hat{q}_1 = \tau_1^B F'(0)$, and $\hat{p}_1 = F'(0)$ is the corresponding asset price of the frontier. As in the baseline economy, the frontier under the common information benchmark is defined by the set of states $\left(\tau_1^L, \pi_1^L\right)$ consistent with $p_1 = \tilde{p}_1 \equiv F'(0)$, which is given by

$$\left(\overline{R}-1\right) - \left[\left(\overline{R}-1\right) + \frac{\tau_1^B}{\tau_1^L}\left(1-\underline{R}\right)\right]\pi_1^L + \frac{\tau_1^B}{\tau_1^L} = \tau_1^B F'(0). \tag{10}$$

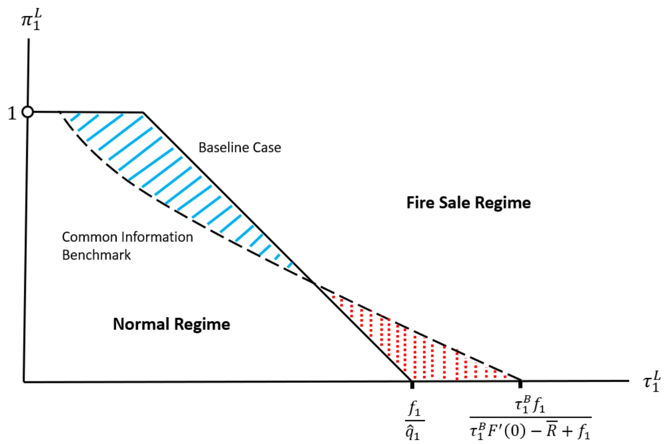

Figure 6: Comparison of Two Regimes under the Baseline Economy and the Common Information Benchmark

Note: This figure illustrates the bisection of the state space into two regimes at date 1 in the baseline case in which the lender's liquidity shock τ_1^L and beliefs π_1^L are private information, and under the common information benchmark case in which this information is directly observable by the borrower. The equilibrium is in the fire sale regime if and only if the equilibrium value of p_1 is below the threshold value $\hat{p}_1$. (For the baseline case, this corresponds to the threshold value of $\hat{q}_1$.) The solid curve in the figure plots the combinations of the states $\left(\tau_1^L, \pi_1^L\right)$ consistent the threshold $\hat{p}_1$, based on the lender's optimality condition for d_1^L, and denotes the frontier between the two regimes. The dashed curve plots the same frontier in the common information benchmark in which the borrower directly observes the lender's private information, and hence $\pi_1^B = \pi_1^L$ all along this curve. For both cases, the region to the southwest of these curves is the normal regime, while the northeast is the fire sale regime. The area shaded with dotted red (solid blue) lines marks the region of the state space in which fire sales occur only in the baseline economy (common information benchmark).

This characterizes the partition $\left(\tau_1^L, \pi_1^L\right) | \tilde{p}_1$, between the fire sale and normal regimes in the common information benchmark. In appendix E.5, we show analytically that the frontier takes the form depicted in figure 6, and that the frontiers in the baseline economy and the common information benchmark intersect in the domain $\pi_1^L \leq 1$.[21]

5.1 Effect of information spillovers on the likelihood of fire sales

How do information spillovers affect the likelihood of fire sales relative to the common information benchmark? Intuititively, this depends on the likelihood of entering the red-shaded region of the state space in figure 6 versus entering the blue shaded region. In general, the likelihood of a fire

[21]Note that, while the slope of $\left(\tau_1^L, \pi_1^L\right) | \hat{p}_1$ in the baseline economy is constant, the slope of $\left(\tau_1^L, \pi_1^L\right) | \tilde{p}_1$ in the common information benchmark is not. In the baseline economy, the frontier between the two regimes is determined entirely by a threshold price of debt q_1 (which maps one-for-one into a threshold asset price p_1). The lender's optimality condition implies that the relationship between the set of π_1^L and τ_1^L consistent with this threshold $\hat{q}_1$ is linear. In the common information benchmark, by contrast, the date 1 regime is determined not by q_1, but by p_1. The relationship between the set of π_1^L and τ_1^L consistent with this threshold $\tilde{p}_1$ is nonlinear.

sale in the baseline economy relative to the common information benchmark is ambiguous and depends on the type of shocks hitting the economy at date 1.

In response to severely adverse cost shocks, the information spillover leaves the borrower excessively pessimistic relative to the common information benchmark. This occurs in the region of figure 6 shaded with dotted red lines. The excessive pessimism which characterizes this region increases the likelihood of a fire sale occurring at date 1. (More precisely, conditional on a severely adverse cost shock, it takes a less severe signal at date 1 to push the economy into the fire sale regime in our baseline economy than in the benchmark.)

By contrast, in response to severely adverse news about the fundamental, the borrower is over-optimistic relative to the benchmark. This optimism, which occurs in the region of the figure shaded by solid blue lines, reduces the likelihood of a fire sale. (That is, conditional on very bad news, it takes a larger adverse cost shock to push the economy into a fire sale regime in the baseline economy compared to the benchmark.)

Thus, information spillovers increase the likelihood of fire sales driven by certain shocks but dampen those driven by others. The overall effect of of the information spillover on the unconditional likelihood of crises is ambiguous and depends on the relative importance of cost shocks versus news in driving asset price fluctuations. These results are formalized in the following proposition.

Proposition 5 (Effect of misinformation on the likelihood of fire sales)**.** *Conditional on a large adverse cost shock, the likelihood of entering a fire sale is higher in the baseline economy than in the common information benchmark. Conditional on a severely adverse news shock, however, the likelihood of entering a fire sale is lower in the baseline economy. The unconditional probability of a fire sale in the baseline economy relative to the common information benchmark is ambiguous.*

Proof. See appendix C.8. ■

5.2 Effect of information spillovers on the severity of fire sales

How do information spillovers affect the severity of a fire sale, conditional on one occurring at date 1? To answer this, let us combine the two partitions of the state space at date 1–the partitions illustrated in figures 4 and 5–to yield a four-way partition of the state space, illustrated in Figure 7. This divides the state space into four regions.

The first region consists of the normal regime characterized by over-optimism. The economy falls in this region at date 1 when the lender's signal is relatively positive and the cost shock is relatively small, leaving the price of debt high. In this region, the borrower holds all of the risky asset in equilibrium and is over-optimistic about the fundamental relative to what it would be in

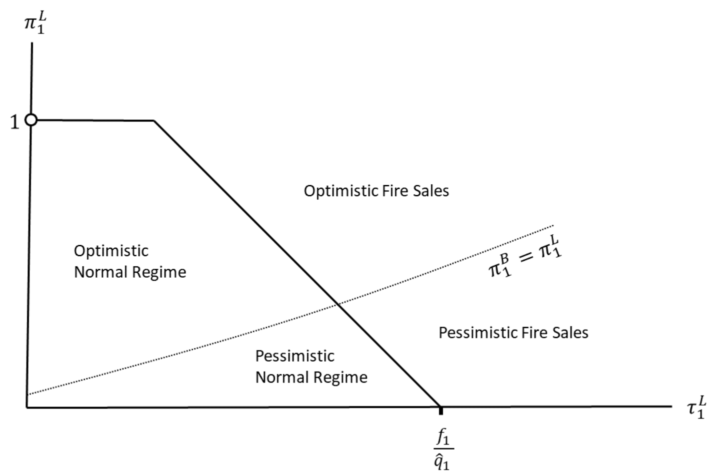

Figure 7: Four-Way Partition of the State Space

Note: This figure illustrates the demarcation of four regions of the state space by whether the equilibrium is in the Normal Regime or Fire Sale Regime, and by whether the borrower is optimistic or pessimistic about the risky asset relative to the better-informed lender.

the common information benchmark in response to the same realization of shocks. Here, over-optimism results in over-investment in the risky asset on the part of the borrower, relative to the common information benchmark.[22]

The second region, the normal regime characterized by pessimism, occurs when the cost shock is relatively small but the lender receives somewhat bad news about the fundamental. Here, the information spillover causes the borrower to be pessimistic relative to the benchmark. Nevertheless, this pessimism is not sufficiently severe to cause the borrower to liquidate any of the risky asset. This pessimism simply distorts the distribution of cash between the lender and borrower at date 1 relative to the common information benchmark.

The third region features fire sales characterized by over-pessimism on the part of the borrower. This occurs when the fire sale is driven by a sufficiently adverse cost shock to the lender. Relative to common information benchmark, the borrower is pessimistic, causing it to liquidate more of the risky asset. Here pessimism distorts allocation relative to the benchmark resulting in a risky asset price more liquidation of the risky asset by the borrower to the traditional sector. Hence, in this region, information spillovers result in pessimism about asset fundamentals, which *amplifies* the severity of the fire sale.

Finally, the fourth region of the state space features fire sales and relative optimism by the

[22]More precisely, optimism does not change the borrower's holdings of the risky asset because, in the normal regime, the borrower holds all the risky asset anyway. But this over-optimism results in a higher risky asset price p_1 relative to the benchmark, and also reduces the net cash payment from the borrower to the lender through the cost of borrowing.

borrower. This obtains when the lender receives bad news but the cost shock is not too severe. Here, the borrowers optimism relative to the common information benchmark limits the extent to which the borrower liquidates the asset and keeps the asset price higher. In this sense, information spillovers *dampen* the severity of the fire sale in this region.

The following proposition formalizes these last two results on the conditions under which information spillovers amplify or dampen the severity of fire sales relative to the benchmark. For these results, we compare the date 1 allocation in our baseline economy to that of the common information benchmark holding date 0 variables constant.

Proposition 6 (Effect of information spillovers on the allocation in the normal and fire sale regimes)**.** *Given the date 0 allocation, the following holds:*

(A) The allocation of the risky asset in the normal regime is identical in the baseline economy and the common information benchmark: $a_1^B = a_0^B$ in both economies.

(B) The information spillover amplifies the severity of fire sales driven by cost shocks and dampens the severity of fire sales driven by bad news at date 1. Formally, for any state $\left(\tau_1^L, \pi_1^L\right)$ in the fire sale regime, a_1^B is lower in the baseline economy compared to the common information benchmark if τ_1^L is sufficiently high (exceeding some threshold $\overline{\tau}_1^L(\pi_1^L)$), and is lower if π_1^L is sufficiently high (exceeding some threshold $\overline{\pi}_1^L(\tau_1^L)$).

Proof. See appendix C.9. ■

Part A shows that information spillovers do no not affect the allocation of the risky asset in the normal regime at date 1. While the beliefs of the lender and borrower may diverge in a way which does not reflect fundamental shocks, this has no bite on the allocation of the risky asset in the normal regime as long as the risky asset is not sold to the traditional sector. (Nevertheless, the allocation of cash between borrowers and lenders may differ, in general.) Part B shows that information spillovers have destabilizing effect on financial markets when a tightening in funding liquidity is driven by adverse cost shocks, but a stabilizing effect when funding illiquidity is driven by bad private news about fundamentals.

5.3 Takeaways on financial stability implications of information spillovers

The confluence of these results suggests that information spillovers have a destabilizing effect in response to liquidity shocks which are orthogonal to fundamentals, amplifying the likelihood and severity of a fire sale. This is because information spillovers result in excessive pessimism in response to adverse cost shocks. By contrast, information spillovers have a stabilizing effect on financial markets in response to bad news, leading to over-optimism about fundamentals which dampens both the likelihood and severity of a fire sale. These results are perhaps counterintuitive, as they imply that financial markets overreact to negative liquidity shocks but under-react to bad news about fundamentals. This is precisely because of the different way that liquidity shocks and private news affect agents' beliefs about fundamentals.

Underlying the model's dynamics is the presence of two externalities: a standard pecuniary externality, and a new 'information externality'. The pecuniary externality is essentially the same as in Lorenzoni (2008): Atomistic borrowers do not internalize how their asset sales at date 1 affects the budget constraints of other borrowers through the asset price p_1. In addition, the economy features an *information externality*: In choosing how much to lend at date 1, lenders do not internalize how their choices affect the information set of borrowers through the price q_1, and therefore how it affects borrowers' beliefs. The interaction of these externalities is what generates the amplification or dampening of financial instability described in section 4. Agents do not internalize how asset prices affect other agents' beliefs through the information externality, and agents' beliefs affect asset prices reinforcing the pecuniary externality.

6 Policy Implications

In this section, we ask what the effects of policy on formation of beliefs are. We show that the effect of central bank interventions on the informational efficiency of the economy depends on the nature of interventions and how they affect the collateralized lending market. In general, interventions do not add information and may even destroy information depending on the type of policy, although there may be cases where this is socially desirable. We first state these results formally in the following proposition.

Proposition 7.

1. *Standard monetary policy tools have no effect on the formation of agents' beliefs.*
2. *Collateralized central bank lending facilities can destroy information and reduce fluctuations in asset prices and the severity of fire sales.*

Proof. See appendix C.10. ■

Standard monetary policy tools For policy tools which affect agents' opportunity cost of lending, typical of more standard monetary policy tools such as open market operations or interest on central bank reserves, central bank interventions do not alter the information set of agents or their beliefs. By affecting the level of interest rates, monetary policy tools affect the opportunity cost of lending. We can therefore map this type of tool into our baseline model as an additional component of the lender's opportunity cost of funds τ_1^G such that the lender's total opportunity cost of funds at date 1 is $\tau_1^L + \tau_1^G$.[23] In appendix C.10, we show that such monetary policy interventions, while supporting funding liquidity, do not not affect the formation of beliefs or affect the ability of market prices to aggregate and reveal private intervention.

Collateralized central bank lending facilities In periods of market turmoil, central banks have often resorted to less conventional measures to support market functioning such as the several lending facilities implemented during the Global Financial Crisis or the onset of the COVID-19 pandemic. These policies effectively turn lenders into intermediaries of collateralized loans from the central bank to borrowers. As such, in our setting they have a more substantial effect on the equilibrium allocation.

In appendix C.10, we model such interventions by assuming that a central bank offers a collateralized loan to the lender at an exogenous price q_1^G, backed by the risky asset. Effectively, the lender intermediates collateralized debt from the central bank to the borrower.[24] As a result, the equilibrium price of date 1 debt form the lender to the borrower is pinned down by the exogenous government lending rate $q_1 = q_1^G$. The lender's optimality condition (1), as a result, drops out of the equilibrium conditions. Fluctuations in the allocation at date 1 (relative to date 0) are therefore driven only by the availability of government liquidity q_1^G.

This implies, however, that the borrower has less information than if there were no policy intervention: The equilibrium price of debt q_1 is no longer informative about the lender's private information. In that sense, the policy intervention *destroys* information, further deteriorating the informational efficiency of the market. Nevertheless, such effects may be socially desirable. This is because such a policy also mitigates volatility of endogenous beliefs, as the borrower's date 1 beliefs about the fundamental are the same as it its date 0 prior. In particular, the policy completely eliminates both liquidity-driven and belief-driven fire sales.

Interpretation Policymakers have frequently cited the effect of emergency interventions on a vague notion of investor 'confidence' as a stabilizing force. While the meaning of confidence is somewhat nebulous, our model provides one conceptualization of this notion. By stabilizing prices at government-administered rates, such policies effectively reduce the informativeness of market prices by disrupting the price-signal mechanism. Our model shows that, rather than being an unwanted side-effect of such interventions, these information effects may be precisely the mechanism through which interventions 'restore confidence' and prevent excessively pessimistic swings in the beliefs of market participants. By temporarily closing the gates to private information, these policies also eliminate pessimistic over-reactions by markets. In this sense, such interventions stabilize

[23] For simplicity, we assume that the level of interest rates affects only the lender's opportunity cost of funds, although the same results would obtain if it affected both agents.

[24] There are no rents from intermediation due to perfect competition between lenders.

markets both directly through the provision of liquidity, and indirectly by presenting investor panic and excessive pessimism about fundamentals.

Whether or not this is welfare-improving depends on the underlying shocks driving the turmoil, which the central bank itself does not typically observe. Our results point to the possibility that it is welfare enhancing to *reduce*, at the margin, the information available to investors. This is an application of the theory of the second best. Namely, the presence of the information externality and the pecuniary externality, described in section 5.3, imply that at the margin, a reduction in the information available to market participants about asset fundamentals may actually improve welfare under certain conditions. Nevertheless, we leave the characterization of optimal policy and its implications for social welfare to future work.

7 Alternative Case with Upstream Information Spillovers

Thus far, we have focused on the case in which the lender gets the cost shock and private signal, but the borrower does not. This implies that information spillovers flow downstream from the lender to the borrower. In this section, we briefly consider the alternative case in which the borrower gets a cost shock and private signal, but the lender does not.[25] In appendix D, we show that while the mechanism differs slightly, the fundamental insights are the same as in the baseline case. This alternative case is illustrated in figure 8.

Suppose the borrower gets a cost shock τ_1^B and private signal s_1^B, but the lender does not (i.e. τ_1^L is a fixed parameter known to both agents). Here, the price of debt q_1 is still pinned down by the lender's optimality condition and therefore conveys no information to the lender about the borrower's private information. However, the price of the risky asset p_1 is pinned down by the borrower's optimality condition and therefore conveys information to the lender about the s_1^B.

Nevertheless, the lender faces an equivalent identification problem as in our baseline case: The lender cannot separately identify the borrower's signal s_1^B and cost shock τ_1^B from the observed price p_1.[26] As a result, information spillovers in this case flow upstream from the borrower to the lender rather than downstream.

In this setting, the beliefs of upstream agents are shaped in equilibrium by the availability of *market liquidity* rather than by funding liquidity, as was the case in our baseline case. Namely,

[25] For simplicity, we assume that only one agent receives both a cost shock and a private signal, while the other agent receives neither. We make this assumption, rather than allowing both agents to receive private signals, to prevent the model from becoming analytically intractable. If, in contrast, both agents received private signals, then the equilibrium would depend not only on how one agent, e.g. the borrower, updates its beliefs in response to the actions of the other, but also on an infinite feedback loop in which also the borrower's actions affect the lender's beliefs, etc. This would make an analytical characterization of equilibrium belief formation extremely difficult.

[26] The borrower cannot credibly reveal its private information to the lender, as it has incentive to inflate the collateral value of its risky asset holdings to obtain cheaper funding.

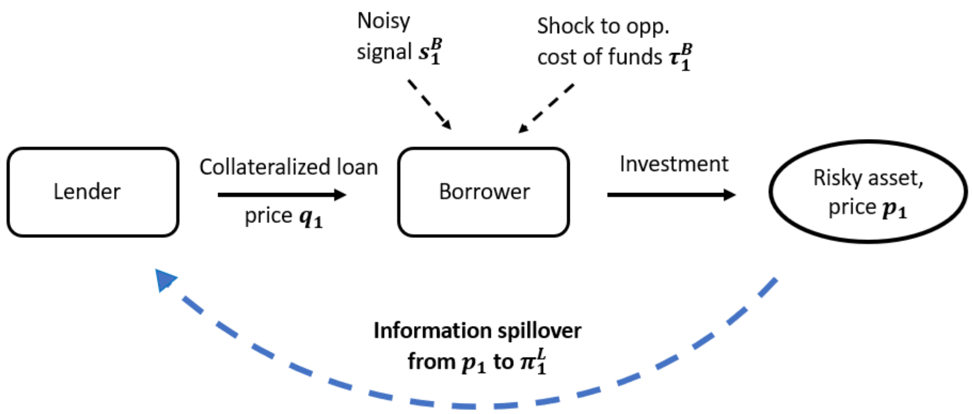

Figure 8: Alternative Case with Upstream Information Spillovers

Note: This figure stylistically illustrates the alternative case in which the borrower receives the private signal and cost shock at date 1, leading to upstream spillovers through the price of the risky asset.

changes in the price of the risky asset p_1 which are driven by cost shocks to the borrower affect lender's optimism or pessimism about the asset fundamental.

In this environment, the availability of market liquidity gives risk to belief-driven booms or busts in the asset market. Cost shocks to the borrower which result in a lower asset price cause the lender to become more pessimistic about the asset. As a result, the lender cuts back funding at date 1. Moreover, this reduction in funding liquidity may cause the borrower to liquidate more of the risky asset, depressing its price. Hence, there may be a feedback loop between market liquidity and funding liquidity. Thus, the nature of information spillovers and the effect of liquidity on beliefs and asset prices are essentially same as in the baseline case.

8 Conclusion

We developed a model of heterogeneous agents, collateralized debt, learning, and fire sales to shed light on how the availability of funding liquidity affects agents' perceptions about fundamentals, and the role that this may play in asset booms or busts. The model shows that beliefs about fundamentals are endogenously shaped by the availability of liquidity in funding and asset markets. This is due to an information spillover through which asset prices affect belief. As a result, liquidity shocks which are orthogonal to asset fundamentals can cause beliefs to become systematically divorced from fundamentals. Moreover, this informational inefficiency may be more acute when liquidity conditions are especially tight or loose, as this impairs the ability of prices to reveal private information about fundamentals. As a result of this mechanism, loose funding liquidity can generate over-optimism about fundamentals leading to over-investment, while tight funding liquidity can lead to excessive pessimism about fundamentals and fire sales. Relative to a counterfactual economy in which all information is publicly observed information spillovers can stabilize or destabilize financial markets, measured by the likelihood and severity of fire sales, depending on the underlying nature of shocks. Finally, we showed that central bank interventions such as liquidity facilities and asset purchases can stabilize financial markets in part by reducing the informativeness of asset prices, thereby reducing the volatility of market participants' beliefs.

A Appendix: Optimization Problems

A.1 Lender's Problem

The representative lender solves a portfolio choice problem, deciding how much unit of collateral to accept, d_t^L for each date t, and how much cash to hold, κ_t^L for each date t, subject to its budget constraint, taking prices p_t, q_t and available contracts as given. B microfounds debt as a collateralized debt contract with c_t units of the risky asset being posted as collateral at t to be repurchased at $t+1$.

Lender's portfolio choice problem at date 1

At date 1, the lender's portfolio choice is to decide how much to lend versus how much cash to hold. The lender's opportunity cost of lending is its date 1 marginal return of cash (which is subject to the liquidity shock) τ_1^L, since this is the date 2 return that the lender gets on its holdings of cash at date 1. If the borrower cannot finance the promised payment to the lender out of its cash holdings or with new borrowing at date 1, then either the borrower has to liquidate some of its holdings of the risky asset to the traditional sector for cash or the lender seizes the collateral and liquidates it to the traditional sector. Both of these actions are payoff-equivalent, so we ignore the confiscation of collateral by the lender and instead assume that the borrower liquidates to the traditional sector in this case.

The lender's problem at date 1 is to maximize its expected utility $E_1^L[C_2^L]$ from date 2 consumption by choosing its portfolio (cash versus a loan) subject to budget constraints at date 1 and date 2. Let d_1^L denote the lender's choice of how many units of loan to invest in (at price q_1) at date 1. (In equilibrium, the condition for market clearing for the loan will be $d_1^L = c_1$ if $c_1 > 0$. If $c_1 = 0$, then $d_1^L > c_1$ is possible because of the reason we discuss later in the the lender's optimal decision. By assumption 1, $d_1^L > 0$ is always possible even when $c_1 = 0$ as we show later.)

Lender's date 0 budget constraint At date 0, the lender allocates its cash endowment between date 0 cash holdings and the date 0 loan to the borrower.

$$\underbrace{\kappa_0^L}_{date\ 0\ cash holdings} + \underbrace{q_0 d_0^L}_{value\ of\ date\ 0\ loan} \leq \underbrace{e_0^L}_{date\ 0\ cash\ endowment}$$

Lender's date 1 budget constraint At date 1, the flow of payments received by the lender is simply $q_0 d_0^L - q_1 d_1^L$, the difference in the value of the loan under the contract terms agreed to at date 0 minus the value of the loan under the new contract terms.

$$\underbrace{\kappa_1^L}_{date\ 1\ cash\ holdings} + \underbrace{q_1 d_1^L}_{value\ of\ date\ 1\ loan} \leq \underbrace{\kappa_0^L}_{date\ 0\ cash\ holdings} + \underbrace{d_0^L}_{value\ of\ date\ 0\ loan}$$

i.e.

$$\underbrace{\kappa_1^L - \kappa_0^L}_{net\ increase\ in\ cash\ holdings} \leq \underbrace{d_0^L - q_1 d_1^L}_{net\ decrease\ in\ loan}$$

Lender's date 2 budget constraint At date 2, the contract is settled or defaulted upon by the borrower, the lender earns a return from its cash holdings, and consumes. Define the date 2 proceeds from the loan as $R_2^d \equiv \min\{R, 1\}$, which accounts for the possibility of default.

$$\underbrace{C_2^L}_{date\ 2\ consumption} = \underbrace{\tau_1^L \kappa_1^L}_{proceeds\ from\ cash\ holdings} + \underbrace{R_2^d d_1^L}_{proceeds\ from\ date\ 1\ loan}$$

Lender's problem at date 1

Taking as given the date 1 contract terms and prices q_1, and p_1, the lender decides how much of its funds to allocate to the loan or cash. The lender's optimization problem is

$$\max_{\kappa_1^L, d_1^L} \ E_1^L \left[C_2^L\right]$$

$$s.t. \quad C_2^L \leq \tau_1^L \kappa_1^L + R_2^d d_1^L$$

$$\kappa_1^L - \kappa_0^L \leq d_0^L - q_1 d_1^L$$

with non-negativity constraints:

$$\kappa_1^L \geq 0, \ d_1^L \geq 0.$$

Both budget constraints will bind at the optimum, so we can replace cash holdings κ_1^L using the date 1 budget constraint.

$$C_2^L = \tau_1^L \left(\kappa_0^L + d_0^L - q_1 d_1^L\right) + R_2^d d_1^L$$

Let $\xi_{d_1}^L$ and $\xi_{\kappa_1}^L$ denote the Lagrange multipliers on the respective non-negativity constraints. The Lagrangian for the lender's date 1 problem is

$$L_1^L = E_1^L \left[U_L \left(\tau_1^L \left(\kappa_0^L + d_0^L - q_1 d_1^L\right) + R_2^d d_1^L\right)\right] + \xi_{d_1}^L d_1^L + \xi_{\kappa_1}^L \left(\kappa_0^L + d_0^L - q_1 d_1^L\right)$$

The lender's first-order condition (FOC) for d_1^L is

$$q_1 = \frac{E_1^L\left[R_2^d\right] + \xi_{d_1}^L}{\tau_1^L + \xi_{\kappa_1}^L}. \tag{11}$$

This says that the lender chooses how much to lend to equalize the discounted marginal return from lending at date 1 to the discounted marginal return on its opportunity cost from holding cash at date 1.

We also have the two complementary slackness conditions:

$$\xi_{d_1}^L d_1^L = 0$$

$$\xi_{\kappa_1}^L \kappa_1^L = 0$$

Note that there is no d_1^L in (11) as the lender's problem is linear. The lender is indifferent across different values of d_1^L as long as the price of the contract is equal to the expected return of the loans, $E_1^L\left[R_2^d\right]/\tau_1^L$, the ratio of the discounted marginal return from lending at date 1 to the discounted marginal return on its opportunity cost from holding cash at date 1. Note that assumption 1 ensures that the lender always holds a strictly positive quantity of cash in equilibrium at date 1, $\kappa_1^L > 0$.[27] Therefore, the lender's Lagrange multiplier for the non-negativity constraint satisfies $\xi_{\kappa_1}^L = 0$ in equilibrium. Moreover, assumption 1 ensures that, lending takes place in equilibrium so that $d_1^L > 0$ and $\xi_{d_1}^L = 0$ in equilibrium.[28] Therefore, given these assumptions, the lender's optimality condition (11) reduces in equilibrium to

$$q_1 = \frac{E_1^L\left[R_2^d\right]}{\tau_1^L}. \tag{12}$$

Thus, in equilibrium, the competitive spot price of the contract at date 1, q_1, will ensure that this condition holds.[29]

[27] The lender's optimality condition implies that, in equilibrium, the lender is just indifferent between holding cash and not, which implies that the lender's Lagrange multiplier from the non-negativity constraint satisfies $\xi_{\kappa_1}^L = 0$ in equilibrium. Otherwise the lender's expected discounted return from lending would exceed its opportunity cost of funds.

[28] Note that assumption 1 ensures that the lender's date 0 endowment e_0^L is large enough to cover all the loans even when the lender is maximally optimistic about the quality of the risky asset and believes $R = \overline{R}$ with probability 1.

[29] To see this intuitively, suppose that q_1 is less than the expected discounted return of a loan, $q_1 < \frac{E_1^L\left[R_2^d\right]}{\tau_1^L}$. Then the lender would want to maximize its holdings of the loans and hold no cash, so that $\kappa_1^L = 0$. But assumption 1 ensured that $\kappa_1^L > 0$, so that we have a contradiction. Suppose instead that we have $q_1 > \frac{E_1^L\left[R_2^d\right]}{\tau_1^L}$. Then, the lender would

A.2 Borrower's Problem

At date 1, the representative borrower solves a portfolio choice and chooses how much to borrow, subject to its budget constraint, collateral constraint, and taking as given the price of the risky asset p_1 and the price of the contract q_1. At date 1, borrowers cannot initiate new risky assets. Moreover, borrowers can exchange their extant holdings of the risky asset to cash by selling it to the traditional sector, at the market price p_1 and investing the proceeds in cash for total date 2 return of $p_1\tau_1^B$. Hence, the borrower's opportunity cost of holding the risky asset is given by τ_1^B/p_1.

At date 1, the borrower must repay its date 0 debt. We show in internet appendix B that debt is microfounded as a collateralized debt contract with c_t of the risky asset being posted as collateral at t to be repurchased at $t+1$ at a unit price of f_t, normalized to $f_t = 1$. Therefore, to repay its debt at date 1, the borrower must buy back from the lender the c_0 units of the risky asset that were posted at collateral. However, the borrower obtains no date 1 cash flow from its holdings of the risky asset at date 1. Therefore, the borrower can finance this repayment of c_0 in three ways: out of any cash holdings at date 1, by raising new debt at date 1, or by liquidating the risky asset in the date 1 spot market for the risky asset.

The borrower can raise new debt at date 1 (i.e. refinance its date-0 debt) using the date-1 contract—that is, by posting c_1 units of the risky asset as collateral in exchange for q_1c_1 units of the consumption good as a loan, where q_1 is the competitive spot price of this contract. If the debt raised at date 1 q_1c_1 is insufficient to cover the repayment c_0 the borrower must make at date 1, this difference must be financed either out of the borrower's cash holdings stored from date 0, or by selling some portion of its holdings of the risky asset in the spot market.

Define κ_0^B as the amount of cash the borrower carries into date 1 from date 0. Now this decision is relevant only in the date-0 optimization problem. For the date-1 problem, the borrower considers it as a predetermined amount of cash κ_0^B.[30]

Collateral constraint at date 1 Let a_0^B denote the borrower's total risky asset holdings brought to date 1 from date 0, and c_0 denote the borrower's date 0 risky asset holdings used as collateral at date 0. (We will later show that, in equilibrium, $c_0 = a_0^B$.) Let a_1^B denote the borrower's total risky asset holdings chosen at date 1, and c_1 denote the amount of the borrower's date 1 asset holdings used as collateral at date 1. (Note that the amount of asset sold by the borrower at date 1 is then $a_0^B - a_1^B$.) The 'collateral constraint' is then $c_1 \leq a_1^B$.

Borrower date 0 budget constraint At date 0, the borrower has a cash endowment and receives a cash loan from the lender, which it can allocate between date-0 cash holdings and in-

want to minimize d_1^L to negative infinity without the non-negativity constraint so that the non-negativity constraint of d_1^L would bind, $d_1^L = 0$. But assumption 1 ensured that $d_1^L > 0$ in equilibrium, so that we again have a contradiction.

[30] We will see in the date-0 problem that the borrowers financed their payment to purchase c_0 assets with the price p_0 at date 0 by their cash endowments e_0^B and the borrowing amount $c_0p_0(1-m_0) = c_0q_0$, where m_0 is the margin of the contract. Therefore, the leftover cash can be defined as $\kappa_0^B \equiv e_0^B + c_0p_0(1-m_0) - c_0p_0 = e_0^B - c_0p_0m_0$.

vestment in the risky asset.

$$\underbrace{q_0 c_0}_{date\ 0\ loan} + \underbrace{e_0^B}_{date\ 0\ cash\ endowment} \geq \underbrace{p_0 a_0^B}_{value\ of\ date\ 0\ asset\ holdings} + \underbrace{\kappa_0^B}_{date\ 0\ cash\ holdings}$$

Borrower date 1 budget constraint At date 1, if the borrower defaults on its date-0 debt obligations, it loses its collateral and has only its cash holdings to find its portfolio choices at date 1. In equilibrium, the borrower never defaults at date 1, because the selling all the collateral would be sufficient in repaying the debt by assumption 2. For this reason, we omit this case. In the event that the borrower does not default at date 1, its date 1 budget constraint is

$$\underbrace{q_1 c_1}_{date\ 1\ loan} + \underbrace{p_1 a_0^B}_{value\ of\ date\ 0\ asset\ holdings} + \underbrace{\kappa_0^B}_{date\ 0\ cash\ holdings} \geq \underbrace{p_1 a_1^B}_{value\ of\ date\ 1\ asset\ holdings} + \underbrace{c_0}_{date\ 0\ loan} + \underbrace{\kappa_1^B}_{date\ 1\ cash\ holdings}$$

i.e.

$$\underbrace{q_1 c_1 - c_0}_{net\ increase\ in\ loan} \geq \underbrace{p_1 \left(a_1^B - a_0^B\right)}_{net\ increase\ in\ asset\ holdings} + \underbrace{\kappa_1^B - \kappa_0^B}_{net\ increase\ in\ cash\ holdings}.$$

Borrower date 2 budget constraint The borrower's date 2 budget constraint limits date 2 consumption C_2^B by the return on the borrower's portfolio of assets held at date 1.

$$\underbrace{C_2^B}_{date\ 2\ consumption} \leq \underbrace{\tau_1^B \kappa_1^B}_{proceeds\ from\ cash\ holdings} + \underbrace{a_1^B R}_{proceeds\ from\ asset\ holdings} - \underbrace{c_1 R_2^d}_{repayment\ of\ debt},$$

where $R_2^d \equiv \min\{R, 1\}$ denotes the realized date 2 repayment of the loan (including the possibility of default).

Each borrower takes the price q_1 of the contract at date 1 as given. In equilibrium, the price q_1 will be informative to the borrower about the lender's private information at date 1. We discuss how the borrower's beliefs evolve in section 3.2. We first characterize the borrower's optimality conditions, conditional on its information set at date 1.

Borrower's optimization problem at date 1

The borrower's optimization problem at date 1 is to make its portfolio and borrowing decision to maximize expected date 2 utility $E_1^B\left[C_2^B\right]$, conditional on its date 1 information set, taking prices as p_1 and q_1 as given.

$$\max_{c_1^B, a_1^B, \kappa_1^B} E_1^B\left[C_2^B\right] \tag{13}$$

$$\begin{aligned} \text{s.t. } & q_1 c_1 - c_0 \geq p_1\left(a_1^B - a_0^B\right) + \kappa_1^B - \kappa_0^B, \\ & C_2^B \leq \tau_1^B \kappa_1^B + a_1^B R - c_1 R_2^d \qquad (14) \\ & c_1 \leq a_1^B, \\ & c_1 \geq 0,\ a_1^B \geq 0,\ \kappa_1^B \geq 0 \end{aligned}$$

The first-order condition for c_1 is

$$\left(\tau_1^B + \xi_{\kappa_1}^B\right) q_1 - E_1^B\left[R_2^d\right] - \mu_1^B + \xi_{c_1}^B = 0, \tag{15}$$

while the first-order condition for a_1^B is

$$-\left(\tau_1^B + \xi_{\kappa_1}^B\right) p_1 + E_1^B[R] + \mu_1^B + \xi_{a_1}^B = 0. \tag{16}$$

The complementary slackness conditions are given by

$$\mu_1^B\left(a_1^B - c_1\right) = 0 \tag{17}$$

$$\xi_{c_1}^B c_1 = 0 \tag{18}$$

$$\xi_{a_1}^B a_1^B = 0 \tag{19}$$

$$\xi_{\kappa_1}^B \kappa_1^B = 0 \tag{20}$$

We can further characterize the borrower's behavior by combining the FOCs for the case with risk-neutrality. Suppose that the collateral constraint is binding in equilibrium so that $\mu_1^B > 0$. (We show in lemma 2 in appendix C.4 that this must be the case.) Then, the FOCs for c_1 and a_1^B hold with equality. Suppose also that $c_1 = a_1^B > 0$ in equilibrium. Combining these binding first order conditions yields

$$\underbrace{E_1^B\left[R_2^d\right] - q_1 \tau_1^B}_{cost\ of\ financing\ asset\ holdings} + \underbrace{\xi_{\kappa_1}^B\left(p_1 - q_1\right)}_{shadow\ value\ of\ cash} = \underbrace{E_1^B[R] - \tau_1^B p_1}_{expected\ net\ return\ of\ risky asset} \tag{21}$$

It is convenient to define $\nu_1\left(\pi_1^B, q_1\right) \equiv E_1^B\left[R_2^d\right] - q_1\tau_1^B$ and $\eta_1\left(\pi_1^B; p_1\right) \equiv E_1^B[R] - \tau_1^B p_1$, so that (21) can be expressed as

$$\underbrace{\nu_1\left(\pi_1^B, q_1\right)}_{\textit{cost of financing asset holdings}} + \underbrace{\xi_{\kappa_1}^B\left(p_1 - q_1\right)}_{\textit{shadow value of cash}} = \underbrace{\eta_1\left(\pi_1^B; p_1\right)}_{\textit{expected net return of risky asset}} \tag{22}$$

If $E_1^B[R] - \tau_1^B p_1 > E_1^B\left[R_2^d\right] - q_1\tau_1^B$, i.e. if the net return from buying asset exceeds the net return from offering collateral (balancing the benefit from a larger loan with the cost of a larger repayment), then we must have that $\xi_{\kappa_1}^B\left(q_1 - p_1\right) < 0$, which requires that $\xi_{\kappa_1}^B > 0$ (non-negativity constraint for κ_1^B binding) and $q_1 < p_1$, which we will show in lemma 2 holds in equilibrium. In this case, the borrower is at a corner solution in its portfolio choice.

A.3 Traditional Sector's Problem

The traditional sector becomes relevant at date 1 only and enters the period with a new endowment of cash $\kappa_1^T = e_0^T$. It can store cash between dates 1 or 2, or buy existing risky assets at date 1 in the spot market. Existing assets need to be managed subject to some increasing costs. The date-2 return of holding a_1^T of the risky asset at date one, net of these management costs, is given by $F(a_1^T)$ units of the consumption good, where $F' > 0$ and $F'' < 0$. Without loss of generality, assume $\tau_1^T = 1$. The traditional sector gets utility only from date-2 consumption C_2^T with linear utility. The traditional sector's optimization problem is to choose how to allocate its endowment between cash or the risky asset to maximize its utility.

$$\begin{aligned} \max_{\kappa_2^T, a_1^T} \ & C_2^T \\ \text{s.t. } & C_2^T = \kappa_2^T + F(a_1^T) \\ & \kappa_1^T = \kappa_2^T + p_1 a_1^T \\ & a_1^T \geq 0 \\ & \kappa_2^T \geq 0 \end{aligned} \tag{23}$$

For simplicity, and without loss of generality, we assume the traditional sector's endowment of cash is sufficiently large that its non-negativity constraint on date-2 cash holdings is never binding, $\kappa_2^T > 0$, by assumption 1. This ensures that, in equilibrium, the traditional sector could buy all existing assets.[31] As a result, the traditional sector will always enter date 2 with positive cash

[31] To see this, note that if the traditional sector holds all the assets, then $a_1^T = a_0^B$. So its date 1 expenditure is $p_1 a_0^B$. Note also that in the eq where the traditional sector is holding assets, we have $p_1 = F'(a_1^T)$, and so this expenditure is $p_1 a_0^B = F'(a_0^B)a_0^B$. Then the above assumption guarantees that the traditional sector has enough cash endowment at

holdings κ_2^T, so the non-negativity constraint $\kappa_2^T \geq 0$ is non-binding. Using binding date-1 and -2 budget constraints, the traditional sector's date-1 problem is to choose a_1^T to maximize $\kappa_1^T - p_1 a_1^T + F(a_1^T)$ subject to $a_1^T \geq 0$. the traditional sector's optimality condition is

$$p_1 = F'(a_1^T) + \xi_{a_1}^T. \tag{24}$$

B Appendix: Microfounding the financial contracts

Here, we microfound the contracting environment in terms a competitive market for repo contracts. At the end of this section, we summarize the environment and show that it is payoff-equivalent to the version in the main text.

Contracting environment

Market segmentation implies that there are gains to trading financial contracts. More precisely, L cannot hold the risky asset at any point, but can gain exposure to the risky asset by lending to B through the use of financial contracts. We assume that, although the risky asset matures after two periods, the borrower finances its holdings of the risky asset using one-period collateralized debt contracts. By financing long-term risky assets with short-term debt, the borrower is exposed to liquidity risk at date 1. We formalize this environment and the nature of liquidity risk below.

Financial contracts

Formally, a date t contract is given by (c_t, f_t), which stipulates a transfer at date t of the risky asset of size c_t from the borrower to the lender as collateral with a promise for the borrower to repurchase the collateral at date $t+1$ at a unit price of f_t, normalized to $f_t = 1$. (That is, the borrower repays the lender $f_t c_t = c_t$ units of the consumption good at date $t+1$.) We assume that the collateral on the loan is held by some outside custodian, not modeled explicitly, who either returns the collateral to the borrower if the contract is honored or sells it and pays the proceeds to the lender if the borrower defaults. This ensures that the lender accepts the asset as collateral for the loan despite not being able to hold the asset itself.

These contracts are traded in a competitive market, where q_t denotes the market price of the contract at $t = 0, 1$. Therefore, the size of the loan from the lender to the borrower at t—that is, the quantity of the consumption good loaned at date t— is given by $q_t c_t$. At date $t+1$, the borrower decides whether to default on the contract and forgo the collateral or not. If the borrower chooses to default on the contract at date $t+1$, the value of this collateral $c_t p_t$ is transferred to the lender.

date one to make this purchase.

Note that both date 0 and date 1 contracts are one-period contracts and involve a repayment of c_t at date $t+1$. One may wonder why the borrower would be willing to enter into the date 0 contract in the first place, given that this will require the borrower to repay the lender c_0 units of the consumption good at date 1 despite the fact that the borrower has no income at date 1. In equilibrium, the borrower will be willing to borrow at date 0 because it anticipates that it can finance this repayment c_0 (at least partially) by refinancing (i.e. rolling over) its debt using the date 1 contract, albeit under different terms. One can interpret this contracting environment as involving long-term (two-period) debt which is renegotiated in the intermediate period.

Note that the lender has full-recourse on the debt at date 1. Thus, the borrower will try to pay the debt in full at date 1 unless it exhausts all of cash and asset holdings. However, at date 2, there is no further recourse to the borrower's balance sheet as the borrower can simply walk away from the collateral. Under the date 1 terms of the contract, the borrower will decide whether to default on the contract and forgo the collateral or not, thus, the payment from the borrower to the lender at date 2 will be determined endogenously as $\min\{p_2, 1\}$, where 1 is the promised payment amount.

Summary and payoff equivalence

More precisely, at date $t = 0,1$, the borrower can sell the lender a repo contract which stipulates that the borrower sells a unit of the risky asset (i.e., collateral) to the lender at date t and repurchases the collateral at t+1 for a unit price of f_t normalized to $f_t = 1$. Hence, the contract is defined at the level of the units of collateral exchanged, c_t. d_t^L is the number of units of the contract demanded by the lender. Therefore, market clearing requires that $c_t = d_1^L$.

What does the flow of payments look like? At date t, the lender buys d_1^L units of the contract (which therefore involves buying d_1^L units of the collateral from the borrower) at a price of q_t per contract. Hence, there is a payment of $q_t d_t^L$ units of the consumption good from the lender to the borrower. At date $t+1$, the borrower either repurchases the collateral from the lender according to the contract or defaults. In the good state, there is no default, so the borrower buys back c_t at a predetermined price $f_t = 1$. Hence the lender receives $c_t f_t = c_t = d_t^L$ units of the consumption good. In the bad state, the borrower prefers to default. In that case, the borrower forgoes its collateral. The lender keeps the c_t units of collateral which yields a gross interest rate of R. Thus, we can express the lender's payoff from loan as $d_1^L R^d$, where $R^d = \min\{R, 1\}$.

Hence, this repo contracting environment is payoff-equivalent to one in which the borrower issues a bond backed by its holdings of the risky asset. In this case, the lender buys d_t^L units of a bond issued by the borrower at unit price of q_t and which promises a gross interest rate of 1. Hence the loan involves $q_t d_t^L$ unit of consumption good. If there is no default at $t+1$, the borrower pays back d_t^L at date 2. If the borrower defaults, it forgoes collateral, and so the lender gets the return of the risky asset, i.e. it receives a payoff of $R^d = \min\{R, 1\}$ per unit of the bond.

C Appendix: Omitted Proofs

C.1 Derivation of the lender's posterior beliefs

First, note that the bounds of the posterior belief of the lender is $\pi_1^L \in (0,1)$. The bounds are derived from the fact that $s_1^L = R + \varepsilon$ with $\varepsilon \in (-\infty, \infty)$, so any realization of s_1^L is possible from both state of the world $R = \bar{R}$ and $R = \underline{R}$. The lender updates posterior beliefs as in (6)

$$\pi_1^L = \frac{\pi_0 \lambda_\varepsilon \left(\varepsilon_1^L = s_1^L - \underline{R}\right)}{\pi_0 \lambda_\varepsilon \left(\varepsilon_1^L = s_1^L - \underline{R}\right) + (1-\pi_0)\lambda_\varepsilon \left(\varepsilon_1^L = s_1^L - \bar{R}\right)}.$$

This expression of π_1^L is possible by using Bayes' rule for events with a positive measure, and then taking limits. For any $\delta > 0$, the events $\left\{s_1^L - \underline{R} \le \varepsilon_1^L \le s_1^L - \underline{R} + \delta\right\}$ and $\left\{s_1^L - \overline{R} \le \varepsilon_1^L \le s_1^L - \overline{R} + \delta\right\}$ are well defined and have positive probability. Therefore, Bayes' rule implies

$$\begin{aligned}
&P(R = \underline{R} | s_1^L - \underline{R} \le \varepsilon_1^L \le s_1^L - \underline{R} + \delta) \\
=&\frac{\pi_0 P\left(s_1^L - \underline{R} \le \varepsilon_1^L \le s_1^L - \underline{R} + \delta\right)}{\pi_0 P\left(s_1^L - \underline{R} \le \varepsilon_1^L \le s_1^L - \underline{R} + \delta\right) + (1-\pi_0) P\left(s_1^L - \overline{R} \le \varepsilon_1^L \le s_1^L - \overline{R} + \delta\right)} \\
=&\frac{\pi_0 \int_{s_1^L - \underline{R}}^{s_1^L - \underline{R} + \delta} \varepsilon' d\Lambda_\varepsilon(\varepsilon')}{\pi_0 \int_{s_1^L - \underline{R}}^{s_1^L - \underline{R} + \delta} \varepsilon' d\Lambda_\varepsilon(\varepsilon') + (1-\pi_0) \int_{s_1^L - \overline{R}}^{s_1^L - \overline{R} + \delta} \varepsilon' d\Lambda_\varepsilon(\varepsilon')},
\end{aligned} \tag{25}$$

where P is the probability function and both the denominator and numerator are positive. As $\delta \to 0$, we have

$$\begin{aligned}
P(R = \underline{R} | s_1^L - \underline{R} \le \varepsilon_1^L \le s_1^L - \underline{R} + \delta) &\to P(R = \underline{R} | \varepsilon_1^L = s_1^L - \underline{R}) = \pi_1^L \\
\int_{s_1^L - \underline{R}}^{s_1^L - \underline{R} + \delta} \varepsilon' d\Lambda_\varepsilon(\varepsilon') &\to \lambda_\varepsilon(\varepsilon_1^L = s_1^L - \underline{R}) \\
\int_{s_1^L - \overline{R}}^{s_1^L - \overline{R} + \delta} \varepsilon' d\Lambda_\varepsilon(\varepsilon') &\to \lambda_\varepsilon(\varepsilon_1^L = s_1^L - \overline{R}).
\end{aligned}$$

Thus, taking the limit of $\delta \to 0$ on both sides of (25) results in (6).

C.2 Proof of Proposition 1

Proof. 1. B' s information at date 1, I_1^B, is given by the set of all variables observable to all agents, which are prices p_1 and q_1. In contrast, L has additional information of τ_1^L and s_1^L, denoted by I_1^L, on top of the publicly available information. By the law of iterated expectation

$$E_1^B\left[E_1^L[x]\right]=E\left[E\left[x\mid I_1^L\right]\mid I_1^B\right]=E\left[x\mid I_1^B\right]=E_1^B[x].$$

Applying the definition of posterior beliefs $\pi_1^i=E_1^i[\mathbb{1}\{R=\underline{R}\}]$ for $i=L,B$ leads to statement 1.

2. By the first statement of proposition 1 and (8), we have

$$\pi_1^B(q_1)=\int_0^1\pi\lambda_T\left(\frac{1-(1-\underline{R})\pi}{q_1}\right)\mathbb{1}\left\{\underline{\tau}<\frac{1-(1-\underline{R})\pi}{q_1}<\bar{\tau}\right\}dG_\pi(\pi)$$

for a given q_1.

First, note that there exists $\hat{\pi}(q)$ such that

$$\frac{1-(1-\underline{R})\hat{\pi}(q)}{q}=\tau_0$$

for any $q<q_0$. Also, note that $\hat{\pi}(q)$ is decreasing in q. From (1), we can rearrange the difference between the prior for τ, τ_0, and realized τ for a given π and equilibrium price q as

$$|\tau-\tau_0|=\left|\frac{1-(1-\underline{R})\pi}{q}-\tau_0\right|,$$

which can be further simplified as

$$\left|\frac{1-(1-\underline{R})\pi}{q}-\tau_0\right|=\frac{1}{q}|1-(1-\underline{R})\pi-1+(1-\underline{R})\hat{\pi}(q)|=\frac{1}{q}|(1-\underline{R})(\pi-\hat{\pi}(q))|. \tag{26}$$

We claim that for any π and $q'>q$,

$$\left|\frac{1-(1-\underline{R})\pi}{q}-\tau_0\right|<\left|\frac{1-(1-\underline{R})\pi}{q'}-\tau_0\right| \tag{27}$$

and therefore,

$$\lambda_T\left(\frac{1-(1-\underline{R})\pi}{q}-\tau_0\right)>\lambda_T\left(\frac{1-(1-\underline{R})\pi}{q'}-\tau_0\right) \tag{28}$$

by the functional form assumption on λ_T.

By equation (26), (27) is true if

$$\frac{1}{q}|(1-\underline{R})(\pi-\hat{\pi}(q))|<\frac{1}{q'}|(1-\underline{R})(\pi-\hat{\pi}(q'))|$$

holds. The previous equation holds if

$$\frac{q'}{q} < \frac{|\pi - \hat{\pi}(q')|}{|\pi - \hat{\pi}(q)|}$$

holds, which is equivalent to

$$\frac{\dfrac{1-(1-\underline{R})\hat{\pi}(q')}{\tau_0}}{\dfrac{1-(1-\underline{R})\hat{\pi}(q)}{\tau_0}} = \frac{\dfrac{1}{1-\underline{R}} - \hat{\pi}(q')}{\dfrac{1}{1-\underline{R}} - \hat{\pi}(q)} < \frac{|\pi - \hat{\pi}(q')|}{|\pi - \hat{\pi}(q)|} \tag{29}$$

by the definition of $\hat{\pi}(q)$.

First, consider the case in which π is either $\pi > \hat{\pi}(q)$ or $\pi < \hat{\pi}(q')$. Then, (29) becomes

$$\frac{\dfrac{1}{1-\underline{R}} - \hat{\pi}(q')}{\dfrac{1}{1-\underline{R}} - \hat{\pi}(q)} < \frac{\pi - \hat{\pi}(q')}{\pi - \hat{\pi}(q)}. \tag{30}$$

Note that for any x and $0 < b < c$, $\dfrac{x-b}{x-c}$ is decreasing in x, because

$$\frac{\partial\left(\dfrac{x-b}{x-c}\right)}{\partial x} = \frac{b-c}{(x-c)^2} < 0.$$

Thus, (30) holds because $\pi \leq 1 < \dfrac{1}{1-\underline{R}}$.

Finally, consider the case in which π is $\hat{\pi}(q') < \pi < \hat{\pi}(q)$. Then,

$$\frac{\pi - \hat{\pi}(q')}{\pi - \hat{\pi}(q)} < \frac{|\pi - \hat{\pi}(q')|}{|\pi - \hat{\pi}(q)|}$$

holds and by (30), (29) also holds. Therefore, (27) and (28) hold.

By the claim, the support of the conditional expectation is decreasing in q_1 for each given realization of π and for the given pdf λ_T as

$$\lambda_T\left(\frac{1-(1-\underline{R})\pi}{q}\right)\mathbb{1}\left\{\underline{\tau} < \frac{1-(1-\underline{R})\pi}{q} < \bar{\tau}\right\} > \lambda_T\left(\frac{1-(1-\underline{R})\pi}{q'}\right)\mathbb{1}\left\{\underline{\tau} < \frac{1-(1-\underline{R})\pi}{q'} < \bar{\tau}\right\}$$

for any $q < q'$. Thus, π_1^B is decreasing in q_1. ■

C.3 Solving the Borrower's Optimization Problem

$$\max_{c_1, f_1, a_1^B, \kappa_1^B} E_1^B \left[C_2^B\right] \tag{31}$$
$$\text{s.t. } q_1 c_1 - c_0 \geq p_1 \left(a_1^B - a_0^B\right) + \kappa_1^B - \kappa_0^B,$$
$$C_2^B \leq \tau_1^B \kappa_1^B + a_1^B R - c_1 R_2^d(f_1)$$
$$c_1 \leq a_1^B,$$
$$c_1 \geq 0,\ a_1^B \geq 0,\ \kappa_1^B \geq 0$$

Note that the date 2 budget constraint implies

$$C_2^B = \tau_1^B \kappa_1^B + a_1^B R - c_1 R_2^d(f_1),$$

while the date 1 budget constraint implies

$$\kappa_1^B = q_1 c_1 - c_0 - p_1 \left(a_1^B - a_0^B\right) + \kappa_0^B.$$

Combining these two yields

$$C_2^B = \tau_1^B \left(q_1 c_1 - c_0 - p_1 \left(a_1^B - a_0^B\right) + \kappa_0^B\right) + a_1^B R - c_1 R_2^d(f_1)$$

Therefore we can write the Lagrangian as

$$L_1^B = E_1^B \left[C_2^B\right] + \mu_1^B \left(a_1^B - c_1^B\right) + \xi_{c_1}^B c_1^B + \xi_{a_1}^B a_1^B + \xi_{\kappa_1}^B \left(q_1 c_1^B - c_0^B + p_1 \left[a_0^B - a_1^B\right] + \kappa_0^B\right) \tag{32}$$

where we have $R_2^d(f_1) \equiv \min\{R, f_1\}$, and $E\left[R_2^d(f_1)\right] = (1-\pi_1) f_1 + \pi_1 \min\{\underline{R}, f_1\}$. Replacing q_1 yields

$$C_2^B = \tau_1^B \left(q_1 c_1^B - c_0^B - p_1 \left(a_1^B - a_0^B\right) + \kappa_0^B\right) + a_1^B R - c_1^B R_2^d(f_1).$$

Then, the expectation is given by

$$E_1^B \left[C_2^B\right] = E_1^B \left[\tau_1^B \left(q_1 c_1^B - c_0^B - p_1 \left(a_1^B - a_0^B\right) + \kappa_0^B\right) + a_1^B R - c_1^B R_2^d(f_1)\right]$$
$$E_1^B \left[C_2^B\right] = \tau_1^B \left(q_1 c_1^B - c_0^B - p_1 \left(a_1^B - a_0^B\right) + \kappa_0^B\right) + a_1^B E_1^B [R] - c_1^B E_1^B \left[R_2^d(f_1)\right]$$

Hence, the Lagrangian is:

$$L_1^B = E_1^B\left[C_2^B\right] + \mu_1^B\left(a_1^B - c_1^B\right) + \xi_{c_1}^B c_1^B + \xi_{a_1}^B a_1^B + \xi_{\kappa_1}^B\left(c_1^B q_1 - c_0^B + p_1\left[a_0^B - a_1^B\right] + \kappa_0^B\right) \tag{33}$$

where $E_1^B\left[C_2^B\right]$ is given by the above.

FOC for c_1^B:

$$\frac{dE_1^B\left[C_2^B\right]}{dc_1^B} - \mu_1^B + \xi_{c_1}^B + \xi_{\kappa_1}^B q_1 = 0$$
$$\tau_1^B q_1 - E_1^B\left[R_2^d(f_1)\right] - \mu_1^B + \xi_{c_1}^B + \xi_{\kappa_1}^B q_1 = 0$$
$$\left(\tau_1^B + \xi_{\kappa_1}^B\right) q_1 - E_1^B\left[R_2^d(f_1)\right] - \mu_1^B + \xi_{c_1}^B = 0$$

FOC for a_1^B:

$$\frac{dE_1^B\left[C_2^B\right]}{da_1^B} + \mu_1^B + \xi_{a_1}^B - \xi_{\kappa_1}^B p_1 = 0$$
$$-\tau_1^B p_1 + E_1^B[R] + \mu_1^B + \xi_{a_1}^B - \xi_{\kappa_1}^B p_1 = 0$$
$$-\left(\tau_1^B + \xi_{\kappa_1}^B\right) p_1 + E_1^B[R] + \mu_1^B + \xi_{a_1}^B = 0$$

C.4 Statement and Proof of Binding Collateral Constraint

Lemma 2 (Collateral Constraint)**.** *In equilibrium, the following holds:*

(A) The borrower's collateral constraint is generically binding, so that $c_1 = a_1^B$.

(B) The price of the asset exceeds the price of the contract, $p_1 > q_1$.

Proof. **Proof of Part (A):**

By replacing μ_1^B in (15) with (16) and using the contract price q_1 that makes L to lend in a positive amount:

$$q_1\left(\tau_1^B + \xi_{\kappa_1}^B\right) + E_1^B[R] - E_1^B[R^d] - \tau_1^B p_1 + \xi_{a_1}^B - \xi_{\kappa_1}^B p_1 + \xi_{c_1}^B = 0,$$

which can be rearranged as

$$E_1^B\left[R-R^d\right]+\xi_{c_1}^B+\xi_{a_1}^B=\left(\tau_1^B+\xi_{\kappa_1}^B\right)(p_1-q_1).$$

Consider the case in which B borrows a positive amount so $a_1, c_1>0$. Then, the above equality can be rearranged as

$$\tau_1^B+\xi_{\kappa_1}^B=\frac{E_1^B\left[R-R^d\right]}{p_1-q_1}=\frac{(1-\pi_1^B)(\overline{R}-1)}{p_1-q_1}, \tag{34}$$

which implies that the expected return of holding the asset with the collateralized debt net of B's shadow value of asset equals to the sum of cash return of B and B's shadow value of cash. The return of the leveraged asset holding on the right-hand side of (34) can be greater than return of B's storage technology, but then B has to exhaust all the cash, $\xi_{\kappa_1}^B>0$, and B cannot pay for additional down payment (or cash collateral or variation margin), $p_1-q_1(f_1)$. In addition, B might want to generate more asset because the return of the leveraged asset holding is exceedingly profitable. If this is the case, then the price will adjust to the point that the equality holds.

Now we show that the collateral constraint is generically binding. If B is borrowing zero amount—that is, $c_1=0$, B does not have enough cash to repay the loans to L unless B liquidates all the assets, implying $a_1^B=c_1=0$. B borrows a positive amount $c_1>0$ only if the return from rolling over the debt is greater than or equal to the cash return. Thus,

$$\tau_1^B+\xi_{\kappa_1}^B=\frac{E_1^B[R-R^d]}{p_1-q_1} \tag{35}$$

holds in any equilibrium with $c_1>0$. If the collateral constraint is not binding, then by (16),

$$\tau_1^B+\xi_{\kappa_1}^B=\frac{E_1^B[R]}{p_1} \tag{36}$$

holds. (35) and (36) imply that

$$\tau_1^B+\xi_{\kappa_1}^B=\frac{E_1^B[R^d]}{q_1}=\frac{(1-\pi_1^B)+\pi_1^B\underline{R}}{q_1},$$

which holds only at non-generic realization of q_1 as π_1^B is decreasing in q_1. The return from leveraging the asset purchase, (35), should exceed the return from the asset purchase without leverage, (36), because otherwise B will not purchase with leverage—that is, $c_1=0$—which is a contradiction. Even if the previous equation holds, which is a not a generic case of parameters, B is indifferent between purchasing the asset with leverage and without leverage. Therefore, we can impose a tie-breaking rule for B, choosing to maximize c_1 when indifferent.[32] Hence, B leverages all of its asset purchase as $c_1 = a_1^B$, and the collateral constraint is binding.

Proof of Part (B):

Intuitively, this condition $p_1 > q_1$ implies that value of the risky asset p_1 exceeds its collateral value q_1. The borrower's optimality condition (21), which can be rearranged as

$$\left(\xi_{\kappa_1}^B + \tau_1^B\right)(p_1 - q_1) = E_1^B[R] - E_1^B\left[R_2^d(f_1)\right]$$

$$p_1 - q_1 = \frac{1}{\xi_{\kappa_1}^B + \tau_1^B}\left[E_1^B[R] - E_1^B\left[R_2^d(f_1)\right]\right].$$

Since $\xi_{\kappa_1}^B \geq 0$ and $\tau_1^B > 0$, we have $p_1 > q_1$ if and only if

$$\begin{aligned} E_1^B[R] &> E_1^B\left[R_2^d(f_1)\right] \\ (1-\pi_1^B)\overline{R} + \pi_1^B\underline{R} &> (1-\pi_1^B)f_1 + \pi_1^B\underline{R} \\ (1-\pi_1^B)\left(\overline{R} - f_1\right) &> 0 \end{aligned}$$

This holds since $\overline{R} > 1$ by assumption. ■

C.5 Proof of Lemma 1

Proof. The proof is based on the full characterization of date-1 equilibrium in proposition 8 in appendix E.1.

As in Case 1 in proposition 8, $p_1 > F'(0)$ implies that the borrower is not selling any assets to the traditional sector, and the asset price equals the fundamental value of the asset based on the borrower's beliefs. Whenever $p_1 < F'(0)$ in equilibrium, the asset is sold to the traditional sector in a positive amount, i.e. there are fire sales, as more sales to the traditional sector depresses prices as $F'(a_1^T) < F'(0)$. ■

C.6 Proof of Proposition 2

Proof. The proof is based on the full characterization of date-1 equilibrium in proposition 8 in appendix E.1.

Proof of Part (A):

[32]This tie-breaking rule can be justified by assuming that B obtains infinitesimally small utility of holding an asset as collateral.

By (6), π_1^L increases as s_1^L decreases. Also, q_1 is decreasing in π_1^L and τ_1^L by (1), implying the first half of the statement.

In Case 2 in proposition 8, lower q_1 results in larger a_1^T and lower p_1, as

$$q_1 + \frac{c_0 - q_1 a_0^B - \kappa_0^B}{a_1^T}$$

is decreasing in q_1 because $a_0^B \geq a_1^T$, and

$$p_1 = F'(a_1^T) = q_1 + \frac{c_0 - q_1 a_0^B - \kappa_0^B}{a_1^T}.$$

Therefore, a decrease in q_1 will increase the liquidity shortage of the borrower, leading to a larger amount of fire sales and lower asset price.

Moreover, the borrower's posterior belief π_1^B is decreasing in q_1 by proposition 1. In Case 3 in proposition 8, higher π_1^B results in lower price through this belief channel. This is because the borrower's lower valuation of the asset should be met by the lower marginal valuation of the traditional sector through the decrease in $F'(a_1^T)$, i.e. larger a_1^T.

Proof of Part (B):

In Case 2 in proposition 8, the borrower sells the asset because the borrower has to repay the date-0 debt contract and their cash holdings are not sufficient as $q_1 a_0^B < c_0^B - \kappa_0^B$. However, the borrower still believes the fundamental value of the asset is above the marginal valuation of the traditional sector evaluated at 0, $F'(0)$, which is always above the market price as $F'(0) > F'(a_1^T) = p_1$. Therefore, we have $p_1^B > F'(0) > p_1$ in a liquidity driven fire sale. In Cases 3 and 4 in proposition 8, the borrower's valuation of the asset is below the marginal valuation of the traditional sector evaluated at 0, as $p_1^B < F'(0)$, which is why the borrower sells the asset, i.e. belief driven fire sale. In Case 3, the borrower has an interior solution, so that $p_1^B = F'(a_1^T) = p_1$. However, in Case 4, the borrower values the asset even less than the traditional sector does, implying that $p_1^B < F'(a_0^B) = p_1$. ■

C.7 Proof of Proposition 4

Proof. We first show the following lemma.

Lemma 3. *Suppose that $\tau_1 = \tau_0$. Then, $E\left[\pi_1^B - \pi_1^L \mid \tau_1 = \tau_0\right] = 0$.*

Proof of Lemma 3. Recall that the expected posterior belief is the prior belief, which is $E\left[E_1^B[\pi_1^L]\right] = E[\pi_1^L] = \pi_0$. It is sufficient to show that the conditional posterior is the same as the prior belief, as $E[\pi_1^B | \tau_1 = \tau_0] = \pi_0$. B updates his belief based on the observation of q_1, which is determined by $q_1 = \dfrac{1-(1-\underline{R})\pi}{\tau}$. Hence,

$$E\left[E_1^B\left[\pi_1^L \mid q_1\right] \mid \tau_1 = \tau_0\right] = \int_0^1 \left[\int_{\underline{\tau}}^{\overline{\tau}} \int_0^1 \pi \mathbb{1}\left\{\pi = \frac{1-\tau\dfrac{1-(1-\underline{R})\hat{\pi}}{\tau_0}}{1-\underline{R}}\right\} dG_\pi(\pi) d\Lambda_T(\tau)\right] dG_\pi(\hat{\pi}), \tag{37}$$

is the expected posterior belief of the borrower when there is no cost of funds (liquidity) shock to the lender. Now, consider the unconditional prior, which is

$$\begin{aligned}
E\left[E_1^B\left[\pi_1^L\right]\right] &= E\left[E_1^B\left[\pi_1^L \mid q_1(\tau_1, \pi_1^L)\right]\right] \\
&= \int_{\underline{\tau}}^{\overline{\tau}} \int_0^1 \int_{\underline{\tau}}^{\overline{\tau}} \int_0^1 \pi \mathbb{1}\left\{\pi = \frac{1-\tau\dfrac{1-(1-\underline{R})\tilde{\pi}}{\tilde{\tau}}}{1-\underline{R}}\right\} dG_\pi(\pi) d\Lambda_T(\tau) dG_\pi(\tilde{\pi}) d\Lambda_T(\tilde{\tau}).
\end{aligned}$$

By change of variable, we obtain

$$\begin{aligned}
&\int_{\underline{\tau}}^{\overline{\tau}} \int_0^1 \int_{\underline{\tau}}^{\overline{\tau}} \int_0^1 \pi \mathbb{1}\left\{\pi = \frac{1-\tau\dfrac{1-(1-\underline{R})\tilde{\pi}}{\tilde{\tau}}}{1-\underline{R}}\right\} dG_\pi(\pi) d\Lambda_T(\tau) dG_\pi(\tilde{\pi}) d\Lambda_T(\tilde{\tau}) \\
=& \int_{\underline{\tau}}^{\overline{\tau}} \int_0^1 \int_0^1 \int_{\underline{\tau}}^{\overline{\tau}} \pi \mathbb{1}\left\{\tau = \frac{\tilde{\tau}\left[1-(1-\underline{R})\pi\right]}{1-(1-\underline{R})\tilde{\pi}}\right\} d\Lambda_T(\tilde{\tau}) dG_\pi(\pi) dG_\pi(\tilde{\pi}) d\Lambda_T(\tau) \\
=& \int_{\underline{\tau}}^{\overline{\tau}} \int_0^1 \int_0^1 \pi \mathbb{1}\left\{\tau = \frac{\tau_0\left[1-(1-\underline{R})\pi\right]}{1-(1-\underline{R})\tilde{\pi}}\right\} dG_\pi(\pi) dG_\pi(\tilde{\pi}) d\Lambda_T(\tau),
\end{aligned}$$

where we use $\int_{\underline{\tau}}^{\overline{\tau}} \tau_1 d\Lambda_T(\tau_1) = \tau_0$, and the last expression coincides with the right-hand side of (37). ■ Lemma 3 implies that when the cost shock is zero, $|\tau_1^L - \tau_0| = 0$, the expected difference between the borrower's posterior belief and the lender's true posterior belief is zero. By independence of the two realizations, the realization of τ_1^L is orthogonal to the realization of π_1^L. Therefore, for a fixed realization of π_1^L, q_1 decreases in τ_1^L, which in turn increases π_1^B by part 2 of proposition 1. Thus, the expected discrepancy between the two beliefs, $E\left[\pi_1^B - \pi_1^L\right]$, is decreasing in cost shocks, τ_1^L. Hence, the expected absolute difference between π_1^B and π_1^L increases as $|\tau_1^L - \tau_0|$ increases. ■

C.8 Proof of Proposition 5

Proof. From lemma 6 and lemma 7 in appendix E.5, the frontiers of the baseline economy and the common information benchmark intersects only once in the domain of $\pi_1^L \leq 1$. Denote that intersection point as $(\tau_1^{L\dagger}, \pi_1^{L\dagger})$. Then, for any $\tau_1^L > \tau_1^{L\dagger}$, there exists π_1^L such that there is a fire sale in the baseline economy but not in the common information benchmark. However, there is no π_1^L for which there is a fire sale in the common information benchmark but not in the baseline economy. (This is represented by the red-shaded region in figure 6.) On the other hand, for any $\tau_1^L < \tau_1^{L\dagger}$, there exists π_1^L such that there is a fire sale in the common information benchmark but not in the baseline economy. However, there is no π_1^L for which there is a fire sale in the baseline economy but not in the common information benchmark. (This is represented by the blue-shaded region in figure 6.)

Whether the unconditional likelihood of entering a fire sale is larger in the baseline economy versus the common information benchmark depends on the relative size of these regions of the state space and the joint distribution of the cost shocks and signal across these regions. Therefore, it is ambiguous whether the unconditional likelihood of a fire sale is larger in the baseline versus the benchmark. ■

C.9 Proof of Proposition 6

Proof. Proof of Part (A): Effects of misinformation on the allocation in the Normal Regime

By comparing a Normal Regime equilibrium in baseline with the Common Information Benchmark, we can see that in both equilibria, $a_1^B = a_0^B$. So in the Normal Regime, beliefs have no effect on the allocation of the risky asset. What they have is an effect on the allocation of cash between the lender vs the borrower, via q_1, and the asset price p_1 (which itself doesn't matter in this regime beyond ensuring $a_1^B = a_0^B$, since pecuniary externality doesn't lead to misallocation here): $\kappa_1^B = q_1 a_0^B - c_0^B + \kappa_0^B$. In both versions, $q_1 = \frac{E_1^L\left[R_2^d(f_1)\right]}{\tau_1^L}$ determiend by beliefs of the lender, which is equivalent to that of the borrower in the common information benchmark. Hence, tighter q_1 is met with cash holdings in this regime, but belief disagreements (learning mechanism) don't affect tighter q_1.

Belief disagreement does affect p_1 (since $p_1 = \frac{E_1^B[R]-E_1^B\left[R_2^d(f_1)\right]}{\tau_1^B} + q_1$), but this has no allocative consequences in this regime. Thus, belief disagreement only affect equilibrium allocation (of asset or cash) at date 1 only to the extent that they affect allocation of asset. (Beliefs in general affect allocation of cash, but this is always determined by q_1 which is pinned down by the lender's belief. So, the borrower's belief matters only to the extent that it affects desired a_1^B).

Proof of Part (B): Effect of misinformation on the severity of fire sale

As we outlined for the Normal Regime, belief disagreement affects the equilibrium allocation (of asset or cash) at date 1 only to the extent that they affect allocation of asset. (This is because in both the baseline case and the common information benchmark, q_1 is pinned down by the lender's beliefs in equilibrium $q_1 = \frac{E_1^L\left[R_2^d(f_1)\right]}{\tau_1^L}$). So at the margin, belief disagreements can affect the equilibrium allocation only to the extent that it affects a_1^B and/or p_1. Recall that in the Fire Sale Regime, we have (61)

$$a_1^B = a_0^B - F'^{-1}\left(\frac{E_1^B\left[R\right] - E_1^B\left[R_2^d(f_1)\right]}{\tau_1^B} + \frac{E_1^L\left[R_2^d(f_1)\right]}{\tau_1^L}\right)$$

while for the benchmark with common information, we have

$$a_1^B = a_0^B - F'^{-1}\left(\frac{E_1\left[R\right] - E_1\left[R_2^d(f_1)\right]}{\tau_1^B} + \frac{E_1^L\left[R_2^d(f_1)\right]}{\tau_1^L}\right) \tag{38}$$

Since $F'^{-1}(\cdot)$ is monotonically decreasing, a_1^B is lower in the baseline case (i.e. the identification problem makes the fire sales more severe) only if the spread $E_1^B\left[R\right] - E_1^B\left[R_2^d(f_1)\right] = \left(1-\pi_1^B\right)\left(\overline{R}-f_1\right)$ is lower. This occurs iff $\pi_1^B > \pi_1^L$. Whether this is true in equilibrium or not depends on the actual realization of π_1^L, τ_1^L given the observed q_1. So, for a given q_1, the identification problem will amplify the fire sales relative to the common info benchmark (lower a_1^B) when π_1^L is low and τ_1^L is high; while it will dampen the fire sales relative to the common info benchmark (higher a_1^B) when π_1^L is high and τ_1^L is low.

The net effect of these two forces determines the overall effect of misinformation on the severity of fire sales. ■

C.10 Policy Interventions

C.10.1 Standard Monetary Policy Tools

For policy tools which affect agents' opportunity cost of lending, typical of more standard monetary policy tools such as open market operations or interest on central bank reserves, central bank interventions does not add information. By affecting the level of interest rates, monetary policy tools affect the opportunity cost of lending. We can therefore map this type of tool into our baseline model as an additional component of the lender's opportunity cost of funds τ_1^G such that the lender's total opportunity cost of funds at date 1 is $\tau_1^L + \tau_1^G$.[33] This captures the level shift in the lender's opportunity cost of founds brought about by changes in interest rates. The lender's optimality condition is still given by (11) with this minor alteration. Therefore, looser monetary policy, captured by a fall in τ_1^G, can bring about a loosening of funding liquidity at date 1. However, as long as the borrower also observes this policy intervention τ_1^G, the policy does not add information. The borrower can identify the component of the lender's opportunity cost of funds attributable to

[33]For simplicity, we assume that the level of interest rates affects only the lender's opportunity cost of funds, although the same results would obtain if it affected both agents.

the government, but cannot infer the lender's private component τ_1^L. As a result, the borrower faces an identical identification problem as in the absence of the policy. Hence, such monetary policy interventions, while supporting funding liquidity, do not not affect the formation of beliefs or affect the ability of market prices to aggregate and reveal private intervention.

C.10.2 Lender of Last Resort Facilities

We consider the effect of policy interventions as a 'lender of last resort'.[34] In this extension, at the beginning of date 1, the government announces its lending program, which specifies the loan terms and the type of agents who can participate. Then, the market clearing price of loans as well as the amount of loans from the government are determined simultaneously, and the borrower updates its belief based on all the available information.

To proceed, we first modify the setup by introducing a government. The government does not observe agent's private information. At date 1, the government has access to lump-sum transfers and two distortionary taxes: the government can tax each unit revenue that the borrower obtains from liquidating the risky asset at date 1 at the rate γ^B; and the government can tax the date-2 return that the lender receives on its cash holdings from date 1 at the rate γ^L. The government can transfer lump-sum amount of cash Γ_1 and Γ_2 at date 1 and 2, respectively. Both Γ_1 and Γ_2 can be either positive (tax) or negative (subsidy).

At date 1, the government provides loans to the market to either lender or borrower at pre-specified rate q_1^G. The promised value of payment per unit of collateral is 1, and denote the amount of collateral posted to the government as c_1^G. The price of the government loan q_1^G is publicly observable at the beginning of date 1, and then the rest of the prices q_1 and p_1 are determined by the competitive markets. The equilibrium prices will be informative to the borrower about the lender's private information.

Thus, the government has three types of tools to intervene in financial markets at date 1: the distortionary taxes γ^L, γ^B allow it to reallocate liquidity at date 1, the government loans at date 1, and the lump-sum transfers allow it to redistribute wealth.

First, we consider the case in which only borrowers can get the loan from the government.

[34] Acharya et al. (2012) present a model of interbank lending and asset sales markets in which banks with surplus liquidity have market power over banks who need liquidity. In their model, frictions arise in lending due to moral hazard, and assets are bank-specific. Surplus banks ration lending and instead purchase assets from needy banks, an inefficiency more acute during financial crises. A central bank acting as a 'lender of last resort' can ameliorate this inefficiency provided it is prepared to extend potentially loss-making loans or is better informed than outside markets. This rationale for central banking finds support in historical episodes, e.g., Park (1991). Our results suggest that the interaction between the identification problem arising from worsening funding liquidity and deterioration in market liquidity due to costly fire sales can be better dealt with the central bank's role of a 'dealer of last resort'.

The government's budget constraint at date 1 is

$$\left(a_0^B - a_1^B\right)\gamma^B + q_1^G c_1^G + \Gamma_1 = \kappa_1^G \tag{39}$$

where γ^B is the proportional tax on the borrower's liquidation of the asset at date 1, c_1^G, q_1^G is the total quantity of collateral posted by the borrower and the price per a unit of collateral, respectively, Γ_1 are lump-sum taxes at date 1, and κ_1^G is the government's date-1 cash holdings. Thus, the government's date-1 cash holdings consist of the taxes collected from the borrower's liquidation of the risky asset and lump-sum taxes minus the cash used to purchase the debt from borrower at q_1^G. Note that κ_1^G can be negative and the government can balance its budget by its date-2 income.

The government's date-2 budget constraint is

$$\kappa_1^G + \Gamma_2 + \gamma^L \kappa_1^L = c_1^G R_2^d. \tag{40}$$

Thus, the government's date-2 income must be met with its negative holdings from date 1, any lump-sum taxes at date 2, or tax revenue on the lender's cash holdings from date 1.

Under this setting, providing cheap government loans when there is already sufficient funding liquidity is inefficient, as the intervention only generates distortions from taxation. Therefore, we focus on the case in which the government provides the loan contract whose price is sufficiently low (or has a sufficiently high interest rate), so the government loans is used only if the loan price q_1 is sufficiently low that induces fire sales, as $p_1 < F'(0)$. This provision of cheap additional funds would decrease 'liquidity driven fire sales' in the case of $p_1^B > F'(0) > p_1$, as the amount of fire sales that borrower has to make to repay its debt would be lower in this case. Nevertheless, this provision of cheap additional funds would not decrease 'belief driven fire sale', which occurs when $p_1^B < F'(0)$. This is because the borrower's information set remains the same as that in the baseline case without the government intervention. Hence, the provision of loans to borrowers does not ameliorate the identification problem and information spillovers issue.

Finally, we consider the case in which only lenders can get the loan from the government. The government's budget constraints at date 1 and date 2 are the same as (39) and (40), respectively. Again, providing cheap government loans when there is already sufficient funding liquidity is inefficient, as the intervention only generates distortions from taxation. Therefore, we focus on the case in which the government provides the loan contract whose price is sufficiently low (or has a sufficiently high interest rate), so the government loans are used only if the loan price q_1 is sufficiently low that induces fire sales, as $p_1 < F'(0)$. This provision of cheap additional funds would be informative only if lender has enough cash to increase its market rate of lending q_1.

In this case, lender receives collateral from borrower after purchasing the collateralized debt contract and then reuses the collateral to borrow from the government. Thus, lender receives intermediation rent through this chain of transactions. The lender's optimization problem becomes

$$
\begin{aligned}
&\max_{\kappa_1^L, d_1^L, c_1^G} E_1^L \left[C_2^L \right] \\
s.t. &C_2^L = \left(\tau_1^L - \gamma^L \right) \kappa_1^L + R_2^d d_1^L - R_2^d c_1^G \\
&\kappa_1^L - \kappa_0^L \leq d_0^L - \hat{q}_1 d_1^L + q_1^G c_1^G \\
&c_1^G \leq d_1^L,
\end{aligned}
$$

with the non-negativity constraints, where the third constraint is the lender's collateral constraint, and $\hat{q}_1$ is the price of loans with the intervention. As previously mentioned, we focus on the case in which lender has incentives to borrow from the government, that is, $q_1 < q_1^G$, where q_1 is the equilibrium loan price without the government intervention. Hence, the FOC for d_1^L is

$$
\hat{q}_1 = \frac{E_1^L \left[R_2^d \right] + \mu_1^L}{\tau_1^L - \gamma^L}, \tag{41}
$$

where μ_1^L is the Lagrange multiplier for the collateral constraint. The FOC for c_1^G is

$$
q_1^G = \frac{E_1^L \left[R_2^d \right] + \mu_1^L}{\tau_1^L - \gamma^L}, \tag{42}
$$

therefore, the equilibrium loan price becomes $\hat{q}_1 = q_1^G$ by competition. Again, the intervention can decrease 'liquidity driven fire sales' in the case of $p_1^B > F'(0) > p_1$, as the amount of fire sales that borrower has to make to repay its debt would be lower in this case.

Importantly, this provision of cheap additional funds to lenders reduce the information spillovers to the borrower. This is because the exogenously given government loan price q_1^G substitutes the loan price q_1 determined by the lender's private information on τ_1^L and π_1^L. Thus, the government intervention as a 'lender of last resort' destroys information in the market. As a result, the amount of 'belief driven fire sales' is less than that in the baseline case. Even though the provision of loans to lenders does not ameliorate the identification problem, it reduces the information spillovers and resulting inefficient fire sales due to excessive pessimism.

D Appendix: Alternative Case of Upstream Information Spillovers

Thus far, we have focused on the case in which the lender gets the cost shock and private signal, but the borrower does not. This implies that information spillovers flow downstream from the lender to the borrower. Moreover, in this case we saw how liquidity conditions in funding markets (measured by the price of debt) affect the borrower's beliefs about fundamentals.

In this section, we consider the alternative case in which the borrower gets a cost shock and private signal, but the lender does not.[35] Because the mechanisms and insights are so similar to our baseline case, we do not solve the entire model under this alternative case. Rather, w e show that, while the mechanism differs slightly, the fundamental insights are the same as in the baseline case. In particular, information spillovers in this case will flow upstream from the borrower to the lender rather than downstream. And, as a result the availability of market liquidity (measured by cost shocks to the borrower which affect the price of the risky asset) will shape the lender's beliefs about fundamentals. Thus, the nature of information spillovers and the effect of liquidity on beliefs and asset prices are essentially same as in the baseline case.

This alternative case is illustrated in figure 8. Suppose the borrower gets a cost shock τ_1^B and private signal s_1^B, but the lender does not (i.e. τ_1^L is a fixed parameter known to both agents). How do the equilibrium conditions differ form our baseline case? In short, all of the equations constituting the real block of the economy take the same form. The only equations that change are those characterizing the evolution of beliefs at date 1.

How would the borrower's choices reflect its private information τ_1^B, s_1^B? In this case q_1 still only reflects the lender's first-order condition according to (12), and is therefore not informative about the borrower's private information. However, the asset price p_1 will reflect the borrower's private information through the equilibrium price (22). Using this equation, the spread between the borrower's valuation and the price of debt at date 1 is

$$\underbrace{p_1 - q_1}_{spread} = \frac{E_1^B[R] - E_1^B\left[R_2^d\right]}{\xi_{\kappa_1}^B + \tau_1^B} \tag{43}$$

(Note also that the quantity of the borrower's investment in the risky asset $a_1^B = c_1$ is just determined residually from this optimality condition and the market clearing condition, and hence contains no additional information about borrower's private information over the spread.)

Identification problem Therefore, the lender uses the observation of $p_1 - q_1$ to form its posterior belief π_1^L, but cannot separately infer π_1^B and τ_1^B due to a similar identification problem as what we had in the case with downstream information spillovers. (Indeed, because q_1 is still pinned down by the lender's first order condition, the asset price p_1 is the variable which con-

[35] For simplicity, we assume that only one agent receives both a cost shock and a private signal, while the other agent receives neither. We make this assumption, rather than allowing both agents to receive private signals, to prevent the model from becoming analytically intractable. If, in contrast, both agents received private signals, then the equilibrium would depend not only on how one agent, e.g. the borrower, updates its beliefs in response to the actions of the other, but also on an infinite feedback loop in which also the borrower's actions affect the lender's beliefs, etc. This would make an analytical characterization of equilibrium belief formation extremely difficult.

veys the borrower's private information. But our analytical expressions are made more simple by characterizing information based on observed spread $p_1 - q_1$.)

(Since the lender has only one observable which is informative about the borrower's private information, it cannot separately identify the borrower's information set at date 1 (i.e. it's private signal) and its date 1 cost shock τ_1^B. Put differently, the lender has only one equation (6) to infer two unobservables s_1^B, τ_1^B. Indeed, there is a continuum of pairs (τ_1^B, π_1^B) that satisfy the relation above for the observed p_1.)

Belief formation Following the same steps to prove proposition 1, we can symmetrically characterize the evolution of the lender's beliefs in this environment. In particular, we have that

1. Lender's belief is the same as the expected posterior of the borrower, i.e. $\pi_1^L = E_1^L[\pi_1^B]$.

2. π_1^L is decreasing in $p_1 - q_1$.

Information spillovers operate upstream In this environment, information spillovers operate *upstream* from the borrower to the lender, rather than downstream as in our baseline case. Suppose that the borrower receives a cost shock (high τ_1^B) or bad private signal so that the asset price is low. The lender then observes that the spread $p_1 - q_1$ is low and will update its belief, which becomes more pessimistic.

Market liquidity affects beliefs Thus, the beliefs of upstream agents are shaped in equilibrium by the availability of *market liquidity* rather than by funding liquidity, as was the case in our baseline case. More precisely, changes in the price of the risky asset p_1 which are driven by cost shocks to the borrower affect lender's optimism or pessimism about the asset fundamental.

Belief-driven booms and busts In this environment, the availability of market liquidity can give risk to belief-driven booms or busts in the asset market. Suppose again that the borrower receives an adverse cost shock, leading to drop in the spread $p_1 - q_1$ and causing the lender to become more pessimistic due to the information spillover (π_1^L goes up). As can be seen from the equilibrium expression for q_1, the equilibrium price of date 1 debt will fall as a result of the higher π_1^L—that is, there will be a tightening of funding liquidity. This worsens the borrower's liquidity position, and forces it to liquidate more of the risky asset at the margin, further lowering the asset price. Hence, market illiquidity gives rise to funding illiquidity through the lender's endogenous pessimism. This feedback from market illiquidity to funding illiquidity gives rise to asset price busts. Symmetrically, in response to a positive cost shock to the borrower, the upstream information spillover would give rise to an asset price boom through the feedback from market liquidity to funding liquidity.

Thus, in this case, same insights hold, with two differences in the mechanism. First, information spillovers propagate upstream from borrowers to lenders. Second, the beliefs of the lender are shaped by the degree of market liquidity, rather than funding liquidity – cost shocks to downstream firms shape the beliefs of upstream firms. As a result, the model still produces belief-driven booms or busts triggered by shocks which affect market liquidity. Moreover, these booms or busts are characterized by a feedback loop between market liquidity and funding liquidity.

E Online Appendix: Omitted Proofs and Results

E.1 Full Characterization of Date-1 Equilibrium

In the Normal Regime, we have

$$a_1^T, \xi_{d_1}^L, \xi_{c_1}^B, \xi_{a_1}^B, \xi_{\kappa_1}^L, \xi_{\kappa_1}^B = 0$$

$$q_1 = \frac{E_1^L\left[R_2^d(f_1)\right]}{\tau_1^L}$$

$$p_1 = \frac{E_1^B[R] - E_1^B\left[R_2^d(f_1)\right]}{\tau_1^B} + q_1$$

$$a_1^B = c_1^B = d_1^L = a_0^B$$

$$\kappa_1^B = q_1 a_0^B - c_0^B + \kappa_0^B$$

$$\kappa_1^L = \kappa_0^L + d_0^L - q_1 a_0^B$$

$$\kappa_1^T = \kappa_0^T - p_1 a_1^T$$

$$C_2^B = \tau_1^B \kappa_1^B + a_1^B R - a_1^B R_2^d(f_1)$$

$$C_2^L = \tau_1^L \kappa_1^L + R_2^d d_1^L$$

$$C_2^T = \kappa_1^T + F(a_1^T)$$

$$\mu_1^B = \left(\tau_1^B + \xi_{\kappa_1}^B\right) p_1 - E_1^B[R] - \xi_{a_1}^B$$

$$\mu_1^T = F'(0) - p_1$$

$$\pi_1^L = Pr\left(R = \underline{R} | s_1^L, I_0\right) = \frac{\pi_0 \lambda_\varepsilon\left(\varepsilon_1^L = s_1^L - \underline{R}\right)}{(1-\pi_0)\lambda_\varepsilon\left(\varepsilon_1^L = s_1^L - \overline{R}\right) + \pi_0 \lambda_\varepsilon\left(\varepsilon_1^L = s_1^L - \underline{R}\right)}$$

$$\pi_1^B = \int_0^1 \pi\lambda_T\left(\frac{1-(1-\underline{R})\pi}{q_1}\right)\mathbb{1}\left\{\underline{\tau} < \frac{1-(1-\underline{R})\pi}{q_1} < \bar{\tau}\right\}dG_\pi(\pi)$$

In the Fire Sale Regime, we have

$$\mu_1^T, \xi_{c_1}^B, \xi_{a_1}^B, \xi_{d_1}^L, \xi_{\kappa_1}^L, \xi_{\kappa_1}^B = 0$$

$$q_1 = \frac{E_1^L\left[R_2^d(f_1)\right]}{\tau_1^L}$$

$$p_1 = F'(a_1^T)$$

$$a_1^B = a_0^B - F'^{-1}\left(\frac{E_1^B[R] - E_1^B\left[R_2^d(f_1)\right]}{\tau_1^B} + \frac{E_1^L\left[R_2^d(f_1)\right]}{\tau_1^L}\right)$$

$$a_1^T = a_0^B - a_1^B$$

$$c_1^B = d_1^L = a_1^B$$

$$\kappa_1^B = q_1 a_1^B - c_0^B + \kappa_0^B - p_1\left(a_1^B - a_0^B\right)$$

$$\kappa_1^L = d_0^L - q_1 d_1^L + \kappa_0^L$$

$$\kappa_1^T = \kappa_0^T - p_1 a_1^T$$

$$C_2^B = \tau_1^B\kappa_1^B + a_1^B R - a_1^B R_2^d(f_1)$$

$$C_2^L = \tau_1^L\kappa_1^L + R_2^d d_1^L$$

$$C_2^T = \kappa_1^T + F(a_1^T)$$

$$\mu_1^B = \tau_1^B p_1 - E_1^B[R]$$

$$\pi_1^L = Pr\left(R = \underline{R}|s_1^L, I_0\right) = \frac{\pi_0 \lambda_\varepsilon\left(\varepsilon_1^L = s_1^L - \underline{R}\right)}{(1-\pi_0)\,\lambda_\varepsilon\left(\varepsilon_1^L = s_1^L - \bar{R}\right) + \pi_0 \lambda_\varepsilon\left(\varepsilon_1^L = s_1^L - \underline{R}\right)}$$

$$\pi_1^B = \int_0^1 \pi \lambda_T \left(\frac{1-(1-\underline{R})\pi}{q_1}\right) \mathbb{1}\left\{\underline{\tau} < \frac{1-(1-\underline{R})\pi}{q_1} < \bar{\tau}\right\} dG_\pi(\pi)$$

Below is the full solution of the date-1 equilibrium and derivation.

Proposition 8. *The equilibrium at date 1 can be characterized as the following:*

1. ***Normal Regime:*** *If $F'(0) < \frac{(1-\pi_1^B)(\bar{R}-1)}{\tau_1^B} + \frac{(1-\pi_1^L)+\pi_1^L \underline{R}}{\tau_1^L}$ and $q_1 a_0^B \geq c_0^B - \kappa_0^B$, then $c_1 = a_1^B = a_0^B = d_1^L$, $a_1^T = 0$, $\kappa_1^B = q_1 a_0^B - c_0^B + \kappa_0^B$, and there will be no fire-sales in the market and the asset price is*

$$p_1 = \frac{(1-\pi_1^B)(\bar{R}-1)}{\tau_1^B} + \frac{(1-\pi_1^L)+\pi_1^L \underline{R}}{\tau_1^L}$$

   or any number less than or equal to p_1 if $q_1 a_0^B - c_0^B = \kappa_0^B$, which implies $\kappa_1^B = 0$.

2. ***Fire Sale Regime with the borrower holding only the risky asset:*** *If $F'(0) < \frac{(1-\pi_1^B)(\bar{R}-1)}{\tau_1^B} + \frac{(1-\pi_1^L)+\pi_1^L \underline{R}}{\tau_1^L}$ but $q_1 a_0^B < c_0^B - \kappa_0^B$, then $c_1 = a_1^B = a_0^B - a_1^T = d_1^L$, a_1^T is determined by*

$$F'(a_1^T) = q_1 + \frac{c_0 - q_1 a_0^B - \kappa_0^B}{a_1^T},$$

   $\kappa_1^B = 0$, and the asset price is $p_1 = F'(a_1^T)$.

3. ***Fire Sale Regime with the borrower at interior solution in portfolio choice:*** *If $F'(0) > \frac{(1-\pi_1^B)(\bar{R}-1)}{\tau_1^B} + \frac{(1-\pi_1^L)+\pi_1^L \underline{R}}{\tau_1^L} > F'(a_0^B)$ holds, then $c_1 = a_1^B = d_1^L$ is determined by*

$$F'(a_0^B - c_1) = \frac{(1-\pi_1^B)(\bar{R}-1)}{\tau_1^B} + \frac{(1-\pi_1^L)+\pi_1^L \underline{R}}{\tau_1^L},$$

   with $a_1^T = a_0^B - c_1$, $p_1 = F'(a_0^B - c_1) = \frac{(1-\pi_1^B)(\bar{R}-1)}{\tau_1^B} + \frac{(1-\pi_1^L)+\pi_1^L \underline{R}}{\tau_1^L}$ and $\kappa_1^B = q_1 a_1^B - c_0^B + \kappa_0^B$.

4. ***Fire Sale Regime with market collapse (B holds only cash):*** *If $F'(a_0^B) > \frac{(1-\pi_1^B)(\bar{R}-1)}{\tau_1^B} +$*

$\dfrac{(1-\pi_1^L)+\pi_1^L\underline{R}}{\tau_1^L}$, *then* $c_1 = a_1^B = d_1^L = 0$, $a_1^T = a_0^B$, *and* $\kappa_1^B = p_1 a_0^B - c_0^B + \kappa_0^B$ *with* $p_1 = F'(a_0^B)$, *so all the assets are sold to* T *and there will be no debt contract between* B *and* L.

Proof. From lemma 2 in appendix C.4, we know that the collateral constraint is binding, and the asset price is greater than the contract price. Now we solve for the equilibrium price p_1.

Case 1. Consider the case in which B holds a positive amount of assets, $a_1 = c_1 > 0$. The reservation asset price p_1^B for B that makes B indifferent between purchasing the asset and holding cash is

$$p_1^B = \frac{(1-\pi_1^B)(\bar{R}-1)}{\tau_1^B} + \frac{(1-\pi_1^L)+\pi_1^L\underline{R}}{\tau_1^L}. \tag{44}$$

Case 1.1. If B is not selling any of its asset due to budget constraint, which is possible only if

$$q_1 a_0^B + \kappa_0^B \geq c_0,$$

then equilibrium price becomes $p_1 = p_1^B$, because otherwise T will be the only marginal buyer of the asset, which implies $a_1 = 0$, a contradiction.

Case 1.2. If B lacks cash to repay the debt as

$$q_1 a_0^B + \kappa_0^B < c_0,$$

then the asset price can be lower than p_1^B and $\xi_{\kappa_1}^B > 0$ as

$$p_1^B > p_1 = \frac{(1-\pi_1^B)(\bar{R}-1)}{\tau_1^B + \xi_{\kappa_1}^B} + \frac{(1-\pi_1^L)+\pi_1^L\underline{R}}{\tau_1^L} = F'(a_1^T),$$

because B liquidates some of the assets to T in order to pay the debt to L even though B prefers not to do so. Therefore, the asset price could be lower than the marginal return of the asset in such equilibrium and determined by the traditional sector's inverse demand. B will liquidate the assets up to the necessary amount needed to match the budget constraint with $\kappa_1^B = 0$,

$$a_1^T F'(a_1^T) = c_0 - q_1(a_0^B - a_1^T) - \kappa_0^B$$

in such equilibrium. Rearranging the terms yields

$$F'(a_1^T) = q_1 + \frac{c_0 - q_1 a_0^B - \kappa_0^B}{a_1^T}, \tag{45}$$

and $p_1 = F'(a_1^T)$. Note that there exists a unique a_1^T in this case, because F' is strictly decreasing.

Finally, the budget constraint determines the cash holdings as

$$\kappa_1^B = q_1 a_0^B - c_0^B + \kappa_0^B,$$

that pins down the optimal decision vector of B if $a_1^T = 0$. If $a_1^T > 0$, the marginal buyer will be T, and the price of the asset will be

$$p_1 = F'(a_0^B - c_1). \tag{46}$$

Combining (46) with (34) yields

$$F'(a_0^B - c_1) = \frac{(1-\pi_1^B)(\bar{R}-1)}{\tau_1^B} + \frac{(1-\pi_1^L) + \pi_1^L \underline{R}}{\tau_1^L}. \tag{47}$$

Therefore, if B is selling a positive amount of asset, the traditional sector's inverse demand function will pin down the asset price as well as the fire-sale amount. If either π_1^L is high or τ_1^L is high, then the right-hand side of (47) decreases and $a_0^B - c_1$ increases, meaning B sells more asset to T.

In the case that $F'(a_0^B) \geq \frac{(1-\pi_1^B)(\bar{R}-1)}{\tau_1^B} + \frac{(1-\pi_1^L) + \pi_1^L \underline{R}}{\tau_1^L}$, then B sells all the asset holdings to T and B does not borrow from L.

On the contrary, if

$$F'(0) < \frac{(1-\pi_1^B)(\bar{R}-1)}{\tau_1^B} + \frac{(1-\pi_1^L) + \pi_1^L \underline{R}}{\tau_1^L},$$

then $c_1 = a_1^B = a_0^B$ assuming $c_0 = a_0^B$, and asset price p_1 is indeterminant as no agent is buying or selling in a positive amount and any value above $F'(0)$ is possible. ∎

E.2 Proof of Proposition 3

Given the date-1 equilibrium, we can derive equilibrium at date 0. We assume $F'(a) = \alpha$ for all $a \geq 0$ from now on. This assumption is for tractability of the solution because otherwise the equilibrium is determined by implicit functions. The only case that disappears with this assumption is case 3 in proposition 8, which is an intermediate equilibrium between full fire sales and zero fire sales.

Proof.

E.2.1 Lender's Problem at Date 0

In section 3.3, we showed that when the lender, L, tightens funding liquidity, L's return for each unit of cash invested (either as cash or lending) is always τ_1^L regardless of the realization of (s_1^L, τ_1^L), because L always holds extra cash at date 1.

Taking date 1 equilibrium outcomes as given, L's problem at date 0 is

$$\max_{\kappa_0^L, d_0^L} \; E_0^L\left[\tau_1^L(\kappa_0^L + d_0^L)\right]$$

$$s.t. \quad \kappa_0^L + q_0 d_0^L \leq e_0^L$$

with non-negativity constraints,

$$\kappa_0^L \geq 0, \; d_0^L \geq 0.$$

Because L's budget constraint binds at the optimum, substitute $\kappa_0^L = e_0^L - q_0 d_0^L$. The first-order condition with respect to the only choice variable, d_0^L, is

$$E_0^L\left[\tau_1^L(1-q_0)\right] + \xi_{d_0}^L = 0. \tag{48}$$

By assumption 1, $d_0^L > 0$ in equilibrium. Therefore, the contract price that makes L indifferent across any amount of d_0^L is

$$q_0 = 1. \tag{49}$$

E.2.2 Borrower's Problem at Date 0

For each realization of q_1 at date 1, the borrower, B, takes the equilibrium market outcome, including B's own optimal decisions, as given. As we have seen in proposition 8, there are different regimes under different realization of q_1. B accounts for that and solves the following optimization problem:

$$
\begin{aligned}
\max_{c_0,a_0,\kappa_0^B} \quad & \underbrace{E_0^B\left[E_1\left[C_2^B \middle| q_1\right]\right]}_{expected\ utility\ for\ each\ realized\ q_1} && (50)\\
= & \underbrace{E_0^B\left[\tau_1^B\left(\kappa_0^B+q_1a_0-c_0\right)+\left(1-\pi_1^B(q_1)\right)a_0(\bar{R}-1)\middle|q_1\geq\bar{q}_1\right]\Pr{}_0^B(q_1\geq\bar{q}_1)}_{no\ liquidation\ case}\\
+ & \underbrace{E_0^B\left[\left(a_0-\frac{c_0-\kappa_0^B-q_1a_0}{\alpha-q_1}\right)\left(1-\pi_1^B(q_1)\right)(\bar{R}-1)\middle|\underline{q_1}\leq q_1<\bar{q}_1\right]\Pr{}_0^B(\underline{q_1}\leq q_1<\bar{q}_1)}_{sells\ the\ asset\ to\ pay\ debt\ but\ no\ sales\ due\ to\ pessimism}\\
+ & \underbrace{E_0^B\left[\tau_1^B\left(\kappa_0^B-c_0+\alpha a_0^B\right)\middle|q_1<\underline{q_1}\right]\Pr{}_0^B(q_1<\underline{q_1})}_{liquidates\ all\ the\ assets}
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t. } & e_0^B\geq p_0a_0^B-q_0c_0+\kappa_0^B,\\
& a_0^B\geq c_0,\\
& \pi_1^B(q_1)=\int_0^1\pi\lambda_T\left(\frac{1-(1-\underline{R})\pi}{q_1}\right)\mathbb{1}\left\{\underline{\tau}<\frac{1-(1-\underline{R})\pi}{q_1}<\bar{\tau}\right\}dG_\pi(\pi),\\
& \underbrace{\bar{q}_1=\frac{c_0-\kappa_0^B}{a_0}}_{cutoff\ for\ liquidity\ induced\ fire\ sales},\\
& \underbrace{\alpha=\frac{(1-\pi_1^B(\underline{q_1}))(\bar{R}-1)}{\tau_1^B}+\underline{q_1},}_{cutoff\ for\ belief\ induced\ fire\ sales}
\end{aligned}
$$

with non-negativity constraints,

$$
c_0\geq 0, a_0^B\geq 0, \kappa_0^B\geq 0.
$$

There are three different regimes for B depending on the realization of q_1, which corresponds to each expected utility representation in the above optimization problem:

1. If $q_1\geq\bar{q}_1$, then B will be able to roll over the debt from date 0 and keep all the assets.

2. If $\underline{q}_1\leq q_1<\bar{q}_1$, then B does not have enough cash to pay the promised debt amount but still optimistic enough to hold the assets as much as possible for the given prices. Therefore, B does liquidity-induced fire sales but sells no more than the necessary amount.

3. If $q_1<\underline{q}_1$, then B is pessimistic about the asset payoff, so T values the asset more than B.

Therefore, there will be belief-induced fire sales. B will sell all the assets to T.

We first confirm that the cutoff for belief-induced fire sales is above the level of q_1 that requires full sales of assets due to liquidity. This is because B can always sell all assets and repay the debt by assumption 2, and thus, any q_1 higher than $\underline{q}_1$ will be enough for B to repay the debt while holding a positive amount of asset.

Lemma 4. *If $q_1 \geq \underline{q}_1$, B can hold a positive amount of assets $c_1 > 0$ while repaying the debt to L in full.*

Proof. Recall that the amount of fire sales is

$$\frac{c_0 - \kappa_0^B - q_1 a_0}{\alpha - q_1},$$

which is decreasing in q_1 because

$$\frac{\partial\left(\frac{c_0 - \kappa_0^B - q_1 a_0}{\alpha - q_1}\right)}{\partial q_1} = \frac{-a_0(\alpha - q_1) + c_0 - \kappa_0^B - q_1 a_0}{(\alpha - q_1)^2} = \frac{-a_0\alpha + c_0 - \kappa_0^B}{(\alpha - q_1)^2} < 0,$$

where the last inequality comes from assumption 2. ■

Denote the Lagrangian multiplier for the collateral constraint as μ. Substituting out κ_0^B and $\overline{q}_1$ using the binding budget constraint and the cutoff equation yields

$$\begin{aligned} L_0^B = &\int_{\frac{(1-q_0)c_0 - e_0^B + p_0 a_0}{a_0}}^{1/\underline{\tau}} \left[\tau_1^B\left(e_0^B + q_0 c_0 - p_0 a_0 + q_1 a_0 - c_0\right) + \left(1 - \pi_1^B(q_1)\right) a_0(\overline{R} - 1)\right] dH(q_1) \\ &+ \int_{\underline{q}_1}^{\frac{(1-q_0)c_0 - e_0^B + p_0 a_0}{a_0}} \left(a_0 - \frac{c_0 - (e_0^B + q_0 c_0 - p_0 a_0) - q_1 a_0}{\alpha - q_1}\right)\left(1 - \pi_1^B(q_1)\right)(\overline{R} - 1) dH(q_1) \\ &+ \int_{\underline{R}/\overline{\tau}}^{\underline{q}_1} \tau_1^B\left(e_0^B + q_0 c_0 - p_0 a_0 - c_0 + \alpha a_0^B\right) dH(q_1) \\ &+ \mu(a_0 - c_0) + \xi_{c_0} c_0 + \xi_{a_0} a_0 + \xi_{\kappa_0^B}(e_0^B + q_0 c_0 - p_0 a_0), \end{aligned} \tag{51}$$

where $H(\cdot)$ is B's subjective distribution function of q_1. Then, we can apply the Leibniz integral rule for derivatives of the Lagrangian function. The first-order conditions of B's optimization problem are

FOC for c_0:

$$
\begin{aligned}
&-\frac{1-q_0}{a_0}\left[\tau_1^B\left(e_0^B+q_0c_0-p_0a_0+\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}a_0-c_0\right)\right. \\
&\left.+\left(1-\pi_1^B\left(\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}\right)\right)a_0(\overline{R}-1)\right] \\
&-\int_{\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}}^{1/\underline{\tau}}\tau_1^B(1-q_0)dH(q_1) \\
&+\frac{1-q_0}{a_0}\left(a_0-\frac{c_0-(e_0^B+q_0c_0-p_0a_0)-\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}a_0}{\alpha-\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}}\right) \\
&\times\left(1-\pi_1^B\left(\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}\right)\right)(\overline{R}-1) \\
&-\int_{\underline{q}_1}^{\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}}\frac{(1-q_0)}{\alpha-q_1}\left(1-\pi_1^B(q_1)\right)(\overline{R}-1)dH(q_1) \\
&-\int_{\underline{R}/\overline{\tau}}^{\underline{q}_1}\tau_1^B\,(1-q_0)\,dH(q_1)-\mu+\xi_{c_0}+\xi_{\kappa_0^B}q_0=0,
\end{aligned}
\tag{52}
$$

FOC for a_0:

$$
\begin{aligned}
&-\frac{p_0a_0-\left((1-q_0)c_0-e_0^B+p_0a_0\right)}{a_0^2}\left[\tau_1^B\left(e_0^B+q_0c_0-p_0a_0+\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}a_0-c_0\right)\right.\\
&\left.+\left(1-\pi_1^B\left(\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}\right)\right)a_0(\overline{R}-1)\right]\\
&+\int_{\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}}^{1/\underline{\tau}}\left[\tau_1^B\left(q_1-p_0\right)+\left(1-\pi_1^B(q_1)\right)(\overline{R}-1)\right]dH(q_1)\\
&+\frac{p_0a_0-\left((1-q_0)c_0-e_0^B+p_0a_0\right)}{a_0^2}\left(a_0-\frac{c_0-(e_0^B+q_0c_0-p_0a_0)-\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}a_0}{\alpha-\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}}\right)\\
&\times\left(1-\pi_1^B\left(\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}\right)\right)(\overline{R}-1)\\
&+\int_{\underline{q}_1}^{\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}}\left(1-\frac{p_0-q_1}{\alpha-q_1}\right)\left(1-\pi_1^B(q_1)\right)(\overline{R}-1)dH(q_1)\\
&+\int_{\underline{R}/\overline{\tau}}^{\underline{q}_1}\tau_1^B\left(\alpha-p_0\right)dH(q_1)+\mu+\xi_{a_0}-\xi_{\kappa_0^B}p_0=0.
\end{aligned}
\tag{53}
$$

E.2.3 Equilibrium Trade and Prices

FOC for c_0, (52), can be further simplified as

$$
\begin{aligned}
& -\frac{1-q_0}{a_0}\left(1-\pi_1^B\left(\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}\right)\right)a_0(\overline{R}-1) \\
& -\int_{\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}}^{1/\underline{\tau}} \tau_1^B(1-q_0)dH(q_1)-\int_{\underline{R}/\overline{\tau}}^{\underline{q}_1} \tau_1^B\,(1-q_0)\,dH(q_1) \\
& +\frac{1-q_0}{a_0}\left(1-\pi_1^B\left(\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}\right)\right)a_0(\overline{R}-1) \\
& -\int_{\underline{q}_1}^{\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}} \frac{(1-q_0)}{\alpha-q_1}\left(1-\pi_1^B(q_1)\right)(\overline{R}-1)dH(q_1)-\mu+\xi_{c_0}+\xi_{\kappa_0^B}q_0 \\
= & -\int_{\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}}^{1/\underline{\tau}} \tau_1^B(1-q_0)dH(q_1)-\int_{0}^{\underline{q}_1} \tau_1^B\,(1-q_0)\,dH(q_1) \\
& -\int_{\underline{q}_1}^{\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}} \frac{(1-q_0)}{\alpha-q_1}\left(1-\pi_1^B(q_1)\right)(\overline{R}-1)dH(q_1)-\mu+\xi_{c_0}+\xi_{\kappa_0^B}q_0 \\
& =-\mu+\xi_{c_0}+\xi_{\kappa_0^B}=0,
\end{aligned}
$$

and the second to last inequality holds because of $q_0=1$, which is from (49).

If B does not borrow at all $c_0=0$, then the collateral constraint should be binding, implying $a_0=c_0=0$. If B is borrowing a positive amount as $c_0>0$, then $\xi_{c_0}=0$ and there can be two different cases. Consider the first case in which $\xi_{\kappa_0^B}$ positive, implying $\kappa_0^B=0$ and $\mu=\xi_{\kappa_0^B}q_0>0$. Thus, the collateral constraint is also binding and $c_0=a_0$. In the second case in which $\xi_{\kappa_0^B}$ is zero, μ should also be zero, implying that the collateral constraint is not binding and B purchases some assets without leverage as $a_0>c_0$.

The result holds because of the linearity of B's utility. If B holds a positive amount of cash, B should be indifferent between holding more or less cash. However, if the return of purchasing more assets by borrowing more from L at date 0 at the cost of less amount of cash at date 1 exceeds the cash return, B borrows with full capacity and spends all the cash to purchase the asset.

FOC for a_0, (53), can be further simplified as

$$
\begin{aligned}
&-\frac{p_0a_0-\left((1-q_0)c_0-e_0^B+p_0a_0\right)}{a_0^2}\left[\left(1-\pi_1^B\left(\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}\right)\right)a_0(\overline{R}-1)\right]\\
&+\int_{\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}}^{1/\underline{\tau}}\left[\tau_1^B(q_1-p_0)+\left(1-\pi_1^B(q_1)\right)(\overline{R}-1)\right]dH(q_1)\\
&+\frac{p_0a_0-\left((1-q_0)c_0-e_0^B+p_0a_0\right)}{a_0^2}\left(1-\pi_1^B\left(\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}\right)\right)a_0(\overline{R}-1)\\
&+\int_{\underline{q}_1}^{\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}}\left(\frac{\alpha-p_0}{\alpha-q_1}\right)\left(1-\pi_1^B(q_1)\right)(\overline{R}-1)dH(q_1)\\
&+\int_{\underline{R}/\overline{\tau}}^{\underline{q}_1}\tau_1^B(\alpha-p_0)\,dH(q_1)+\mu+\xi_{a_0}-\xi_{\kappa_0^B}p_0\\
=&\int_{\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}}^{1/\underline{\tau}}\left[\tau_1^B(q_1-p_0)+\left(1-\pi_1^B(q_1)\right)(\overline{R}-1)\right]dH(q_1)\\
&+\int_{\underline{q}_1}^{\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}}\left(\frac{p_0-\alpha}{q_1-\alpha}\right)\left(1-\pi_1^B(q_1)\right)(\overline{R}-1)dH(q_1)\\
&+\int_{\underline{R}/\overline{\tau}}^{\underline{q}_1}\tau_1^B(\alpha-p_0)\,dH(q_1)+\mu+\xi_{a_0}-\xi_{\kappa_0^B}p_0=0.
\end{aligned}
$$

We finally derive the date-0 equilibrium allocation and prices.

From the FOCs for c_0 and a_0,

$$
\begin{aligned}
&\int_{\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}}^{1/\underline{\tau}}\left[\tau_1^B(q_1-p_0)+\left(1-\pi_1^B(q_1)\right)(\overline{R}-1)\right]dH(q_1)\\
&+\int_{\underline{q}_1}^{\frac{(1-q_0)c_0-e_0^B+p_0a_0}{a_0}}\left(\frac{p_0-\alpha}{q_1-\alpha}\right)\left(1-\pi_1^B(q_1)\right)(\overline{R}-1)dH(q_1)\\
&+\int_{\underline{R}/\overline{\tau}}^{\underline{q}_1}\tau_1^B(\alpha-p_0)\,dH(q_1)+\xi_{c_0}+\xi_{a_0}-\xi_{\kappa_0^B}(p_0-1)=0.
\end{aligned}
$$

From the borrower's optimality condition under $a_0=c_0>0$, B holds zero amount of cash, $\kappa_0^B=0$.

Under this case, B's return from this leveraged asset purchase becomes

$$\begin{aligned}\xi_{\kappa_0^B} = \Bigg[& \int_1^{1/\underline{\tau}} \left[\tau_1^B (q_1 - p_0) + \left(1 - \pi_1^B(q_1)\right)(\overline{R} - 1)\right] dH(q_1) \\ & + \int_{\underline{q}_1}^1 \left(\frac{p_0 - \alpha}{q_1 - \alpha}\right)\left(1 - \pi_1^B(q_1)\right)(\overline{R} - 1) dH(q_1) \\ & + \int_{\underline{R}/\overline{\tau}}^{\underline{q}_1} \tau_1^B (\alpha - p_0)\, dH(q_1)\Bigg] / (p_0 - 1).\end{aligned} \tag{54}$$

It is sufficient to show that $\xi_{\kappa_0^B} > \tau_1^B$, so B prefers to purchase the asset with leverage to holding cash.

If B uses up all the cash endowments to purchase the asset with leverage, the asset price is

$$p_0 = \frac{A + e_0^B}{A}.$$

The price should be higher than T's willingness-to-pay for the asset, which is also above the debt payment amount at date 1, so B is able to repay the debt in full if B wants to liquidate the assets. Therefore,

$$p_0 = \frac{A + e_0^B}{A} > \alpha \geq 1. \tag{55}$$

From assumption 1,

$$\begin{aligned} & e_0^B \tau_1^B < A(E_0^B[R] - 1) \\ & \Rightarrow \tau_1^B < \frac{A}{e_0^B}(E_0^B[R] - 1) = \frac{E_0^B[R] - 1}{p_0 - 1}, \end{aligned} \tag{56}$$

where the last equality comes from (55). Hence, if the marginal return of purchasing the asset with leverage, $\xi_{\kappa_0^B}$, is greater than or equal to $\dfrac{E_0^B[R]-1}{p_0 - 1}$, then $a_0 = c_0 = A$ and $\kappa_0^B = 0$ is the equilibrium portfolio of B. The difference between the two returns is

$$\begin{aligned}\xi_{\kappa_0^B} - (E_0^B[R] - 1) = & \int_1^{1/\underline{\tau}} \left[\pi_1^B(q_1)(1 - \underline{R}) + \tau_1^B (q_1 - p_0)\right] dH(q_1) \\ & + \int_{\underline{q}_1}^1 \left[\pi_1^B(q_1)(1 - \underline{R}) - \left(\frac{p_0 - q_1}{\alpha - q_1}\right)\left(1 - \pi_1^B(q_1)\right)(\overline{R} - 1)\right] dH(q_1) \\ & + \int_{\underline{R}/\overline{\tau}}^{\underline{q}_1} \left[\pi_1^B(q_1)(1 - \underline{R}) + \tau_1^B(\alpha - p_0) - (1 - \pi_1^B(q_1))(\overline{R} - 1)\right] dH(q_1).\end{aligned} \tag{57}$$

We will show that this difference is positive. Because $\dfrac{(1-\pi_1^B(q))(\overline{R}-1)}{\tau_1^B}+q$ is increasing in q and $\alpha = \dfrac{(1-\pi_1^B(\underline{q}_1))(\overline{R}-1)}{\tau_1^B}+\underline{q}_1 \geq 1$,

$$\alpha - q < \frac{(1-\pi_1^B(q))(\overline{R}-1)}{\tau_1^B}$$

for any $q \in [\underline{q}_1, 1]$. Thus, we have

$$\begin{aligned}
&\int_{\underline{q}_1}^{1}\left[\pi_1^B(q_1)(1-\underline{R}) - \left(\frac{p_0-q_1}{\alpha-q_1}\right)\left(1-\pi_1^B(q_1)\right)\left(\overline{R}-1\right)\right]dH(q_1)\\
>&\int_{\underline{q}_1}^{1}\left[\pi_1^B(q_1)(1-\underline{R}) - \left(\tau_1^B\frac{p_0-q_1}{\left(1-\pi_1^B(q_1)\right)\left(\overline{R}-1\right)}\right)\left(1-\pi_1^B(q_1)\right)\left(\overline{R}-1\right)\right]dH(q_1)\\
=&\int_{\underline{q}_1}^{1}\left[\pi_1^B(q_1)(1-\underline{R}) + \tau_1^B\left(q_1-p_0\right)\right]dH(q_1). \qquad (58)
\end{aligned}$$

Again, because $\alpha = \dfrac{(1-\pi_1^B(\underline{q}_1))(\overline{R}-1)}{\tau_1^B}+\underline{q}_1$ and $(1-\pi_1^B(q))$ is increasing in q,

$$\begin{aligned}
&\int_{\underline{R}/\overline{\tau}}^{\underline{q}_1}\left[\pi_1^B(q_1)(1-\underline{R}) + \tau_1^B(\alpha-p_0) - (1-\pi_1^B(q_1))(\overline{R}-1)\right]dH(q_1)\\
>&\int_{\underline{R}/\overline{\tau}}^{\underline{q}_1}\left[\pi_1^B(q_1)(1-\underline{R}) + \tau_1^B\left(\frac{(1-\pi_1^B(\underline{q}_1))(\overline{R}-1)}{\tau_1^B}+\underline{q}_1-p_0\right) - (1-\pi_1^B(\underline{q}_1))(\overline{R}-1)\right]dH(q_1)\\
>&\int_{\underline{R}/\overline{\tau}}^{\underline{q}_1}\left[\pi_1^B(q_1)(1-\underline{R}) + \tau_1^B\left(q_1-p_0\right)\right]dH(q_1). \qquad (59)
\end{aligned}$$

Combining (57), (58), and (59) implies

$$\begin{aligned}
\xi_{\kappa_0^B} - (E_0^B[R]-1) >& \int_{1}^{1/\underline{\tau}}\left[\pi_1^B(q_1)(1-\underline{R}) + \tau_1^B\left(q_1-p_0\right)\right]dH(q_1)\\
&+\int_{\underline{q}_1}^{1}\left[\pi_1^B(q_1)(1-\underline{R}) + \tau_1^B\left(q_1-p_0\right)\right]dH(q_1)\\
&+\int_{\underline{R}/\overline{\tau}}^{\underline{q}_1}\left[\pi_1^B(q_1)(1-\underline{R}) + \tau_1^B\left(q_1-p_0\right)\right]dH(q_1)\\
=&\int\left[\pi_1^B(q_1)(1-\underline{R}) + \tau_1^B q_1 - \tau_1^B p_0\right]dH(q_1)\\
=&\pi_0(1-\underline{R}) + \tau_1^B E_0^B[q_1] - \tau_1^B p_0. \qquad (60)
\end{aligned}$$

Recall that $q_1 = \dfrac{(1-\pi_1^L)+\pi_1^L\underline{R}}{\tau_1^L}$ from (1), which implies

$$E_0^B[q_1] = E_0^B\left[\frac{(1-\pi_1^L)+\pi_1^L\underline{R}}{\tau_1^L}\right] = E_0^B\left[(1-\pi_1^L)+\pi_1^L\underline{R}\right]E_0^B\left[\frac{1}{\tau_1^L}\right] = \left[(1-\pi_0)+\pi_1^0\underline{R}\right]E_0^B\left[\frac{1}{\tau_1^L}\right],$$

where the last two equalities hold because of the independence between τ_1^L and s_1^L and the law of iterated expectations, respectively. Because $f(x) = 1/x$ is a convex function,

$$E_0^B\left[\frac{1}{\tau_1^L}\right] > \frac{1}{E_0^B\left[\tau_1^L\right]} = \frac{1}{\tau_0^L}$$

by Jensen's inequality. Finally, from (56), $E_0^B[R] > \tau_1^B p_0$.

Therefore, (60) becomes

$$\begin{aligned}\xi_{\kappa_0^B} - (E_0^B[R]-1) &> \pi_0(1-\underline{R}) + \tau_1^B E_0^B[q_1] - \tau_1^B p_0 \\ &> \pi_0(1-\underline{R}) + (1-\pi_0) + \pi_0\underline{R} - (1-\pi_0)\overline{R} - \pi_0\underline{R} = 0.\end{aligned}$$

Therefore, the return from investing in the asset with leverage $\xi_{\kappa_0^B}$ is greater than the return from cash holdings τ_1^B. Finally, this also shows that the equilibrium with $\kappa_0^B = 0$ is the only equilibrium. If $\kappa_0^B > 0$, then the price of the asset p_0 will be even lower, increasing the asset return even higher. ■

E.3 Counterfactual on Learning

To understand how learning interacts with funding and market liquidity, we first compare the response of our baseline economy to shocks to the response of an alternative benchmark economy in which beliefs are stale—that is, we assume agents' date-0 beliefs are never updated in response to new information. We measure the severity of a fire sale by the extent to which the borrower fire sells its holdings of the risky asset to the traditional sector: the greater (and, equivalently, the lower the asset price p_1), the more severe the fire sale at date 1. This result is summarized in lemma 5 below.

Lemma 5 (Learning amplifies the severity of fire sales)**.** *For any set of shocks (π_1^L, τ_1^L) such that the equilibrium is in the Fire Sale Regime at date 1, the price of the risky asset p_1 is lower and the extent of misallocation of the asset a_1^T is higher in equilibrium compared to an alternative benchmark economy in which agents' posterior beliefs are exogenously set to their date-0 priors.*

Proof. We show how learning amplifies misallocation during a fire sale for any q_1 (i.e. shock pair (π_1^L, τ_1^L)), relative to a benchmark in which beliefs don't change—that is, agents have private information but do not update their beliefs in response to new information. Note that in the Fire Sale Regime, we have

$$a_1^B = a_0^B - F'^{-1}\left(\frac{E_1^B[R] - E_1^B\left[R_2^d\right]}{\tau_1^B} + \frac{E_1^L\left[R_2^d\right]}{\tau_1^L}\right) \tag{61}$$

$$p_1 = F'(a_1^T)$$

Since $F'(\cdot)$ is monotonically decreasing, $F'^{-1}(\cdot)$ is also monotonically decreasing. Therefore, holding the borrower's beliefs constant for a moment, a lower $q_1 = \frac{E_1^L[R_2^d]}{\tau_1^L}$ implies that a_1^B is lower. Note that the spread $E_1^B[R] - E_1^B\left[R_2^d\right] = \left(1-\pi_1^B\right)\left(\overline{R}-1\right)$ is decreasing in π_1^B (since $\overline{R} > 1$ by assumption). A lower q_1 implies that π_1^B is higher (B is more pessimistic) by proposition 1, which also makes a_1^B lower. Thus, a lower q_1 leads to a lower a_1^B in two ways: the direct effect of a lower q_1 on a_1^B and the effect of lower q_1 on a_1^B through greater pessimism and a lower spread.

Since a larger decrease in funding liquidity leads to lower a_1^B, this also implies that a_1^T is higher and hence p_1 is lower. Hence, the fire sale is more severe in the Fire Sale Regime when the funding illiquidity is more severe. Hence, pessimism/the information externality is associated with greater misallocation/worse fire sales during a fire sale. ■

We showed in section 3.3 that pessimism and optimism on the fundamental value of the asset arises endogenously in our model as a result of funding liquidity. Lemma 5 above shows that this increased pessimism contributes to market illiquidity, by depressing asset prices and making it more costly to raise funds through liquidation. Therefore, the model yields an adverse feedback between pessimistic beliefs about fundamentals, funding liquidity, and market illiquidity—a dynamic, which exacerbates crises.

E.4 Date-1 Equilibrium under Common Information Benchmark

In the Normal Regime under the Common Information Benchmark we have

$$a_1^T, \xi_{d_1}^L, \xi_{c_1}^B, \xi_{a_1}^B, \xi_{\kappa_1}^L, \xi_{\kappa_1}^B = 0$$

$$q_1 = \frac{E_1\left[R_2^d(f_1)\right]}{\tau_1^L}$$

$$p_1 = \frac{E_1[R] - E_1\left[R_2^d(f_1)\right]}{\tau_1^B} + q_1$$

$$a_1^B = c_1^B = d_1^L = a_0^B$$

$$\kappa_1^B = q_1 a_0^B - c_0^B + \kappa_0^B$$

$$\kappa_1^L = \kappa_0^L + d_0^L - q_1 a_0^B$$

$$\kappa_1^T = \kappa_0^T - p_1 a_1^T$$

$$C_2^B = \tau_1^B \kappa_1^B + a_1^B R - a_1^B R_2^d(f_1)$$

$$C_2^L = \tau_1^L \kappa_1^L + R_2^d d_1^L$$

$$C_2^T = \kappa_1^T + F(a_1^T)$$

$$\mu_1^B = \left(\tau_1^B + \xi_{\kappa_1}^B\right) p_1 - E_1\left[R\right] - \xi_{a_1}^B$$

$$\mu_1^T = F'(0) - p_1$$

$$\pi_1 := \pi_1^L = Pr\left(R = \underline{R} | s_1^L, I_0\right) = \frac{\pi_0 \lambda_\varepsilon\left(\varepsilon_1^L = s_1^L - \underline{R}\right)}{(1-\pi_0)\lambda_\varepsilon\left(\varepsilon_1^L = s_1^L - \overline{R}\right) + \pi_0 \lambda_\varepsilon\left(\varepsilon_1^L = s_1^L - \underline{R}\right)}$$

$$\pi_1^B = \pi_1$$

In the Fire Sale Regime under the Common Information Benchmark, we have

$$\mu_1^T, \xi_{c_1}^B, \xi_{a_1}^B, \xi_{d_1}^L, \xi_{\kappa_1}^L, \xi_{\kappa_1}^B = 0$$

$$q_1 = \frac{E_1\left[R_2^d(f_1)\right]}{\tau_1^L}$$

$$p_1 = F'(a_1^T)$$

$$a_1^B = a_0^B - F'^{-1}\left(\frac{E_1[R] - E_1\left[R_2^d(f_1)\right]}{\tau_1^B} + \frac{E_1\left[R_2^d(f_1)\right]}{\tau_1^L}\right)$$

$$a_1^T = a_0^B - a_1^B$$

$$c_1^B = d_1^L = a_1^B$$

$$\kappa_1^B = q_1 a_1^B - c_0^B + \kappa_0^B - p_1\left(a_1^B - a_0^B\right)$$

$$\kappa_1^L = d_0^L - q_1 d_1^L + \kappa_0^L$$

$$\kappa_1^T = \kappa_0^T - p_1 a_1^T$$

$$C_2^B = \tau_1^B \kappa_1^B + a_1^B R - a_1^B R_2^d(f_1)$$

$$C_2^L = \tau_1^L \kappa_1^L + R_2^d d_1^L$$

$$C_2^T = \kappa_1^T + F(a_1^T)$$

$$\mu_1^B = \tau_1^B p_1 - E_1[R]$$

$$\pi_1 := \pi_1^L = Pr\left(R = \underline{R} | s_1^L, I_0\right) = \frac{\pi_0 \lambda_\varepsilon\left(\varepsilon_1^L = s_1^L - \underline{R}\right)}{(1-\pi_0)\lambda_\varepsilon\left(\varepsilon_1^L = s_1^L - \overline{R}\right) + \pi_0 \lambda_\varepsilon\left(\varepsilon_1^L = s_1^L - \underline{R}\right)}$$

$$\pi_1^B = \pi_1$$

E.5 Defining the Partition in the Common Information Benchmark

The following lemma shows how this frontier compares to that under the Baseline Economy.

Lemma 6 (Frontier in the Common Information Benchmark)**.** *The frontier $\left(\tau_1^L, \pi_1^L\right)|\tilde{p}_1$ in the Common Information Benchmark is strictly convex and has a negative slope in the domain for τ_1^L. Moreover, the y-intercept $\frac{1}{1-\underline{R}}$ of the frontier $\left(\tau_1^L, \pi_1^L\right)|\tilde{p}_1$ is the same as the y-intercept for* *the frontier* $\left(\tau_1^L, \pi_1^L\right)|\hat{p}_1$ *in the Baseline Economy, and the x-intercept* $\frac{\tau_1^B}{\tau_1^B F'(0)-\overline{R}+1}$ *in the Common Information Benchmark exceeds that in the Baseline Economy* $\frac{1}{\hat{q}_1}$.

Proof. See appendix E.6. ■

For the given $\hat{q}_1$ of the frontier in the Baseline Case, there exists a point $\hat{\tau}_1^L$ such that the corresponding $\pi_1^L = 1$ as

$$\pi_1^L = 1 = \frac{1-\hat{\tau}_1^L \hat{q}_1}{1-\underline{R}}.$$

Similarly, there exists a point $\tilde{\tau}_1^L$ such that the corresponding $\pi_1^L = 1$ as

$$\pi_1^L = 1 = \frac{\tau_1^B f_1 + \tilde{\tau}_1^L \left(\overline{R} - f_1 - \tau_1^B F'(0)\right)}{\tilde{\tau}_1^L \left(\overline{R} - f_1\right) + \tau_1^B \left(f_1 - \underline{R}\right)}.$$

Some of the results, which follow, depend on whether the frontier partitioning the state space into the normal and fire sale regimes in the baseline economy intersects the frontier in the common information benchmark. We define the following condition related to the y-intercepts:

Condition 1.

$$\hat{\tau}_1^L > \tilde{\tau}_1^L \tag{62}$$

If condition 1, which is $\hat{\tau}_1^L > \tilde{\tau}_1^L$ holds, then the frontier in the baseline economy is above the frontier under the common information benchmark up to some point $(\tau_1^{L*}, \pi_1^{L*})$ such that $\pi_1^{L*} < 1$, where the two frontiers meet. The following lemma shows that condition 1 is necessary and sufficient for the frontiers in the baseline and benchmark to meet in the relevant domain, and that this condition always holds in equilibrium.

Lemma 7.

A. *Condition 1 is a necessary and sufficient condition for the frontiers in the baseline case and the common information benchmark to meet in the domain such that* $\pi_1^L \leq 1$.

B. *Condition 1 always hold in equilibrium.*

Proof. See appendix E.7. ■

Condition 1 implies that the frontier in the baseline case starts declining linearly at a point $(\hat{\tau}_1^L, 1)$, whereas the frontier in the common information benchmark goes through the point $(\tilde{\tau}_1^L, 1)$. Because both frontiers started at the same y-intercept, the average slope of the frontier in the common information benchmark is above that in the baseline case. Because the slope of the frontier in the common information benchmark is increasing (by strict convexity shown in lemma 6), the frontier in the baseline economy is always below the frontier in the common information benchmark. The proof shows that this condition is always satisfied in equilibrium.

E.6 Proof of Lemma 6

Here we characterize the threshold between the Normal and Crisis Regimes in the Common Information Benchmark.

Proof.

Frontier in Baseline Economy

Recall the equilibrium asset price.

$$p_1 = \frac{E_1^B[R] - E_1^B\left[R_2^d(f_1)\right]}{\tau_1^B} + q_1.$$

The threshold price is defined by asset price at threshold of Normal and Fire Sale Regimes. This threshold asset price is $\hat{p}_1 = F'(0)$ satisfying

$$\begin{aligned}
\frac{E_1^B[R] - E_1^B\left[R_2^d(f_1)\right]}{\tau_1^B} + \hat{q}_1 &= F'(0) \\
\hat{q}_1 + \frac{\left(1-\pi_1^B(\hat{q}_1)\right)\left(\overline{R} - f_1\right)}{\tau_1^B} &= F'(0) \\
\left(1-\pi_1^B(\hat{q}_1)\right)\left(\overline{R} - f_1\right) + \tau_1^B \hat{q}_1 &= \tau_1^B F'(0). \qquad (63)
\end{aligned}$$

This defines the threshold value $\hat{q}_1$. Given this, the set of $\left(\tau_1^L, \pi_1^L\right)$ consistent with $\hat{q}_1$ is given by

$$\frac{\left(1-\pi_1^L\right) f_1 + \pi_1^L \underline{R}}{\tau_1^L} = \hat{q}_1.$$

Solve for π_1^L :

$$\pi_1^L = \frac{f_1 - \tau_1^L \hat{q}_1}{f_1 - \underline{R}} = \frac{1 - \tau_1^L \hat{q}_1}{1 - \underline{R}}.$$

This defines the curve of the frontier partitioning the state space into the Normal and Crisis Regimes.

The y-intercept of the curve (when $\tau_1^L = 0$, though never occurs) is $\frac{1}{1-\underline{R}} > 1$, while the x-intercept (when $\pi_1^L = 0$) is $\frac{1}{\hat{q}_1} > 0$.

The slope of the curve is negative and constant:

$$\frac{d\pi_1^L}{d\tau_1^L} = \frac{-\hat{q}_1}{1-\underline{R}} < 0.$$

Frontier in Common Information Benchmark

When $\pi_1^B = \pi_1^L$, the threshold in benchmark is defined by

$$\left(1-\pi_1^L\right)\left(\overline{R}-f_1\right)+\tau_1^B\hat{q}_1=\tau_1^B F'(0)$$

i.e.

$$\left(\overline{R}-f_1\right)-\left[\left(\overline{R}-f_1\right)+\frac{\tau_1^B}{\tau_1^L}\left(f_1-\underline{R}\right)\right]\pi_1^L+\frac{\tau_1^B}{\tau_1^L}f_1=\tau_1^B F'(0) \tag{64}$$

We now trace out the frontier of all $\left(\tau_1^L,\pi_1^L\right)$ such that this is satisfied. First, we derive the x-intercept of the frontier by supposing that $\pi_1^L=0$:

$$\begin{aligned}
\left(\overline{R}-f_1\right)+\frac{\tau_1^B}{\tau_1^L}f_1 &= \tau_1^B F'(0)\\
\left(\overline{R}-f_1\right)\tau_1^L+\tau_1^B f_1 &= \tau_1^L\tau_1^B F'(0)\\
\tau_1^B f_1 &= \tau_1^L\left(\tau_1^B F'(0)-\overline{R}+f_1\right)\\
\tau_1^L &= \frac{\tau_1^B f_1}{\tau_1^B F'(0)-\overline{R}+f_1}.
\end{aligned}$$

Claim 1. *This x-intercept is greater than the x-intercept in the Baseline case.*

Proof. We want to show that the x-intercept of the frontier in the Common Info benchmark is larger than that in the Baseline economy, i.e.

$$\frac{\tau_1^B f_1}{\tau_1^B F'(0)-\overline{R}+f_1}>\frac{f_1}{\hat{q}_1}$$

$$\hat{q}_1>\frac{\tau_1^B F'(0)-\left(\overline{R}-f_1\right)}{\tau_1^B}$$

where $\hat{q}_1$ is defined by (63) in the Baseline case. Recall that $\hat{q}_1$ is defined by

$$\hat{q}_1=F'(0)-\frac{\left(1-\pi_1^B(\hat{q}_1)\right)\left(\overline{R}-f_1\right)}{\tau_1^B}$$

So we want to show that

$$F'(0)-\frac{\left(1-\pi_1^B(\hat{q}_1)\right)\left(\overline{R}-f_1\right)}{\tau_1^B}>\frac{\tau_1^B F'(0)-\left(\overline{R}-f_1\right)}{\tau_1^B}.$$

The inequality is equivalent to

$$
\begin{aligned}
F'(0)-\frac{\left(1-\pi_1^B(\hat{q}_1)\right)\left(\overline{R}-f_1\right)}{\tau_1^B} >& F'(0)-\frac{\left(\overline{R}-f_1\right)}{\tau_1^B}\\
\left(1-\pi_1^B(\hat{q}_1)\right)\left(\overline{R}-f_1\right) <& \left(\overline{R}-f_1\right)\\
\left(1-\pi_1^B(\hat{q}_1)\right) <& 1,
\end{aligned}
$$

which holds because $\pi_1^B>0$. ■

Hence, we have that the x-intercept is larger in the Common Info Benchmark compared to the Baseline case. (Note that this automatically implies that the x-intercept is positive, since it's obviously positive in the Baseline case.)

What is the y-intercept of the frontier in the Common Info Benchmark? (i.e. what happens to π_1^L as τ_1^L approaches zero?): The frontier equation (64) can be rearranged as

$$
\begin{aligned}
\left(\overline{R}-f_1\right)-\left[\left(\overline{R}-f_1\right)+\frac{\tau_1^B}{\tau_1^L}\left(f_1-\underline{R}\right)\right]\pi_1^L+\frac{\tau_1^B}{\tau_1^L}f_1 &= \tau_1^B F'(0)\\
-\left[\left(\overline{R}-f_1\right)+\frac{\tau_1^B}{\tau_1^L}\left(f_1-\underline{R}\right)\right]\pi_1^L &= \tau_1^B F'(0)-\frac{\tau_1^B}{\tau_1^L}f_1-\left(\overline{R}-f_1\right)\\
\pi_1^L &= \frac{\frac{\tau_1^B}{\tau_1^L}f_1+\left(\overline{R}-f_1\right)-\tau_1^B F'(0)}{\left(\overline{R}-f_1\right)+\frac{\tau_1^B}{\tau_1^L}\left(f_1-\underline{R}\right)}\\
\pi_1^L &= \frac{\tau_1^B f_1+\tau_1^L\left(\overline{R}-f_1\right)-\tau_1^L\tau_1^B F'(0)}{\tau_1^L\left(\overline{R}-f_1\right)+\tau_1^B\left(f_1-\underline{R}\right)}\\
\pi_1^L &= \frac{\tau_1^B f_1+\tau_1^L\left(\overline{R}-f_1-\tau_1^B F'(0)\right)}{\tau_1^L\left(\overline{R}-f_1\right)+\tau_1^B\left(f_1-\underline{R}\right)}.
\end{aligned}
$$

The y-intercept (when $\tau_1^L=0$) is $\frac{f_1}{f_1-\underline{R}}$. Hence, this is the same y-intercept as in the Baseline case.

The slope of this frontier is defined by taking the derivative of π_1^L with respect to τ_1^L:

$$
\begin{aligned}
\frac{\partial\pi_1^L}{\partial\tau_1^L} =& \frac{\partial\left(\frac{\tau_1^B f_1+\tau_1^L\left(\overline{R}-f_1-\tau_1^B F'(0)\right)}{\tau_1^L\left(\overline{R}-f_1\right)+\tau_1^B\left(f_1-\underline{R}\right)}\right)}{\partial\tau_1^L}\\
=& \frac{\left(\overline{R}-f_1-\tau_1^B F'(0)\right)\left(\tau_1^L\left(\overline{R}-f_1\right)+\tau_1^B\left(f_1-\underline{R}\right)\right)-\left(\overline{R}-f_1\right)\left(\tau_1^B f_1+\tau_1^L\left(\overline{R}-f_1-\tau_1^B F'(0)\right)\right)}{\left(\tau_1^L\left(\overline{R}-f_1\right)+\tau_1^B\left(f_1-\underline{R}\right)\right)^2}\\
=& \frac{\left(\overline{R}-f_1-\tau_1^B F'(0)\right)\left(\tau_1^B\left(f_1-\underline{R}\right)\right)-\left(\overline{R}-f_1\right)\left(\tau_1^B f_1\right)}{\left(\tau_1^L\left(\overline{R}-f_1\right)+\tau_1^B\left(f_1-\underline{R}\right)\right)^2}<0,
\end{aligned}
$$

which holds because $\left(\overline{R}-f_1-\tau_1^B F'(0)\right) < \left(\overline{R}-f_1\right)$ and $\tau_1^B f_1 > \tau_1^B(f_1-\underline{R})$. Denote $\Psi \equiv \left(\overline{R}-f_1-\tau_1^B F'(0)\right)\left(\tau_1^B\left(f_1-\underline{R}\right)\right)-\left(\overline{R}-f_1\right)\left(\tau_1^B f_1\right)$. Also, the second derivative becomes

$$\frac{\partial^2 \pi_1^L}{\partial\left(\tau_1^L\right)^2}=\frac{-2\left(\overline{R}-f_1\right)\left(\tau_1^L\left(\overline{R}-f_1\right)+\tau_1^B\left(f_1-\underline{R}\right)\right)\Psi}{\left(\tau_1^L\left(\overline{R}-f_1\right)+\tau_1^B\left(f_1-\underline{R}\right)\right)^4}>0,$$

where the last inequality holds by $\overline{R} > f_1 > \underline{R}$ and $\Psi < 0$. Therefore, the frontier is strictly convex with a negative slope (in our relevant domain). ■

E.7 Proof of Condition for Intersection of the Frontiers

Using the results from the proof of lemma 6 in appendix E.6, we derive the necessary and sufficient condition for the existence of the intersection of the frontiers in the Baseline Economy and the Common Information Benchmark.

Proof.

Part A. In the Baseline Economy, the slope is always $\frac{d\pi_1^L}{d\tau_1^L}=\frac{-\hat{q}_1}{f_1-\underline{R}}<0$. In the Common Info Benchmark, the slope is

$$\frac{\left(\overline{R}-f_1-\tau_1^B F'(0)\right)\left(\tau_1^B\left(f_1-\underline{R}\right)\right)-\left(\overline{R}-f_1\right)\left(\tau_1^B f_1\right)}{\left(\tau_1^L\left(\overline{R}-f_1\right)+\tau_1^B\left(f_1-\underline{R}\right)\right)^2}$$

which is increasing as shown in lemma 6. Therefore, once the frontier in the Common Information Benchmark is at the same point $(\tau_1^{L^*}, \pi_1^{L^*})$ or above that point as $(\tau_1^{L^*}, \pi_1^L)$ with $\pi_1^L \geq \pi_1^{L^*}$ and has the same slope as the frontier in the Baseline Case at $\tau_1^{L^*}$, the two frontiers would never meet for any $\tau_1^L > \tau_1^{L^*}$ (see figure 6 for a graphical example).

First, suppose that $\hat{\tau}_1^L \leq \tilde{\tau}_1^L$. We claim that the two frontiers will never meet for $\pi_1^L \leq 1$. Since the two frontiers have the same y-intercept, $\dfrac{1}{1-\underline{R}}$, the average slope of the frontier in the Common Information Benchmark within the interval $[0, \hat{\tau}_1^L]$ has to be greater than that of the frontier in the Baseline Case within the same interval. Because of strict convexity, the slope of the frontier in the Common Information Benchmark at $\hat{\tau}_1^L$ has to be greater than that of the frontier in the Baseline Case as well. Thus, the two frontiers will never meet for the states with $\pi_1^L \leq 1$.

Now suppose that the two frontiers never meet for the states with $\pi_1^L \leq 1$. We claim that $\hat{\tau}_1^L \leq \tilde{\tau}_1^L$ should hold and prove this by contradiction. Suppose the contrary, $\hat{\tau}_1^L > \tilde{\tau}_1^L$. Because the x-intercept in the Common Information Benchmark is greater than the x-intercept in the Baseline case, the frontier in the Baseline case continuously goes from $\pi_1^L = 1$ to $\pi_1^L = 0$ within an interval that is contained in the interval between $\tilde{\tau}_1^L$ and the x-intercept in the Common Information Benchmark. Since the frontier is continuously decreasing and convex, there exists a point that the two frontiers meet by the intermediate value theorem.

Part B. Recall from appendix E.5 that the intercept $\hat{\tau}_1^L$ is defined by $\pi_1^L = 1 = \frac{1-\hat{\tau}_1^L \hat{q}_1}{1-\underline{R}}$, which we can rewrite as $\hat{\tau}_1^L = \frac{\underline{R}}{\hat{q}_1}$. Also, the intercept $\tilde{\tau}_1^L$ is defined by $\pi_1^L = 1 = \frac{\tau_1^B f_1 + \tilde{\tau}_1^L \left(\overline{R} - f_1 - \tau_1^B F'(0)\right)}{\tilde{\tau}_1^L \left(\overline{R} - f_1\right) + \tau_1^B (f_1 - \underline{R})}$, which we can rewrite as $\tilde{\tau}_1^L = \frac{\underline{R}}{F'(0)}$. Therefore, condition 1, $\hat{\tau}_1^L > \tilde{\tau}_1^L$, holds if and only if $F'(0) > \hat{q}_1$. We showed in section 5 that $F'(0) = \hat{p}_1$, where $\hat{p}_1$ is the threshold asset price below which the fire sale regime obtains at date 1. Therefore, condition 1 is identical to $\hat{p}_1 > \hat{q}_1$. Recall also from lemma 2 that, for each realization of shocks $\left(\tau_1^L, s_1^L\right)$, we have $q_1 < p_1$ in equilibrium. This implies that $\hat{p}_1 > \hat{q}_1$, and hence, condition 1 always holds in equilibrium. ■

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